MODEL DRIVEN DESIGN AND CHARACTERIZATION OF MICROWAVE SYSTEMS
Thesis submitted in fulfillment of the requirements for the degree of Doctor in Engineering (Doctor in de Ingenieurswetenschappen) by

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Model Driven Design and Characterization of Microwave Systems

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List of Abbreviations

ACPR  Adjacent Channel Power Ratio
ADS   Advanced Design System
BLA   Best Linear Approximation
CP    Circuit Parameters
DGS   Defected Ground Structure
DP    Design Parameters
DUT   Device Under Test
EM    electromagnetic
EVM   Error Vector Magnitude
FRF   Frequency Response Function
LBTLS Local Bootstrapped Total Least Squares
LDS   Low Discrepancy Sampling
LHS   Latin Hypercube Sampling
LTE   Long Term Evolution
LTI   Linear Time-Invariant
MC    Monte Carlo
MIMO  Multiple-Input Multiple-Output
NPR   Noise Power Ratio
OFDM  Orthogonal Frequency Division Multiplexing

PCB  Printed Circuit Board
pdf  probability density function
PDN  Power Distribution Network
PISPO Periodic-In Same Period-Out
PQ   Performance Quantities
PSD  Power Spectral Density

QMC  Quasi Monte Carlo

RBF  Radial Basis Functions
RF   Radio Frequency
RMS  Root Mean Square
RMSE Root Mean Square Error

SC   Stochastic Collocation
SINAD Signal-to-Noise and Distortion ratio
SISO Single-Input Single-Output
SNR  Signal-to-Noise Ratio

TL   transmission line
TOI  Third Order Intercept point

VF   Vector Fitting
VNA  Vector Network Analyzer

WLAN Wireless Local Area Network
1. Introduction

Over the last years there has been an exponential increase in the amount of Radio Frequency (RF) devices and applications. Each new application comes with its own transmission requirements and its own frequency band. The presence of more applications results in the need for more allocated frequency bands. As each application also tends to be used by more devices, there is an increased requirement for more bandwidth [Chan 04].

In the development of the 5G communication technology, the increase in applications and devices will only become more extensive. The 5G communication standard will bring higher data rates, lower latency and a higher reliability to the end users. All of this will result in a higher bandwidth of the wireless communication. Designers face significant challenges to meet these strong design requirements for linear as well as nonlinear components. [Bayl 14, Peto 18]

To achieve a higher bandwidth, new 5G applications are pushed into frequency bands that are close to bands that are already reserved for other existing applications. Hence, the RF frequency band becomes more and more congested. Filter specifications therefore become even more challenging. The reduction of the guard bands between allocated frequency bands reduces the transition zone between stopband and passband, hereby increasing the selectivity that is needed. However, attenuation specifications are to be kept to avoid interference with neighboring channels that jeopardizes transmission quality. The race towards more selective filters is therefore even faster.

To realize the demanding specifications of modern telecommunication equipment, more complex filter structures are required. This complicates the design of the filters significantly. However, the design procedures that are used to realize these filters have not evolved at the same pace. Massive numerical optimization based on electromagnetic (EM) simulations is often used to solve the problem. This
process is both time-consuming and blocks the intuitive insight in the operation of the device that is so useful to any designer. This insight helps a designer to ensure robust and reliable designs.

Of course, the design of nonlinear components of an RF chain is as important as the design of linear components. The 5G technology demands both a more efficient transmission and a higher reliability while consuming less power. The nonlinear behavior of a device has a significant impact on the overall performance of the communication system. This nonlinear behavior can be a desired effect, such as in a mixer, or it can be a distortion, such as in a power amplifier. Either way, characterization of the nonlinear behavior of a modulated device is necessary.

In amplifier design, more and more complex models are being used. Although intended differently, they only hamper the designer to realize this nonlinear device. Besides, the characterization of the device is based on a large set of measures that are extracted using different measurement setups and require different measurements using different excitation signals. This leads to a testing time that grows proportionally to the number of measures to be determined.

In the context of 5G, an increasing testing time becomes more and more problematic. RF devices comprising the high demands of 5G networks require even more diverse tests at different stages of the design process. Testing these highly integrated RF devices can become too complex and time-consuming resulting in a significant increase in device cost [Walk 19].

It is clear that designers face many challenges within the upcoming 5G communication technology. The design of linear as well as nonlinear devices will become more complicated and there is a need for new approaches towards design techniques as well as measurements.

1.1. Research goal and outline

The goal of this work is to help designers get more insight into the realization of complicated designs of both the linear as well as the nonlinear components that are present in an RF telecommunication system.

The thesis is divided into two parts: Part I: Model driven filter design, where our goal is examined for a linear component and Part II: Figures of merit for characterizing nonlinear devices, where our goal is examined for a nonlinear component.

In Part I, we improved on the current design procedure of filters while keeping its simplicity. To this end, we introduced models into the design process. As the "one
model does it all” was already proven to be unsuccessful in previous attempts, we used a different paradigm, where a specifically tailored model was extracted for specific design tasks.

We design a microwave filter in three different ways. First, Chapter 2 thoroughly discusses the existing filter design approach by EM optimization. Then, Chapters 3 and 4 introduce two new design approaches where identification and modeling is used to overcome the disadvantages that are present in the EM optimization approach. Chapter 3 uses metamodels to this end, while Chapter 4 is based on a scalable equivalent circuit model. Chapter 5 thoroughly compares these three filter design approaches. Finally, Chapter 6 extends the equivalent circuit model and takes a first step in incorporating these extensions within the framework of the two newly proposed design approaches.

Note that each of the three design approaches are illustrated on an example filter structure: the Defected Ground Structure (DGS). The DGS and its properties are therefore thoroughly discussed in Section 1.3.

The models that were introduced in the new design approaches reduce the design time significantly and increase the physical insight into the EM operation of the filter. Note that these new design techniques can be generalized to the design of other linear devices.

In Part II, we improved on the current characterization techniques for assessing the nonlinear behavior of a power amplifier. Here too, we introduced a model to simplify the test procedure.

Chapter 7 extracts multiple figures of merit to assess the nonlinearity of a device based on measurements by using the Best Linear Approximation (BLA) model.

Using the BLA allows to perform multiple characterization tests at once. On top of that, the required testing time is reduced significantly. Note that this technique can be generalized to the characterization of other nonlinear devices.

1.2. Preliminaries and definitions

This section introduces some preliminaries and definitions that are used throughout this work. First, power waves and S-parameters are introduced. Then, the concept of a filter template is discussed. Finally, the errors that pop up in the design procedures of Part I are defined.
1. Introduction

S-parameters

In both parts of the thesis, we will perform the measurements and the simulations in the frequency domain. Two-port systems considered in this work are described by power waves and S-parameters. Different definitions of the power waves and S-parameters exist. We use the definitions as defined in [Kuro 65].

Filter template

Part I mainly focuses on filter design. We only consider filters with design specifications imposed on the amplitude of the Frequency Response Function (FRF). The magnitude approximation problem consists of finding a suitable magnitude function, whose magnitude characteristics satisfy a given set of filter specifications, and which is realizable in practice [Raut 10]. These filter specifications are also called the filter template.

The selectivity, $k$, of a filter is a measure for the slope of the transition between bands. Together with the relative bandwidth, it is a measure for the complexity of the filter design. For a bandpass filter, the selectivity is given by

$$k = \frac{f_{p2} - f_{p1}}{f_{s2} - f_{s1}}$$

with $f_p$ the passband edge frequencies and $f_s$ the stopband edge frequencies, as defined in the bandpass filter template shown in Figure 1.1.

Similarly, the filter template for a bandstop filter is shown in Figure 1.2. The selectivity of a bandstop filter is given by

$$k = \frac{f_{s2} - f_{s1}}{f_{p2} - f_{p1}}$$

In this work, we define the stopband as the frequency band(s) for which the transmission factor $|S_{21}|$ is below the stopband amplitude $A_s$. The passband is defined as the frequency band(s) for which the transmission factor $|S_{21}|$ is above $A_p = -3\,\text{dB}$. The passband edge frequencies $f_p$ are also called the cut-off frequencies $f_c$. They determine the point where only half of the incident power is transmitted. From now on, we use the term cut-off frequencies for $f_{p1}$ and $f_{p2}$ rather than passband edge frequencies.

Note that only the magnitude characteristics of a filter are considered in this work to discuss the methodology of filter design. However, for some modulation
Figure 1.1.: A filter template imposes specifications on $|S_{21}|$. For a passband filter $|S_{21}|$ should be above $A_p$ in the passband $[f_{p_1}, f_{p_2}]$ and below $A_s$ in the stopband.

Error definitions

Each step performed in the design procedures that will be discussed in Part I of the thesis (Chapters 2-4) introduces an error. The different errors that are used in these chapters are the design error, the model error and the simulation error. They are defined here.

Throughout this thesis we examine three different filter design techniques. To validate and compare the three design approaches, we rely on how well the obtained design fits the required filter template. This is done by evaluating the design errors, $E_{\text{spec}_i}$. A design error represents the norm of the difference between the required filter specification, $\text{spec}_i$, and the obtained filter specification, $\hat{\text{spec}}_i$, relative to the norm of the required specification:

$$E_{\text{spec}_i} = \frac{|\text{spec}_i - \hat{\text{spec}}_i|}{|\text{spec}_i|}$$  \hspace{1cm} (1.1)

with $\text{spec}_i$ the i-th filter specification.

To validate the model quality, we evaluate the residual errors using the absolute and relative model error. The absolute model error is defined as the norm of the
1. Introduction

Figure 1.2.: For a stopband filter $|S_{21}|$ should be below $A_s$ in the stopband $[f_{s_1}, f_{s_2}]$ and above $A_p$ in the passband.

The absolute model error is defined as the absolute difference between the simulated complex S-parameter data, $S_{kl}(j\omega_i)$, and the modeled complex S-parameter data, $\hat{S}_{kl}(j\omega_i)$, evaluated at a frequency $\omega_i$:

$$\epsilon_{abs,kl}(j\omega_i) = |S_{kl}(j\omega_i) - \hat{S}_{kl}(j\omega_i)|$$  \hspace{1cm} (1.2)

The relative model error is defined as the absolute model error divided by the norm of the simulated S-parameter data, $S_{kl}(j\omega_i)$:

$$\epsilon_{rel,kl}(j\omega_i) = \frac{|S_{kl}(j\omega_i) - \hat{S}_{kl}(j\omega_i)|}{|S_{kl}(j\omega_i)|}$$ \hspace{1cm} (1.3)

Indices $k,l = 1 : 2$ since we only consider two-port networks.

We will also consider the absolute and relative Root Mean Square Error (RMSE), which are defined as:

$$\text{RMSE}_{abs,kl} = \sqrt{\frac{\sum_{i=1}^{F} |\epsilon_{abs,kl}(j\omega_i)|^2}{F}}$$ \hspace{1cm} (1.4)

$$\text{RMSE}_{rel,kl} = \sqrt{\frac{\sum_{i=1}^{F} |\epsilon_{rel,kl}(j\omega_i)|^2}{F}}$$ \hspace{1cm} (1.5)

with $F$ the number of frequency points and $\epsilon_{abs}$, $\epsilon_{rel}$ the absolute and relative model error, related to the norm of the complex difference between the simulated and the modeled S-parameter data.
For all extracted models in this work, we aim for an absolute RMSE of at most $\text{RMSE}_{\text{abs}} = -25\text{dB}$ for all $S$-parameters. Of course, the absolute model error should always be verified at each frequency point separately, because a low RMSE does not necessarily mean that the model error is low over the whole frequency band. The relative RMSE should also be verified to check the relative fit. The model error should be low enough in order not to influence the design error.

To validate the simulations, we evaluate the simulation error by using the absolute and relative error (defined in Equations (1.2) and (1.3)) and the absolute and relative RMSE (defined in Equations (1.4) and (1.5)), with $S_{kl}(j\omega_i)$ the measured $S$-parameter data and $\hat{S}_{kl}(j\omega_i)$ the simulated $S$-parameter data. The simulation error is the same for the three design approaches discussed in this work. The absolute RMS simulation error reaches a minimum of approximately $-25\text{dB}$ for the selected simulation settings (discussed in the next section). Note that this error also includes the fabrication tolerances of the design. Since this is a limiting factor, the target of a maximum absolute RMS model error of $\text{RMSE}_{\text{abs}} = -25\text{dB}$ is a logical choice.

### 1.3. The Defected Ground Structure

As an example structure in **Part I** we opt for a Defected Ground Structure (DGS) as this structure has multiple applications in the microwave field. For example, it can be used to improve or easily shape the FRF, to reduce the physical size or to improve the performances of a microwave circuit. Its potential is already demonstrated in the design of couplers [Lim 00], power dividers [Lim 01a, Woo 05], power amplifiers [Lim 01b, Choi 06], antennas [Sung 03, Arya 10, Chun 04, Guha 05] and filters [Vela 04, Park 02, Lin 16, Sabr 15]. This thesis only focuses on the applications in filter design but the results obtained in this thesis can be used in other applications as well.

The literature shows that a DGS can suppress the spurious frequency responses in coupled-line band-pass filters [Vela 04, Park 02] as it can be used to realize bandstop filters. Furthermore, it behaves as a filtenna [Lin 16, Sabr 15] since the slot, present in the ground plane of the filter, radiates.

This section consists of 4 parts. First, we explain the structure and its properties in more detail. Next, the settings that are needed to simulate a DGS are discussed, since they require special care. Then, the precautions that must be taken when measuring a DGS are discussed. Finally, the substrate settings are listed for the DGS designs and simulations used throughout this thesis.
1. Introduction

Properties of a DGS

![Diagram of DGS with cross-section and top-views with single and multiple slots.](image)

A DGS is a microstrip structure that consists of a transmission line on the top layer with one or more resonating slots (called "defects") in the metallic ground-plane (Figure 1.3). The slots are usually located directly underneath the transmission line for coupling to occur between the line and the slot. The coupling introduces a transmission zero, or an antiresonance, in the FRF of the filter as energy is radiated when the slot resonates. This implies that the DGS behaves as a bandstop filter and a bandpass antenna in the same frequency band. The fundamental or bandstop frequency of the filter depends on the shape and the size of the slot. In the remainder of the text, we will focus on the bandstop filter characteristics of the DGS.

Besides the bandstop behavior, the DGS has two other characteristics. The slots cause a slow-wave effect, that can result in a size reduction of the designed circuits [Garg 13], and increase the characteristic impedance of the transmission line, which is useful to design filters with an increased selectivity [Garg 13, Arya 10]. Note that these "defects" are intentional, and hence controllable.

A DGS can consist of a single slot or can have multiple slots. If the slots are equally spaced, the structure is called a periodic DGS. The latter allows more pronounced characteristics when compared to a structure with only a single slot [Garg 13]. In Chapters 2-5 we focus on a DGS with a single slot. Chapter 6 discusses the extension of the proposed techniques to a periodic DGS (Figure 1.3(c)).
1.3. The Defected Ground Structure

Figure 1.4.: Slots of a DGS can take different shapes. [Garg 13]

The slots can take different shapes [Bree 08, Garg 13] (Figure 1.4). Different shapes impose a different level of expression of the slot characteristics. Depending on the targeted application, the selection of a different slot shape results in different properties [Weng 08]. Since we cannot analyze all possible shapes, we will focus here on one in particular: a rectangular slot. Note that the methods developed in this work can also be applied to DGSs with other slot shapes.

In the example used in Chapters 2-5, the defect in the ground-plane consists of a simple rectangular slot that is positioned perpendicularly to and symmetrically around the transmission line, as shown in Figure 1.3 (b). This DGS has only three geometrical design parameters: the width of the transmission line, \( W \), and the width and length of the slotline, \( W_s \) and \( L_s \).

As mentioned before, the fundamental resonating frequency of the slot depends on the shape and the size of the slot. In this text, the shape of the slot is fixed to be a rectangle. The dimensions of the slot, its width \( W_s \) and its length \( L_s \), are used as design parameters that can be used to tune the fundamental frequency and the impedance of the slot. The slot is a half-wavelength resonator so a longer slot will lead to a lower fundamental frequency. Hence, \( L_s \) is dominant in determining the fundamental frequency since
1. Introduction

\[ L_s \propto \frac{\lambda_0}{2} = \frac{v_p}{2f_0} \]  

with \( v_p \) the phase velocity and \( \lambda_0 \) the wavelength corresponding to the fundamental frequency \( f_0 \).

Beside the fundamental frequency, the FRF of the DGS can display antiresonances at multiples of the fundamental frequency. Hence, these antiresonances are harmonically related and are therefore called harmonic responses from now on.

Whether these harmonic responses are present in the FRF or not depends, to some extent, on the position of the coupling point between the slotline and the transmission line. This coupling point is called the feedpoint of the slot.

Figure 1.5: The position of the feedpoint between slotline and transmission line selects which modes of the electric field are excited and which are not. Left: the even modes, \( m = 2k \), are zero at the feedpoint. Right: the odd modes, \( m = 3k \), are zero at the feedpoint.

Figure 1.5 shows the electric field of the first 4 modes of the resonating slot. When the transmission line is positioned at \( \frac{1}{2} \) of the slotline (left), the electric field of the even modes (\( m = 2k \) with \( k = 1 : \infty \)) is zero at the feedpoint. At these modes, the slot does not resonate because no energy coupling occurs between the transmission line and the slotline. In this case, the harmonic responses corresponding to these modes will be present in the FRF. Only the modes at which the slot resonates results in an antiresonance in the FRF (Figure 1.6).

When the transmission line is positioned at \( \frac{1}{3} \) of the slotline (right), the electric field at the feedpoint is zero for the modes with a modal index that are a multiple of 3 (\( m = 3k \) with \( k = 1 : \infty \)). In this case, the harmonic responses corresponding to
1.3. The Defected Ground Structure

Figure 1.6.: FRF of a simulated DGS with a feedpoint at \( \frac{1}{2} \) of the slot’s length.

These modes will be present in the FRF (Figure 1.7). Note that the third mode is slightly excited as a small bump is visible in the FRF, despite the positioning of the feedpoint at \( \frac{1}{3} \) of the slotline.

Figure 1.7.: FRF of a simulated DGS with a feedpoint at \( \frac{1}{3} \) of the slot’s length. The position of the feedpoint selects which modes of the electric field are excited and which are not.

Figure 1.8 shows the general filter template (•••) that will be used for a DGS in this work. Note that there are no stopband frequencies \( f_s \) since the DGS has a very narrow stopband. Therefore, we only consider the fundamental frequency, or the center frequency \( f_0 \), as stopband frequency: \( f_s = f_0 \). For the DGS example, we consider 4 filter specifications:

- the center frequency, \( f_0 \)
1. Introduction

- the cut-off frequencies, \( f_{c_1} \) and \( f_{c_2} \)
- the stopband amplitude, \( A_s \)

Note that the passband amplitude \( A_p \) is fixed to \(-3\)dB so it is not considered to be a filter specification. The filter template for a DGS (\( \cdots \)) then imposes that \(|S_{21}|\) should be below the stopband amplitude \( A_s \) at the center frequency \( f_0 \) (\( \bullet \)) and above \(-3\)dB in the passband, defined by the cut-off frequencies \( f_{c_1} \) and \( f_{c_2} \).

![Diagram of filter template](image)

**Figure 1.8:** A filter template (\( \cdots \)) for a DGS : \(|S_{21}|\) should be below \( A_s \) at the stopband frequency \( f_s = f_0 \) (\( \bullet \)) and above \(-3\)dB in the passband, defined by the cut-off frequencies \( f_{c_1} \) and \( f_{c_2} \).

The DGS is a passive structure and its S-parameters are symmetrical. A 2-port is called symmetric if it is electrically equivalent to itself when port 1 and port 2 are interchanged [Bele 68]. In terms of S-parameters, symmetry requires that \( S_{11} = S_{22} \) and \( S_{12} = S_{21} \). Hence, a symmetric 2-port is also reciprocal (\( S_{12} = S_{21} \)). For simplicity, we will therefore only consider S-parameters \( S_{11} \) and \( S_{21} \) when we discuss the S-parameter response throughout the rest of this work.

**Simulation settings for a DGS**

All EM simulations shown in this thesis are performed in Advanced Design System (ADS) [Keys 14a] using the Momentum simulator [Keys 14b]. Momentum is a 3-dimensional planar EM solver that uses the Method of Moments (MoM) [Harr 93] in the frequency domain to simulate microstrip structures.

Simulating a DGS requires specific simulation settings in order to achieve accurate simulation results. For a general microstrip structure simulation, a substrate definition with infinite ground plane is often used, as illustrated in Figure 1.9.
Since we want to simulate a structure with an aperture in the ground plane, we cannot use the infinite ground plane any longer. We need to define a substrate with a finite ground plane instead, as shown in Figure 1.10.

![Figure 1.9.: Definition of a substrate with an infinite ground plane.](image1)

![Figure 1.10.: Definition of a substrate with a finite ground plane.](image2)

Besides the substrate definition it is also important to choose the calibration type wisely. Usually, transmission line calibration, called a TML-calibration, is used. Momentum [Keys 14b] then automatically adds a transmission line feed line to the excitation port and thereby assumes the presence of an infinite ground plane. This leads to unrealistic results when using a finite ground plane, which is necessary when simulating a DGS. Hence, the calibration should be set to ‘none’ when using a finite ground plane substrate definition.

Using a finite ground plane substrate, however, has the disadvantage that the simulation time increases drastically when compared with a simulation with an infinite ground plane. For an infinite ground plane, the return currents are modeled by impedance and conductivity models. For a finite ground plane, the return currents are calculated by solving the Maxwell equations instead of using models, as is always the case for the calculation of the currents on the signal layer. Hence, more calculations are needed. Keeping the simulation time reasonably low implies that the surface area of the finite ground plane should be restricted. The dimensions of the finite ground plane are mainly defined by the length of the
1. Introduction

Table 1.1.: Substrate settings for a RO4003\textsuperscript{TM}.

<table>
<thead>
<tr>
<th>$\varepsilon_r$</th>
<th>$\tan\delta$</th>
<th>$h$ [mm]</th>
<th>$T$ [µm]</th>
<th>$\sigma_{Cu}$ [mS]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.55</td>
<td>0.0021</td>
<td>1.524</td>
<td>35</td>
<td>5.8e7</td>
</tr>
</tbody>
</table>

slot, $L_s$. Keeping $L_s$ reasonably small will put a lower bound on the fundamental frequency, $f_0$, as $L_s \propto 1/(2f_0)$ (Equation (1.6)).

However, the simulation time also increases with the highest simulation frequency. This is because the mesh density of the mesh points at which the Maxwell equations are calculated increase with the highest simulation frequency. To keep the simulation time reasonably low, we will try to restrict the highest simulation frequency to 5GHz and thereby putting an upper bound on the fundamental frequency.

Taking both limitations into account will give us a range of operational frequencies. To simplify the modeling in Chapters 2-5, we only consider the fundamental frequency of the example structure (shown in Figure 1.3). Chapter 6 discusses the extension to a wide-band model, where we consider multiple antiresonances. To guarantee that the third harmonic response falls below 5GHz, we set the absolute maximum of the fundamental frequency to 1.6GHz. To avoid that the dimensions of the finite ground plane become too large, we set the absolute minimum of the fundamental frequency to 1.1GHz. These absolute limits are taken into account in the design examples in Chapters 2-5. The frequency band of interest is therefore $[0.1 - 2.5]$ GHz so that the fundamental frequency is fully captured. The frequency band of interest will be extended to $[0.1 - 5]$ GHz in Chapter 6 to capture up to the third harmonic response.

Precautions for measuring a DGS

Measuring a DGS requires extra precautions. The structure should be measured in open air rather than on a massive conductive table or plane. If not, the conductive table behaves as an infinite ground plane, altering the measurement response that is expected.
1.3. **The Defected Ground Structure**

**Substrate settings**

The substrate used here is a Rogers 4003 (RO4003\(^{TM}\)). It has a relative permittivity \(\varepsilon_r = 3.55\), a dissipation factor \(\tan \delta = 0.0021\) at 2.5 GHz and a thickness \(h\) of 60mil (= 1.524 mm). The conducting material is copper with a conductance \(\sigma_{Cu}\) of 5.8e7 mS. The thickness of the conduction layer \(T\) is 35 \(\mu\)m. These substrate settings are summarized in Table 1.1. This microstrip substrate is used for all designs discussed throughout this work. All designs use a microstrip technology with a characteristic impedance of the transmission line of \(Z_c = 50\Omega\). This corresponds to a transmission line width of 3.4 mm.
Part I.

Model driven filter design
Radio Frequency (RF) and microwave filters are currently designed and/or fine-tuned using electromagnetic (EM) optimization because classical filter synthesis techniques tend to fail when the filter selectivity increases [Swan 07]. EM optimization is very accurate but also holds several disadvantages: it provides no physical insight to designers, the optimization time increases drastically with the complexity and with the accuracy level one wants to achieve, etc.

Section 2.1 describes the general concept of, and the need for, EM optimization in filter design. Section 2.2 discusses the positive and negative aspects of this approach in more detail. Finally, Section 2.3 designs a Defected Ground Structure (DGS) example using this technique.
2. Filter design using electromagnetic optimization

2.1. General concept

Over the last years there has been an exponential increase in the amount of RF devices and applications. Each application has its own transmission requirements and its own frequency band. More applications result in more allocated frequency bands. More devices make these allocated frequency bands to require more bandwidth. In other words, the RF frequency band becomes more and more congested [Chan 04, Bayl 14].

We try filling up the empty space that is still left in the RF frequency band, which makes filter specifications extra challenging. There is less space between allocated frequency bands, which reduces the transition area between stopband and passband. However, attenuation specifications are to be kept in order to avoid interference with neighboring channels. Together with a higher bandwidth this results in more selective filters.

The higher the selectivity of the filter, the more complex the filter structure should be to realize the desired specifications. Simple filter structures can easily be designed using the classical filter design. Designing complex structures, however, proves to be a challenge when using this design procedure. The different steps of the classical filter design process are discussed here, and are shown graphically in Figure 2.1.

1. User settings

Starting from the filter specifications and the application at hand, the user should select a proper microstrip transmission line (TL) structure. The user should also specify the frequency range (which will also depend on the application at hand) and the design space (which can depend on the application but other aspects can also lead to limitations of the boundaries).

2. Approximation

The designer should select a suitable rational function that approximates the desired frequency response as best as possible. Examples are Butterworth, Chebyshev or Bessel polynomial approximations [Pram 16, Rhea 94].

3. Synthesis

Next, the rational function is synthesized into a prototype filter with the same response as the rational function. For commensurate TL structures, applying the Richards’ transformation [Rich 48] allows the use of the classical lumped-element synthesis techniques, e.g. Darlington [Darl 39].

4. Realization

The synthesized prototype filter is then realized in microstrip technology
using the Richards’ transformation for the correspondence between lumped and distributed elements.

5. **Variability analysis**
Finally, a variability analysis (also called a sensitivity analysis) should be performed to verify the robustness of the design.

![Diagram of filter design process]

**Figure 2.1.** The classical filter design approach is not accurate enough in the realization phase to realize the required filter response.

The more demanding filter specifications cannot be fulfilled any more by using a classic design based on basic, ideal TL structures, consisting of simple delay lines. The classical design procedure was later extended to allow synthesis of more selective filters by use of coupled TLs. [Cohn 57] introduced a widely used approximation technique for narrow-band filters using impedance inverters. This technique is satisfactory for designing simple resonating structures, e.g. coupled line resonator filters [Cohn 58]. However, it will still not be sufficient to achieve accurate results when designing more complex resonating structures, such as Defected Ground Structures (DGS).

Dealing with complex filter specifications requires more complex TL structures (e.g. non commensurate structures, DGSs) or even combinations of distributed and lumped components (e.g. a TL that is loaded by a capacitance along the
2. Filter design using electromagnetic optimization

In general, the classical filter design procedure is, even with its extensions, not adapted to the use of these complex structures and their associated models in the design. It uses idealized circuit models, consisting of ideal, reactive elements only. In the realization step, these ideal lumped components are transformed into ideal TLs or ideal coupled TL sections. The complex structures however, cannot be approximated by a single ideal element. They require non-ideal components and/or subsystems to obtain a good match of the desired and realized frequency response. Approximating these complex structures by the empirical design equations obtained for standard, ideal TL structures is too crude. Hence, in the realization phase the classical design procedure is not sufficiently accurate any more to realize the required frequency response accurately according to the desired filter specifications.

Numerical EM optimization is used to cope with the lack of accuracy in classical filter design due to idealization. In numerical electromagnetics, the Maxwell equations are solved by means of an EM field simulator. Field-based simulations require more computation time and memory than circuitry-based simulations, but they are much more accurate because they take parasitics and other second-order effects into account [Swan 03].

In all EM simulations performed in this thesis, Momentum [Keys 14b] was used in microwave or full-wave mode. This means that the general frequency dependent Green functions are used that fully characterize the substrate [Keys 14b]. Hence, no simplifications to the Maxwell equations are made, thereby maximizing the accuracy achieved in an EM simulation for a fixed frequency grid and mesh density.

Two types of EM optimization exist: direct optimization and hybrid optimization. Direct EM optimization works directly on the design parameters of the microwave structure [Bila 01, Band 94]. The optimization parameters are independent from one another, so they can be adjusted separately. This is interesting when the designer wants to know the influence on the Frequency Response Function (FRF) of each parameter separately.

Hybrid EM optimization includes an analytical model in the optimization process [Garc 04, Kahr 02, Bakr 99]. The use of the analytical model in hybrid optimization is twofold. It allows to find a good starting point for the EM optimization. This brings the optimizer already closer to its convergence, speeding up the process compared with direct optimization. The model is also used after each iteration to evaluate the new state and correct the error vector accordingly, also leading to a faster convergence. Note that nonetheless, the optimization still requires time-expensive EM simulations for each iteration. The analytical model can be a
rational function, a circuit network, or a model defined by artificial neural networks, space mapping techniques, other numerical techniques or a combination of the before mentioned options [Garc 04, Kahr 02, Bakr 99].

In general, when designing a filter using EM optimization filter design, the designer should follow a series of steps:

1. **User settings**
   Starting from the filter specifications and the application at hand, the user should select a proper TL structure (or microstrip structure), the frequency range and the design space.

2. **EM optimization**
   The microstrip structure should be optimized within the selected design space using EM simulations and, additionally, using analytical models in case of hybrid optimization.

3. **Variability analysis**
   After finding an optimized design, a variability analysis should be performed to verify the robustness of the design. However, variability analysis based on EM-simulations is extremely time-consuming.

This filter design procedure is shown graphically in Figure 2.2. The optimization of the structure works directly on the filter specifications. The design flow is therefore much shorter and provides less physical insight than the classical design approach. The next section discusses the advantages and disadvantages of the EM optimization in more detail.

![Diagram](image_url)

Figure 2.2.: EM optimization filter design is accurate but very slow and lacks the physical insight that is provided by the classical filter synthesis.
2. Filter design using electromagnetic optimization

2.2. Advantages and drawbacks

Although some advantages and disadvantages were already briefly discussed in the previous section, we want to collect them here for accessibility. We also go a bit more into detail about the consequences of each of them.

An EM optimization process is based on multiple EM field-based simulations. Using EM-simulations instead of circuitry-based simulations has several advantages:

- They can cover a wider parameter range compared with circuits or models, that often make use of linearization.
- They take indirect couplings and other second order effects into account, thereby considering all physical and electrical effects that can occur in a structure.
- They can achieve a very high accuracy over a wide frequency range.
- They can simulate more complex, irregular structures without any loss of accuracy.

However, there are also a lot of drawbacks:

- They require a huge amount of memory space and computation time due to the time-consuming EM simulations.
- The more complex the structure or the design space becomes, the longer the simulation time will be. However, even small, simple structures can take a relatively long time to simulate.
- The designer loses the physical insight of the link between the TL section and the prototype that is provided in the classical design approach. Making proper design choices therefore becomes hard or even impossible.
- Variability analyses based on EM-simulations also become extremely time-consuming. However, they are important to check the effect of fabrication tolerances on an optimized design thereby evaluating the reliability of the obtained design.
- No information about the structure can be reused in future designs.
- The design process must be repeated completely when the specifications change.

In conclusion, EM optimization is a powerful technique for designing complex microwave structures. Unfortunately, this approach has many drawbacks. The
2.3. Example: Defected Ground Structure

The direct EM optimization design approach is now illustrated for the design of a DGS shown in Figure 1.3 (Section 1.3). We consider 3 design cases with different filter specifications for this example structure and discuss the results of each of them. For the convenience, we follow the design flow of Section 2.1, Figure 2.2.

User settings

Before starting the optimization process, the user needs to specify the frequency range and the design space. As a structure, we select a DGS (Section 1.3).

The frequency range will strongly depend on the application at hand. In general for a selected microstrip structure, selecting the frequency range will automatically put some restrictions on the design space. In the case of the DGS, we have set an absolute upper and lower bound for the fundamental frequency, \( f_0 \in [1.1 \text{ - } 1.6] \text{GHz} \), to restrict the simulation time (as already discussed in Section 1.3: Simulation settings for a DGS).

The design parameters can be geometrical parameters as well as material properties. In the numerical example used here, only the geometrical parameters \( W_s \) and \( L_s \), the width and the length of the slotline (Figure 1.3), are considered as design parameters.

Both design parameters, \( W_s \) and \( L_s \), should be bounded. Large \( W_s \)-values lead to high reflections in the passband so that almost no signal arrives at the output. Small \( W_s \)-values are not physically realizable. The selected range for \( W_s \) is \([0.3 \text{ - } 3] \text{mm} \) (Table 2.1). The limits of \( L_s \) mainly depend on the absolute limits of \( f_0 \) since \( L_s \) and \( f_0 \) are strongly related: \( f_0: L_s \propto 1/(2f_0) \) (see Equation (1.6)). Small \( L_s \)-values lead to large \( f_0 \)-values, and that results in a simulation time that becomes too long, as explained in Section 1.3. Large \( L_s \)-values increase the size of the finite ground plane, and hence the simulation time becomes too large. Therefore, the selected range for \( L_s \) is \([80 \text{ - } 101] \text{mm} \).
The boundaries of the design parameters indirectly determine the feasible filter specifications, or the *performance quantities*. We will use the terms "filter specifications" and "performance quantities" interchangeably. When talking about the design procedure we use the term "filter specifications", while "performance quantities" will more likely be used when we talk about the filter specifications in the metamodeling optimization context.

For the DGS example, we consider 4 performance quantities:

- the center frequency, \( f_0 \)
- the cut-off frequencies, \( f_{c1} \) and \( f_{c2} \)
- the stopband amplitude, \( A_s \)

The boundaries of the performance quantities are defined by the boundaries on the design space and are given in Table 2.1. The boundaries of \( f_0 \) are set to \([1.19 - 1.53]\) GHz. These fall within the absolute limits of \([1.1 - 1.6]\) GHz that were set in Section 1.3: *Simulation settings for a DGS*.

We simulated the S-parameter data over a frequency range of \([0.1 - 2.5]\) GHz with a fixed resolution of 10MHz resulting in 241 equidistant samples. Simulating up to 2.5GHz fully captures the feasible center frequencies (Table 2.1). The 10MHz-resolution is a trade-off between accuracy and simulation time.

Table 2.1.: The user needs to limit the design space. The boundaries of the design parameters indirectly determine the feasible filter specifications.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotline width, ( W_s ) [mm]</td>
<td>0.30</td>
<td>3.00</td>
</tr>
<tr>
<td>Slotline length, ( L_s ) [mm]</td>
<td>80.00</td>
<td>101.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feasible filter specifications</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency, ( f_0 ) [GHz]</td>
<td>1.19</td>
<td>1.53</td>
</tr>
<tr>
<td>Cut-off frequency 1, ( f_{c1} ) [GHz]</td>
<td>0.85</td>
<td>1.11</td>
</tr>
<tr>
<td>Cut-off frequency 2, ( f_{c2} ) [GHz]</td>
<td>1.45</td>
<td>2.25</td>
</tr>
<tr>
<td>Stopband amplitude, ( A_s ) [dB]</td>
<td>-32.2</td>
<td>-23.0</td>
</tr>
</tbody>
</table>

The three considered design cases should be selected to remain within these boundaries, otherwise the design will not be feasible. The filter specifications are randomly selected within these boundaries for the three DGS design cases and are given in Table 2.2.
2.3. Example: Defected Ground Structure

Table 2.2.: We used three design cases with different filter specifications to discuss filter design by EM optimization.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency, $f_0$ [GHz]</td>
<td>1.24</td>
<td>1.35</td>
<td>1.50</td>
</tr>
<tr>
<td>Cut-off frequency 1, $f_{c1}$ [GHz]</td>
<td>0.92</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>Cut-off frequency 2, $f_{c2}$ [GHz]</td>
<td>1.55</td>
<td>1.81</td>
<td>2.10</td>
</tr>
<tr>
<td>Stopband amplitude, $A_s$ [dB]</td>
<td>−28.0</td>
<td>−26.0</td>
<td>−22.0</td>
</tr>
</tbody>
</table>

**Optimization**

Starting from the filter specifications, the DGS is optimized directly in Advanced Design System (ADS) [Keys 14a] using the random optimizer. This optimizer uses the random search method, where a new set of parameter values is obtained by using a pseudo-random generator, called a positive perturbation of the initial values. A Weighted Least Square cost function is evaluated for the positive as well as the negative perturbation, when the sign of the positive perturbation is reversed. For each function evaluation, an EM simulation of the DGS is performed using Momentum. The error vectors of these two points are compared with the error vector of the initial point. If one of the new error vectors is smaller than the initial error vector, the set of parameter values that leads to the smallest error vector becomes the initial set for the next iteration. Otherwise, the random generator finds another set of parameter values from the same set of initial values. The random optimizer proved to have the best and fastest convergence.

The starting values are the lower boundaries of our design space and are identical for the three cases: $W_s = 0.3$ mm and $L_s = 80$ mm. Better results would be obtained if the starting values were selected randomly within the design space to use the random optimizer in a more efficient way.

**Results**

The EM optimization provides the desired results, shown in Figures 2.3, 2.4 and 2.5 for Case 1, 2 and 3 respectively. The S-parameters of the optimal designs (—) fulfill the required filter template (・・・) for each of the design cases. The initial design, obtained with the starting values, is shown in blue (—). The optimal design parameters are given in Table 2.3.
2. Filter design using electromagnetic optimization

Figure 2.3.: The S-parameters of the optimal design (—) by EM optimization fulfill the required filter template (---) for Case 1 (Table 2.2) starting from the initial design (—).

Table 2.3.: The EM optimization approach provides the desired results. However the computation effort is quite high.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotline width, $W_s$ [mm]</td>
<td>0.81</td>
<td>2.74</td>
<td>2.32</td>
</tr>
<tr>
<td>Slotline length, $L_s$ [mm]</td>
<td>99.2</td>
<td>94.7</td>
<td>82.0</td>
</tr>
<tr>
<td>Number of function evaluations</td>
<td>24</td>
<td>18</td>
<td>26</td>
</tr>
<tr>
<td>Total computation time [s]</td>
<td>1883</td>
<td>1562</td>
<td>2033</td>
</tr>
</tbody>
</table>

All simulations were performed in a Windows environment with 8GB RAM and Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz. The numerical results of the EM optimization are given in Table 2.3.

The average CPU time for one frequency-domain EM simulation (or one function evaluation) over 241 frequency points is 70s. The total computation time is obtained by multiplying this CPU time by the number of function evaluations for each case and adding some additional time that is required for making the decision of taking a step.

The total computation time is already quite high even though we have only a 2-dimensional design space. Increasing the number of design parameters would lead to an exponential increase of the total computation time. Alternative methods are
2.3. Example: Defected Ground Structure

![Figure 2.4](image1.png)

Figure 2.4.: The S-parameters of the optimal design (--) by EM optimization fulfill the required filter template (---) for Case 2 (Table 2.2) starting from the initial design (--).

![Figure 2.5](image2.png)

Figure 2.5.: The S-parameters of the optimal design (--) by EM optimization fulfill the required filter template (---) for Case 3 (Table 2.2) starting from the initial design (--).
required to keep the computation time reasonably low. In the next two chapters, new techniques are introduced to circumvent the drawbacks of EM optimization.
3. Filter design using metamodels

The results of this chapter are obtained through close collaboration with Francesco Ferranti and are published in [Van 17].

To circumvent the drawbacks of electromagnetic (EM) optimization, we introduce models into the design procedure to improve the current design method and make it more directly applicable. We can use metamodels to link the design parameters directly to the filter template. Hereby, we gain some physical insight that was not provided by the EM optimization approach, but more importantly we can achieve a significant gain in terms of simulation speed.

Section 3.1 describes the general concept of the proposed technique. Section 3.2 discusses the positive and negative aspects of the proposed technique in more detail. Section 3.3 gives a small overview of different sampling techniques, metamodeling techniques and validation methods that can be used. Section 3.4 applies the proposed approach to the Defected Ground Structure (DGS) as was used in the previous chapter (Section 2.3).
3. **Filter design using metamodels**

### 3.1. General concept

To overcome the disadvantages of using EM optimization in design, discussed in Chapter 2, we introduce metamodels to empower fast simulation and verification in the filter design process. First, we explain what metamodels are. Then, we explain how they are introduced in the design procedure by use of the graphical design flow.

Metamodels are a very efficient representation that provide a functional relationship between input and output variables [Klei 08]. In our case, these variables correspond to physical design parameters (of the microstrip structure) and filter performance quantities (or filter specifications) respectively. Hence, a metamodel is built for each of the performance quantities, linking that particular performance quantity to all design parameters. For example, \( f_0(W_s, L_s) \) is the metamodel that represents the dependence of the center frequency of the filter on the width and the length of the slot. It is one of the four metamodels for the selected DGS example structure that will be discussed in Section 3.4.

The metamodels are introduced as an intermediate step between the filter specifications and the microstrip structure. This is shown graphically in Figure 3.1. They return the filter specifications as a function of the design parameters (shown in gray). However, the inverse models are needed to be used in a design context. Inverting the metamodels is done by optimizing them to find the design parameters as a function of a set of given filter specifications (shown in green). Note that constructing the metamodels should be done only once for one particular transmission line (TL) structure (or microstrip structure) with a specific set of user settings (frequency range and design space) since the metamodels are reusable. Therefore, it is shown in gray since it is not always needed for the standard design procedure by metamodeling:

1. **User settings**
   
   Starting from the filter specifications and the application at hand, the user should select a proper microstrip structure, the frequency range and the design space.

2. **Construction of the metamodels**
   
   If metamodels do not yet exist for a selected microstrip structure and the set of user settings, new metamodels should be built using EM simulations over the design space.

3. **Optimization using metamodels**
   
   The metamodels should be optimized within the selected design space.
3.2. Advantages and drawbacks

3.2. Advantages and drawbacks

The newly proposed design method using metamodels has again some advantages as well as some drawbacks. They are discussed below and illustrated by use of an example design in Section 3.4.

Design by metamodeling is much faster than design by EM optimization because this design approach does not rely on costly EM simulations assuming the metamodels are at disposal. The filter optimization uses metamodels instead to find the design parameters corresponding to the required set of filter specifications.

4. Variability analysis

Once an optimized design is obtained, a variability analysis should be performed to verify the robustness of that design. Using the metamodels can speed-up the variability analysis by some orders of magnitude, as here a reliable result requires a statistically meaningful set of test cases.

Figure 3.1.: Filter design by metamodeling is accurate and fast. It provides more physical insight than EM optimization and allows a time-efficient variability analysis.

Section 3.2 discusses the advantages and drawbacks of the design procedure proposed above. The decisions made for the constructing the metamodels are based on the small literature study given in Section 3.3. The different steps of the proposed design procedure are illustrated on a DGS example in Section 3.4.

3.2. Advantages and drawbacks
of the costly EM simulations of the microstrip structure. Metamodels are much more time-efficient to evaluate when compared with an EM simulation. The low function evaluation cost makes variability analyses also become time-efficient and feasible when using metamodels instead of EM simulations.

Unfortunately, constructing the metamodels requires some additional CPU time. We call this additional time the modeling time. The prominent contribution to this modeling time is the time-consuming EM simulation that should be performed for each design space sample. Note that the number of EM simulations required to construct the metamodels is still much less than the number of EM simulations (function evaluations) when EM optimization is used.

Luckily, the metamodels are reusable in future filter designs based on the same technology and general filter structure. Hence, the additional modeling time mentioned above is only required once when constructing new metamodels. New metamodels should be generated for each new microstrip structure, or when the set of user settings change.

The metamodels provide more physical insight in the link between filter specifications and design parameters. Although the new approach still uses optimization, it is no longer a blind optimization. The metamodels provide knowledge to the designer about the influence of the design parameters on the filter specifications. This feeds the intuition of the designer and empowers problem solving based on "educated guesses". Note that even more physical insight can be obtained when using an equivalent circuit model. This will be done in Chapter 4.

Design by metamodeling is also accurate and the level of accuracy can be assessed. The level of accuracy depends on three main factors: the sample size for a specific sampling technique with respect to its convergence rate, the accuracy of the EM simulations (which mainly relates to the number of frequency samples) and the accuracy of the selected metamodeling technique. This points in the direction of using more samples to ensure a high accuracy. However, remember that more samples (or a higher sample size) means more time-consuming EM simulations are required to construct the metamodels and more frequency samples per EM simulation means a higher simulation time per simulation. Moreover, more accurate metamodeling techniques are often less time-efficient (see Section 3.3). Hence, there is a trade-off between design time and accuracy. Note that this is something that also holds for EM optimization!

The trade-off between accuracy and design time only affects the modeling time. Once the metamodels are constructed, the achievable level of accuracy is fixed and remains fixed during the design optimization. If the designer wants a more accurate design, new improved metamodels need to be constructed.
Although there is a trade-off between accuracy and modeling time, there are some ways to lower the modeling time for a fixed level of accuracy. The choice of the sampling technique plays an important role in this aspect. Adaptive sampling techniques typically reduce the sample size, hence also the modeling time. The same can be said for the density of the frequency grid used in the EM simulation: using adaptive frequency sampling requires less frequency samples to obtain the same amount of useful information in one simulation. Furthermore, the use of a global rational (pole-residue) model, e.g. Vector Fitting (VF) [Gust 99], for the S-parameters can be used to quickly evaluate the S-parameter data achieved in step 3 over a much denser frequency grid to allow a more efficient and accurate estimation of the performance quantities out of the S-parameter data. To reduce the modeling time even more, this interpolation to extract the performance quantities can be performed by a local rational modeling technique, e.g. based on the Local Bootstrapped Total Least Squares (LBTLS) [Peum 19]. These possible improvements of the modeling time for a fixed accuracy level are discussed in more detail throughout this chapter.

### 3.3. Metamodeling and design space sampling

Building the metamodels requires also some CPU time, called the modeling time. This seems to be similar to the design by EM optimization, as building metamodels also requires some time-consuming EM simulations. Hence, it seems as if the calculation time is moved from the optimization to the modeling. However, modeling requires much less of these expensive simulations when compared with EM optimization of the filter. Besides, the CPU time required for building the metamodels is only required once for a specific microstrip structure with a specific set of user settings since the metamodels are reusable.

The metamodels are constructed following the next steps:

1. **Design space sampling**
   The design space is sampled to find a set of data samples on which the metamodels are built. Selecting a proper sampling technique is very important, especially for high dimensional problems. When the number of design parameters increases for fixed sampling technique, the curse of dimensionality leads to an exponential growth of the design space with its dimensionality and hence an exponential growth of the design space samples. Advanced techniques such as design space reduction [Kozi 14] or segmentation [Loll 09] can be used to circumvent this.
3. Filter design using metamodels

2. Metamodeling
Metamodels are built on underlying, often time-consuming, simulations. In our case, EM simulations are used as input data to the algorithms that build the metamodels. An EM simulation is performed to compute the S-parameter data at each design space sample. From each of these S-parameter data samples, the corresponding filter performance quantities are extracted. Based on these extracted data, metamodels are built that link the design parameters (metamodels’ input) to the performance quantities (metamodels’ output).

3. Model validation
Finally, the identified metamodels need to be validated to check the robustness and the quality of the obtained design. The most common technique is cross-validation [Hast 09]. Cross-validation predicts the accuracy of the model at a new data sample or interpolation point.

This section gives an overview of some sampling techniques, metamodeling techniques and validation methods that can be used. The advantages and disadvantages of these techniques are discussed to help the designer select a suitable technique. The procedure is applied to a DGS example in Section 3.4, where we discuss which of the proposed techniques is suitable for this example, depending on the different choices available to the designer, as well as the alternatives that are available for the different steps.

Design space sampling

The first step in modeling is to sample the user defined design space. Different sampling techniques exist for this purpose. The idea is to sample in a most informative way using a minimal number of samples. The most commonly used sampling techniques are space-filling, where a uniform sample distribution is proposed to cover the design space.

The most simple space-filling technique is to sample the design space over an equidistant or regular grid [Wang 07]. This is called a regular tensor-product sampling. It is a quite straightforward technique and works well for small, 2-dimensional design spaces. Suppose you want to divide each dimension in \( p \) bins. Sampling a 2D design space over \( p \) samples per dimension then gives a total number of samples or a sample size of \( n = p^2 \). However, if the dimension of the design space, \( N \), increases, the number of samples increases exponentially, \( n = p^N \). This is clearly not an efficient sampling scheme. This is visible in Figure 3.2, which shows a regular tensor-product sampling of a 2-dimensional design.
3.3. Metamodeling and design space sampling

Figure 3.2.: This figure shows a regular tensor-product sampling of a 2-dimensional design space ($N = 2$) where each dimension is divided into $p = 16$ bins. This sampling method requires a large number of samples ($n = p^N = 256$) to achieve uniformity.

For high-dimensional design spaces, **Monte Carlo (MC) sampling** [Wang 07] is a popular, simple alternative that unfortunately is also known to be inefficient. MC is a random sampling method that achieves the same amount of information as a uniform grid only with less samples. In MC sampling, every point is sampled independently. Unfortunately, this often leads to clustering as well as gaps in the design space. This is visible in Figure 3.3 (top figures), which shows a MC sampling of 2-dimensional design space ($N = 2$) where each dimension is divided into $p = 16$ bins (right). For comparison with the techniques that will be discussed next, we sample this design space using $n = p = 16$ samples. Note that this technique does not necessarily require this exact number of samples. The $N$-dimensional uniformity is poor: clustering and gaps are clearly visible. Furthermore, even the uniformity of the 1-dimensional projections is poor: some bins have no projected samples, while other bins have 3 or more. Hence, MC has poor uniformity properties.

**Latin Hypercube Sampling (LHS)** [Loh 96] is a popular alternative for MC, aiming at a more uniform distribution of the samples. Suppose, again, that you want to divide each dimension in $p$ bins. LHS only uses $n = p$ samples for these
$p^N$ bins. They are randomly selected in the design space such that exactly 1 sample is present in each bin for all 1-dimensional projections on the axes of the N-dimensional space. Hence, LHS imposes uniformity for all 1-dimensional projections, thereby having better uniformity properties than MC. This is shown in Figure 3.3 (middle figures), which shows a LHS sampling of a 2-dimensional design space ($N = 2$) where each dimension is divided into $p = 16$ bins (right). As said before, LHS only uses $n = p = 16$ samples to sample this design space and exactly 1 sample is present in each bin for the two 1-dimensional projections. However, LHS still shows poor N-dimensional uniformity properties which can possibly lead to sample clustering and design space gaps when the complete design space is considered. This is visible when we look at the left middle figure of Figure 3.3: not every square contains a sample. Hence, LHS is not the best space-filling technique, other techniques exist with a more uniform distribution of the samples in a multidimensional design space.

To achieve a more uniform distribution of the samples for multidimensional problems, the **Quasi Monte Carlo (QMC) sampling method based on Sobol sequences** can be used [Bran 14]. This is a quasi-random method that shares the properties of a random sample distribution. It shows low discrepancy properties, discrepancy being a quantitative measure for the level of deviation from the uniform sample distribution. **Low Discrepancy Sampling (LDS)** techniques lead to a sample distribution without clustering or design space gaps. This is shown in Figure 3.3 (bottom figures), which shows a QMC sampling based on Sobol sequences of a 2-dimensional design space ($N = 2$) where each dimension is divided into $p = 16$ bins (right). As LHS, QMC uses only $n = p = 16$ samples to sample this design space and exactly 1 sample is present in each bin for the two 1-dimensional projections. Additionally, QMC also imposes N-dimensional uniformity: exactly 1 sample is present in each square as shown in the left bottom figure of Figure 3.3.

In general, the information level increases with an increased sample size: the more samples that are taken, the smaller the model error becomes. However, the number of design space samples is proportional to the number of time-consuming simulations that are required to build the models. An additional problem pops up as not all the samples necessarily contain useful information. In other words, adding a sample point might not contribute a lot to the accuracy. There have been many attempts to define optimality criteria to select the optimal sample size and sample set. Unfortunately, these criteria are not always practical to determine an optimal set to reach a certain accuracy level.

Finding the optimal sample size and sample set is a not straightforward problem to solve. The required number of design space samples strongly depends on the
Figure 3.3.: This figure shows three sampling techniques to sample a 2-dimensional design space ($N = 2$) where each dimension is divided into $p = 16$ bins (right). LHS and QMC both impose uniformity of the 1-dimensional projections. Only QMC sampling imposes N-dimensional uniformity (left). This figure is copied from [Kuch 15].
3. Filter design using metamodels

smoothness CHARACTERISTICS of the system under study. Adaptive or sequential sampling and/or adaptive metamodeling can be used to reach an optimal sample size and sample set iteratively. In adaptive sampling, new samples are sequentially generated while verifying the accuracy of the achieved metamodels. MC and QMC are both sampling methods that allow the use of an adaptive sampling. However, LHS cannot be used for this purpose because it does not allow samples to be generated sequentially. Experience learns that adaptive sampling succeeds to keep the CPU time that is required to build the metamodels reasonably low.

Metamodeling

A metamodel is an approximation of a function and is constructed from underlying, often time-consuming, simulation models [Klei 08]. It is used to replace these costly simulations for a variety of reasons: design optimization, sensitivity analysis, design space exploration. For all these applications, the goal mainly remains the same: finding a global metamodel that is as accurate as possible at a reasonable cost.

Different metamodeling techniques exist. The three most used global metamodels are briefly discussed in this section: polynomial regression, Kriging and Radial Basis Functions (RBF). Which technique is preferred above another mainly depends on three factors: the dimension of the design space, the smoothness of the function to be approximated with respect to the design parameters and the degree of nonlinearity of the function to be approximated.

Polynomial regression [Forr 09, Jin 01, Klei 08] uses linear combinations of global polynomial basis functions that are usually fitted using a (weighted) least squares method. It is a widely used technique and results in models that are fairly easy to construct. The obtained model is very time-efficient to evaluate. It easily provides insight into the design space since the effect of each design parameter on the function is simply visible from looking at the size of the coefficients in the linear combinations.

Polynomial regression provides accurate and robust results if the factors are lax: functions that are smooth with respect to the design parameters, with a low order of nonlinearity and a low dimensionality. Even for three-dimensional problems, the number of basis functions can already become quite large due to the high number of possible mixed interactions, depending on the order of the polynomial model. Sparse basis functions can be used to circumvent this problem but it is very hard to know beforehand which mixed interactions will be weak enough to allow to omit them. In general, for moderate to high-dimensional problems,
strongly nonlinear problems or non-smooth problems, other techniques will be more appropriate.

**Kriging** [Wang 07, Jin 01, Forr 09] models use linear combinations of fixed basis functions that are known beforehand. Overall, Kriging is a very accurate and robust method, even for higher dimensional or strongly nonlinear problems that remain smooth with respect to the parameters. However, it performs very badly for non-smooth functions. Its main disadvantage is the time-efficiency when compared with other metamodeling techniques. Another drawback is that this technique is not straightforward to implement.

As a trade-off between Kriging and polynomial regression, **Radial Basis Functions (RBF)** modeling [Wang 07, Jin 01, Huss 02] is a good alternative. RBF uses weighted linear combinations of simple basis functions. It is fairly easy to implement and remains accurate and robust enough for high dimensional problems as well as for non-smooth problems. It performs even better than Kriging for nonlinear problems. It is also more time-efficient than Kriging, but less efficient than polynomial regression. In general, it is less accurate than Kriging, but more accurate than polynomial regression. All these properties show that RBF is indeed a good trade-off between the two former methods.

In general, the performance of a metamodeling technique depends on the sampling method. Especially Kriging and RBF are very sensitive to the selected sampling technique.

**Model validation**

Whatever sampling or metamodeling method the designer selects to build a metamodel, the quality of the estimated model needs to be assessed through a validation. The choice of a suitable validation method depends on the selected metamodeling technique. For regression models (e.g. polynomial regression), the model error returned by the (weighted) least squares estimations that were performed during the model extraction can be used to validate the achieved metamodel. When using an interpolation model (e.g. Kriging, RBF), however, a real validation is required because a measure for the model error is not available as it was never evaluated in the extraction procedure. Sometimes it might be convenient to employ a separate validation sample set, with samples that are not part of the selected design space. However, in many cases, drawing on a separate validation set is too costly and alternative methods are required.

Cross-validation is the simplest and most widely used technique to assess the quality of a model. In cross-validation, a part of the design space samples is used to
build the metamodel (estimation sample set), while another part is used to validate this model (validation sample set). There exist different types of cross-validation techniques.

*Leave-p-out* cross-validation selects \( p \) samples from the design space and uses them as a validation set. This validation is then repeated for all the possible ways to split the original sample set (the design space samples) into an estimation set and a validation set. It requires \( C^n_p \) cross-validation tests to be performed. Therefore, it is a computationally expensive method.

The *k-fold* cross-validation is a less computationally expensive alternative. It is an approximation of the *leave-p-out* method. In *k-fold* cross-validation, the sample set is randomly partitioned into \( k \) sets of approximately equal cardinality size. Then, for \( i = 1,...,k \), a metamodel is built considering all but the \( i-th \) data partition. This excluded dataset is used to evaluate the corresponding validation model error. An estimate of the metamodel error is then calculated by taking the average over the validation model errors of all the \( k \) iterations. All data samples of the original sample set (or the design space samples) were then used to achieve this estimated model error. [Hast 09]

### 3.4. Example: Defected Ground Structure

The design by metamodeling is illustrated on the DGS shown in Figure 1.3 (Section 1.3). As in Chapter 2, we again consider 3 design cases with different filter specifications for this example structure and discuss the results of each of them. We follow the design flow from Section 3.1, thereby assuming that no metamodels are available yet for the selected application. The necessity of a variability analysis is shortly discussed in this section, although performing one is out of the scope of this thesis.

**User settings**

The user needs to specify the frequency range of interest as well as the design space to be considered. As a microstrip structure we use a DGS (Section 1.3).

We use the same design space as in Chapter 2, Section 2.3. The design parameters are the width and the length of the slot (Figure 1.3), \( W_s \) and \( L_s \). The boundaries are chosen to be \([0.3 - 3]\) mm and \([80 - 101]\) mm respectively (Table 3.1). For the reasoning of these limitations we refer to Section 2.3.
Table 3.1.: The user needs to limit the design space. The boundaries of the design parameters indirectly determine the feasible filter specifications. This table gives the boundaries for the DGS example used here.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotline width, $W_s$ [mm]</td>
<td>0.30</td>
<td>3.00</td>
</tr>
<tr>
<td>Slotline length, $L_s$ [mm]</td>
<td>80.00</td>
<td>101.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feasible filter specifications</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency, $f_0$ [GHz]</td>
<td>1.19</td>
<td>1.53</td>
</tr>
<tr>
<td>Cut-off frequency 1, $f_{c1}$ [GHz]</td>
<td>0.85</td>
<td>1.11</td>
</tr>
<tr>
<td>Cut-off frequency 2, $f_{c2}$ [GHz]</td>
<td>1.45</td>
<td>2.25</td>
</tr>
<tr>
<td>Stopband amplitude, $A_s$ [dB]</td>
<td>$-32.2$</td>
<td>$-23.0$</td>
</tr>
</tbody>
</table>

Table 3.2.: We used three design cases with different filter specifications to discuss filter design by EM optimization.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency, $f_0$ [GHz]</td>
<td>1.24</td>
<td>1.35</td>
<td>1.50</td>
</tr>
<tr>
<td>Cut-off frequency 1, $f_{c1}$ [GHz]</td>
<td>0.92</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>Cut-off frequency 2, $f_{c2}$ [GHz]</td>
<td>1.55</td>
<td>1.81</td>
<td>2.10</td>
</tr>
<tr>
<td>Stopband amplitude, $A_s$ [dB]</td>
<td>$-28.0$</td>
<td>$-26.0$</td>
<td>$-22.0$</td>
</tr>
</tbody>
</table>

The boundaries of the design space indirectly determine the feasible filter specifications or performance quantities $f_0$, $f_{c1}$, $f_{c2}$ and $A_s$. These boundaries are given in Table 3.1.

The three considered design test cases have to be selected to lie within these boundaries as otherwise the design will not be feasible. We consider the same design cases as in Chapter 2 to allow a straightforward comparison of the obtained results. The selected filter specifications are repeated in Table 3.2 for the three DGS design cases.

We simulated the S-parameters of the DGS structure over a frequency range of $[0.1 – 2.5]$ GHz in 27 equidistant samples. To be able to easily extract metamodels using these simulation data, a much denser frequency grid is required to achieve accurate results. However, to reduce the simulation time we opted for a sparse
3. **Filter design using metamodels**

frequency grid for the EM simulations and the use of an intermediate model of the S-parameters. In the next subsection, a rational model is extracted for the simulated S-parameter data using the VF algorithm [Gust 99]. This model is then evaluated over a denser grid of 1001 frequency samples. Extraction and evaluation of one VF model over 1001 frequency samples takes 80ms, while one EM simulation over 1001 samples takes 300s. Hence, using VF is much more time-efficient than the time-consuming EM simulations, thereby reducing the total simulation time. To extract a rational model with an accuracy of minimum −50dB at least 27 equidistant frequency samples are required in this example.

**Construction of the metamodels**

The selection of the sampling and metamodeling technique depends on three main factors: the dimension of the design space, the smoothness of the performance quantities as a function of the design parameters and the nonlinearity of the performance quantities as a function of the design parameters. The first factor is easily identifiable since this is set by the designer in the previous step. The level of nonlinearity depends on how well the performance quantities can be modeled by a hyperplane in the design space. The smoothness of the function to be approximated is determined by the presence or absence of discontinuities in the dependence relation and/or its derivatives. Both nonlinearity level and smoothness of the function are often not well known beforehand and therefore they need extra consideration.

In our case, we have a two-dimensional design space. Since the number of dimensions is very small, we opt for a simple design space sampling with a regular or equidistant grid. We sample the design space \((W_s, L_s)\) over an equidistant grid of 49 \((7 \times 7)\) samples.

Besides the low dimensionality, the problem also proves to have a smooth dependence on the design parameters and a low-order of the nonlinearity (as shown in Figures 3.4-3.5). The conditions to use polynomial regression models to construct the metamodels are therefore met. Hence, we opt for this metamodeling technique since it is very simple to understand and easy to implement.

As mentioned in Section 3.1, the idea is to construct a metamodel for each performance quantity, linking the design parameters of the structure (input variables of the metamodels) to this particular filter performance quantity (output variable of the metamodel). As in Chapter 2, we consider 4 performance quantities. Hence, we need to construct 4 metamodels. Each of them depends on the design parameters \(W_s\) and \(L_s\), we obtain: \(f_0(W_s, L_s)\), \(f_{c1}(W_s, L_s)\), \(f_{c2}(W_s, L_s)\) and \(A_s(W_s, L_s)\).
To construct the metamodels, we need to extract the performance quantities from the S-parameter data for each of the design space samples. Hence, we need a very dense frequency grid for the S-parameters to estimate accurate values for the performance quantities. Especially $f_0$ and $A_s$ prove to require very high sample densities as the DGS has a very narrow stopband. However, performing EM simulations over a very dense frequency grid leads to an excessive growth of the simulation time.

To reduce the high simulation time, the data samples of the performance quantities are acquired in a two-step estimation. First, we use the EM-simulator to compute the S-parameter data at a well-chosen number of 27 equidistant frequency samples for each of the 49 design space samples. To this end, we use the Momentum [Keys 14b] EM solver of Advanced Design System (ADS) [Keys 14a]. Then, we extract a rational (pole-residue) model of the frequency-dependent S-parameter data for each of the 49 design space samples using a VF technique [Gust 99]. The rational VF models have the following form:

$$S_{model}(j\omega) = \sum_{q=1}^{Q_{poles}} \frac{\text{Residues}_q}{j\omega - \text{poles}_q} + D$$

(3.1)

with $Q_{poles}$ the number of poles and $D$ the residual. The residues $\text{Residues}_q$ and poles $\text{poles}_q$ are either real or come in complex conjugate pairs, while $D$ is real [Gust 99].

We evaluate these rational models over a much denser frequency grid of 1001 equidistant frequency samples to allow a more efficient and accurate estimation of the 49 data sample sets of the performance quantities. We achieve an absolute Root Mean Square Error (RMSE) (Equation (1.4)) of $-50$ dB.

Finally, we build a metamodel for each of the 4 performance quantities, $\hat{y}_j$ ($j = 1 : 4$), by using the multidimensional polynomial regression model of order 3:

$$\hat{y}_j = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1^2 + a_4 x_2^2 + a_5 x_1 x_2 + a_6 x_1^3 + a_7 x_2^3 + a_8 x_1^2 x_2 + a_9 x_1 x_2^2$$

(3.2)

In our example case, $x_1$ and $x_2$ represent the 2 design parameters $W_s$ and $L_s$ respectively.

The global polynomial basis functions are fitted using a least squares method. The resulting polynomial regression models are shown in Figures 3.4-3.5 as a function of the two design parameters $W_s$ and $L_s$. The data samples of the performance...
quantities prove to vary smoothly with respect to the design parameters and show a dependence which is close to linear. The resulting metamodels fit these samples quite well. The only exception is the metamodel \( A_s(W_s, L_s) \). There, the samples did not vary in a way that is smooth enough to allow a perfect fit by a third order polynomial regression model. However, for our purpose, this model fit is acceptable.

To validate the models, we use the model error of the least squares fit of the polynomial models. Table 3.3 gives the relative model error for each of the constructed metamodels.

<table>
<thead>
<tr>
<th>Metamodel</th>
<th>Relative model error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0(W_s, L_s) )</td>
<td>0.20</td>
</tr>
<tr>
<td>( f_{c1}(W_s, L_s) )</td>
<td>0.30</td>
</tr>
<tr>
<td>( f_{c2}(W_s, L_s) )</td>
<td>0.19</td>
</tr>
<tr>
<td>( A_s(W_s, L_s) )</td>
<td>0.37</td>
</tr>
</tbody>
</table>

**Optimization of the metamodels**

The obtained metamodels return the performance quantities or filter specifications, \( f_0, f_{c1}, f_{c2} \) and \( A_s \), as a function of the design parameters, \( W_s \) and \( L_s \). However, in a design context, we need to find the design parameters leading to a filter response that fulfills the desired specifications. Hence, the metamodels need to be inverted to be used in the design procedure. Here, the models are inverted by optimization. To this end, the optimization function \texttt{fminimax} of \texttt{MATLAB R2014a} was used.

The function \texttt{fminimax} minimizes the worst-case (largest) error value of a set of multi-variable functions, starting from an initial estimate. This is typically used when having multiple objective functions or cost functions to minimize. In our case, we have 4 objective functions, one for each of the performance quantities. The boundaries of the design space (Table 3.1) are also given to the \texttt{fminimax} function.

The starting values are the lower boundaries of our design space and are identical for the three cases: \( W_s = 0.3 \text{mm} \) and \( L_s = 80 \text{mm} \). Again, better results might be obtained if the starting values were selected randomly within the design space.
Figure 3.4.: The polynomial regression models are fit on the performance quantities at the design space samples (○). Top: $f_0 (W_s, L_s)$. Bottom: $f_{c1} (W_s, L_s)$
Figure 3.5.: The polynomial regression models are fit on the performance quantities at the design space samples (○). Top: $f_{c2}(W_s, L_s)$. Bottom: $A_s(W_s, L_s)$
3.4. Example: Defected Ground Structure

Figure 3.6.: The S-parameters of the optimized design (-----) by metamodeling fulfill the required filter template (•••) for Case 1 (Table 3.2) starting from the initial design (——).

Results

The metamodeling optimization provides the desired results, as is shown in Figures 3.6, 3.7 and 3.8 for Case 1, 2 and 3 respectively (Table 3.2). The S-parameters of the optimized designs (-----) fulfill the required filter template (•••) for each of the design cases. The initial design, that is obtained with the starting values, is shown in blue (——). The optimized design parameters are given in Table 3.4.

Table 3.4.: The metamodeling-based design optimization approach provides the desired results.

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotline width, ( W_s ) [mm]</td>
<td>0.851</td>
<td>2.74</td>
</tr>
<tr>
<td>Slotline length, ( L_s ) [mm]</td>
<td>99.9</td>
<td>94.1</td>
</tr>
<tr>
<td>Number of function evaluations</td>
<td>153</td>
<td>109</td>
</tr>
<tr>
<td>Total computation time [ms]</td>
<td>3.52</td>
<td>2.51</td>
</tr>
</tbody>
</table>

All simulations were performed in a Windows environment with 8GB RAM and Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz. The numerical results of the EM optimization are given in Table 3.4.
3. Filter design using metamodels

Figure 3.7: The S-parameters of the optimized design (—) by metamodeling fulfill the required filter template (⋯) for Case 2 (Table 3.2) starting from the initial design (—).

Figure 3.8: The S-parameters of the optimized design (—) by metamodeling fulfill the required filter template (⋯) for Case 3 (Table 3.2) starting from the initial design (—).
3.4. Example: Defected Ground Structure

Metamodeling optimization is much faster than EM optimization. The average CPU time required for one frequency-domain EM simulation (1 EM function evaluation) over 1001 frequency samples is 300s, while one function evaluation of the four metamodels takes only 23 μs on average, assuming that the metamodels are at our disposal. Multiplying this CPU time by the number of function evaluations (see Table 3.4) confirms the significant computational resources that are saved by using the proposed metamodeling technique.

Generating the metamodels requires some additional CPU time, that contains three main contributions:

1. The acquisition of the 49 design space samples requires 49 EM simulations of 10s each to generate the S-parameters over 27 frequency samples. This step requires 490s.

2. The generation of a VF model and its evaluation over 1001 frequency samples takes 80ms per design space sample. This step requires 3.92s.

3. The CPU time needed for the extraction of the data samples of the performance quantities from the S-parameters data (obtained by VF models) and the construction of the corresponding metamodels takes 0.5s.

Hence, the total CPU time needed to generate the metamodels is \((490 + 3.92 + 0.5)\)s = 494.42s = 8.24 minutes. The time needed to evaluate the metamodels lies in the order of a few milliseconds, e.g. 3.52ms for Case 1, and is therefore insignificant.

Once the metamodels are generated they can be reused in the next optimization designs of this filter type. This time the new design requires only 3.52ms, 2.51ms and 3.34ms for Case 1, 2 and 3 respectively (see Table 3.4). Hence, the time needed to generate the metamodels can be definitely accepted considering the saved computational resources and the fact that the models are reusable.

As we discussed before, the time cost for generating the metamodels can be reduced by: 1) using adaptive sampling schemes in the design space, which reduces the number of design parameter samples (and hence EM simulations) that is required to obtain accurate metamodels; 2) by using adaptive frequency sampling when simulating the S-parameters with the EM solver.

Replacing the time-consuming EM optimization by optimization of the metamodels reduces the design time a lot. Comparing Tables 2.3 and 3.4 shows a speed-up of more than a factor 50000. Filter design using metamodels solves a major drawback of design by EM optimization. However, neither of these two techniques provides much physical insight to the designer. In the next chapter, a design technique is proposed that solves this drawback.
### Variability analysis

Before the optimized design is actually fabricated, the designer needs to check its robustness or its reliability. In other words, the designer needs to verify the influence of small variations of each design parameter on the desired characteristics, e.g. the Frequency Response Function (FRF). This validation is called a *variability analysis* or a *sensitivity analysis*. In general, the sensitivity analysis estimates how the uncertainty on the output of a model, a system or a device is affected by the presence of an uncertainty on the input variables.

Calculating the variability of the FRF for all design parameters is extremely useful to check the effect of fabrication tolerances on the optimized designs. It can also help the designer to better understand the relation between the input and the output variables.

The Monte Carlo (MC) analysis has been used traditionally to conduct a variability analysis in the design flow of EM systems. This analysis is easy to implement and provides accurate results. However, it requires a large number of EM simulations to be statistically significant, which leads to a very high computational cost. In the literature, an alternative class of variability analysis techniques have already been proposed that is based on the Stochastic Collocation (SC) method [Xiu 05, Babu 07].

The metamodels that we extracted could also be used in a variability analysis setting. The metamodels can be used for a design space exploration where a small variation around one variable is considered. Of course, the metamodels should be accurate enough for this purpose, because the design space exploration is performed very locally. Many studies can already be found concerning the use of metamodels for variability analyses or global design space explorations purposes [Wang 07, Klei 08, Oste 18].

In this work, however, we only focus on the use of metamodels for optimization purposes. Studies about variability analyses are therefore not considered here. We do however find it important to mention them, because it is an important part of a design process.
4. Filter design using scalable circuit models

The results of this chapter are obtained through close collaboration with Francesco Ferranti. This chapter is published as a paper in the Special Issue of the MTT transactions linked to IMaRC [Van 18b].

In the previous chapter we used metamodeling to design a filter. The metamodels have the advantage to be reusable for different filter specifications and can also be used to conduct a variability analysis. This approach solves many drawbacks of electromagnetic (EM) optimization except for one: it does not provide much physical insight into the behavior of the structure to potential designers. To gain more physical insight, circuit models can be used in the design procedure. However, a trade-off must be made between design speed and insight when replacing metamodels by circuit models.

Section 4.1 describes the general concept of the circuit-based filter design approach used in this chapter. Section 4.2 discusses the advantages and the drawbacks of the proposed technique. Section 4.3 discusses the extraction of an equivalent circuit model for a specific example, namely the Defected Ground Structure (DGS). Section 4.4 describes the scalability of the circuit model, which is necessary for the model to be useful in a design context. Finally, Section 4.5 applies the proposed technique to the DGS example structure that was used in the previous chapters.
4. Filter design using scalable circuit models

4.1. General concept

In the literature, surrogate-assisted techniques [Band 04, Quei 05, Forr 09, Kozi 16] have been proposed for the efficient optimization of computationally expensive models, e.g. EM models of filters. In Chapter 3, we introduced filter design by metamodels that work directly on the performance quantities (or the filter specifications) of the filter of interest. This technique proves to speed up the design process enormously when compared to EM optimization without compromising on the reliability or the accuracy of the results. Constructing the metamodels requires some CPU time. Fortunately, this CPU time is only required once for a specific microstrip structure with a specific set of user settings since the metamodels are reusable in a design context. Hence, the metamodels should be constructed only once for the same application where the same microstrip structure, frequency range and design space is used. Unfortunately, the metamodels provide only a little bit more physical insight to designers when compared to design by EM optimization.

To achieve more physical insight, we introduce an equivalent circuit model into the design procedure. From a modeling point of view, the circuit representation is not the only possibility. For example, we have shown in Chapter 3 that metamodels can be built immediately on the different design quantities of interest. However, designers often desire that a model also provides a physical interpretation to feed their intuition. In the case of the EM phenomena they can really benefit from a circuit model interpretation that is more easily linked to the filter behavior.

The equivalent circuit model is an important concept in several communities: it is used from filter design to signal and power integrity. An equivalent circuit often lies at the core of the design process. Circuit interpretation is very often used to help decide on the layout of a filter by looking at the resonators, for layout corrections as tuning a resonator in circuit form is more intuitive than tuning a microstrip structure, or to tackle power and signal integrity issues in Printed Circuit Boards (PCBs) and Power Distribution Networks (PDNs). A physical interpretation helps the understanding of the kind of phenomena that are needed to achieve/suppress (anti)resonances or to achieve/suppress certain zeros/poles in the transfer function behavior.

The circuit-based design approach that we propose in this chapter is based on scalable lumped-element equivalent circuit models. The circuit model needs to be scalable to be useful in filter design. By scalable, we mean that the circuit parameters of the equivalent circuit are linked to the physical design parameters by a set of mathematical functions. To this end, we will again use a metamodeling
4.1. General concept

technique, but this time the output variables will be the circuit parameters instead of the performance quantities. The input variables are again the design parameters, as in the previous chapter. Hence, the metamodels return the circuit parameters in function of the design parameters.

The scalable circuit model is introduced in the design procedure as an intermediate step between the filter specifications and the transmission line (TL) structure (or microstrip structure), shown graphically in Figure 4.1. Note that from now on we will use the term scalable circuit model when we mean the combination of the circuit model and the metamodels in the design flow. The circuit model can be optimized to find the optimized circuit parameters as a function of the desired filter specifications (shown in green). The metamodels return the circuit parameters as a function of the design parameters. Hence, they need to be inverted to be able to use them in the design procedure. Inverting the metamodels is performed numerically to find the design parameters as a function of a set of circuit parameters (shown in green) that at their turn correspond to a set of desired filter specifications via the circuit model. Note that constructing the scalable circuit model should be done only once for one particular microstrip structure and a particular set of user settings. Therefore, this is shown in (gray) since it is not always part of the standard design procedure using scalable circuit models. The different steps that are used in the design by scalable circuit modeling are listed below:

1. **User settings**
   Starting from the filter specifications and the application at hand, the user should select a proper microstrip structure, the frequency range and the design space.

2. **Construction of the scalable circuit model**
   If a scalable circuit model does not yet exist for the selected microstrip structure, a new scalable circuit model should be extracted. The extraction of an equivalent circuit model is discussed in Section 4.3 for the DGS. The scalability of this model is discussed in Section 4.4.

3. **Optimization of the scalable circuit model**
   The circuit model should be optimized to obtain an accurate fit within the selected design space. This is mandatory to find the design parameters corresponding to the required set of filter specifications. This modeling can be split into two steps. First, the circuit model is optimized to find the circuit parameters corresponding to the required filter specifications. Next, the metamodels are optimized to find the design parameters that
correspond to the optimized circuit parameters that were obtained in the previous optimization.

4. Variability analysis
After finding an optimized design, a variability analysis should be performed to verify the robustness of the design. Using the metamodels can also speed-up the variability analysis significantly.

Figure 4.1.: Filter design using scalable circuit models is accurate and fast. Furthermore, it provides physical insight to the designer into the internal EM operation of the filter.

To make the equivalent circuit model scalable, metamodeling will be used. A more suitable metamodeling technique is required here than the ones discussed before in Section 3.3. The data extraction is not as simple as in the previous chapter because here an independent optimization is involved: the number of degrees of freedom of the model is larger than the number of physical design parameters, since the number of circuit parameters of the equivalent circuit model is larger than the number of design parameters of the microstrip structure. An independent optimization process often leads to noisy data. For a global metamodel to take into account this noisy data, you need a large amount of basis functions and coefficients. Therefore, a local metamodeling technique would be more suitable since it is piece-wise continuous and hence, can handle the noisy data better.
However, the techniques discussed in Section 3.3 are all global metamodeling techniques. Section 4.4 therefore explains the local metamodeling technique and the associated sampling method used in this chapter.

In Section 4.5, the proposed design procedure is applied to a DGS example. In the previous chapters (2 and 3), a 2-dimensional DGS example was discussed. In order to properly validate the scalable circuit model, we opt for a 4-dimensional design space in this chapter. The design flow of Figure 4.1 is thereby followed.

### 4.2. Advantages and drawbacks

The scalable circuit model has an important advantage over the previously discussed techniques. It provides the physical insight the designer needs concerning the EM filter’s internal operation, while EM optimization [Step 01] (Chapter 2) or metamodeling of performance quantities [Van 17] (Chapter 3) do not provide this essential design support tool.

The scalable circuit model is much faster to evaluate when compared to the time-consuming EM simulations. However, it is somewhat slower to evaluate than the metamodels that were used in Chapter 3.

Constructing the scalable circuit model also takes more time than constructing the metamodels used in the previous chapter. However, the scalable circuit model is also reusable for different filter designs that use the same microstrip structure and the same user settings (frequency range and design space). Hence the extra CPU time is only needed once for a particular structure and a particular set of user settings. The construction and the identification of the equivalent circuit model are discussed in Section 4.3.

### 4.3. Extraction of the equivalent circuit model

In order to use a circuit model in a filter design procedure, it should be able to represent the EM behavior of the selected filter structure with high accuracy. Equivalent circuit models can often be found in the literature for many microstrip structures. If such a model is accurate enough to meet the design requirements of the user, it can be used immediately in the proposed design procedure. However, these models are often oversimplified and their accuracy is therefore often very limited. Most of these models predict only a part of the filter’s behavior, thereby neglecting all parasitic effects. Luckily, an existing model can often be used as a
starting point and can then be fine-tuned/adjusted to achieve the wanted accuracy level. The minimum accuracy level we want to achieve is given by an absolute Root Mean Square Error (RMSE) of at most \(-25\) dB (Chapter 1).

In this section we analyze how an equivalent circuit model was extracted for the DGS described in Section 1.3. More in particular we will focus here on the DGS as shown in Figure 1.3, where the feedpoint of the slotline is positioned in the middle of the slot. In this chapter (Section 4.5), we study the same example design case as in Chapters 2 and 3. There we considered a frequency band of interest of \([0.1 – 2.5]\) GHz. However, since we plan to use the circuit model for wide-band modeling in Chapter 6, we will already analyze the structure’s behavior over a wider frequency range of \([0.1 – 5]\) GHz here.

To extract an equivalent circuit model for the DGS, we start from a model that is commonly used in the literature. However, the models suffer a lack of accuracy and is therefore not suitable to be used in a design context. As a first step, we improve this literature model to improve its accuracy.

**The literature model**

As a starting point, we use an equivalent circuit topology that was proposed in the literature [Calo 04]. This model is shown in Figure 4.2. It is used as an initial guess for the accurate circuit model we want to achieve.

![Equivalent circuit model proposed in the literature [Calo 04].](image)

This literature model describes an elementary section of the DGS. This elementary section is shown in Figure 4.3 on the right. The S-parameters of this section are obtained after a de-embedding step to remove the delay contribution of the signal transmission line that is present before and after the elementary section (shown in Figure 4.3 on the left).

The delay complicates the modeling of the complete DGS. Therefore, all models that are discussed in these work model the elementary section instead of the
4.3. Extraction of the equivalent circuit model

De-embedding of a DGS is necessary to achieve an elementary section (right) where the delay contribution of the signal transmission line has been removed. Complete DGS. However, note that the results shown in the remainder of this thesis always show the S-parameters of the complete DGS and of the model embedded in the delay lines. The embedding and de-embedding was calculated in MATLAB as a mathematical pre- and post-multiplication of the T-matrix of the microstrip structure or the model with the T-matrix of an ideal delay line with an electrical length corresponding to the physical length of the transmission line in the DGS. Figures 4.4 and 4.5 show the S-parameters of the complete DGS (—) and of an elementary section of the DGS where the delay is removed (—).

The transmission line that is present in the DGS is represented by a series inductance, \( L_0 \), and a shunt capacitance, \( C_0 \). Note that this is nothing else than an elementary section representation of a lossless transmission line by the Telegrapher’s equations [He 93].

An ideal transformer with turns ratio \( n \) represents the coupling between the transmission line and the slotline of the DGS. The slotline itself is represented by an ideal transmission line that is connected to ground on both ends and tapped along the line to the transformer. The position of the tap corresponds to the position of the feedpoint of the slotline in the DGS. The ideal transmission line that represents the slot is described by its characteristic impedance, \( Z_{\text{slot}}^{0} \), and its electrical length, \( \theta = \theta_1 + \theta_2 \), where \( \theta_1 \) and \( \theta_2 \) define the position of the tap. In the
4. Filter design using scalable circuit models

![Graph showing S11 vs Frequency](image)

Figure 4.4: \( S_{11} \) of the complete DGS (---) and of an elementary section of the DGS where the delay contribution is removed (----). Top: amplitude response. Bottom: phase response.

example case considered in this chapter (Figure 1.3) we have \( \theta_1 = \theta_2 \), since the slotline is positioned symmetrically with respect to the transmission line.

The DGS is simulated over a frequency band from 0.1GHz up to 5GHz. The geometrical parameters of the DGS are the transmission line width, \( W \), the slot width, \( W_s \), and the slot length, \( L_s \) (Section 1.3). The geometrical parameter values used in the EM simulation can be found in Table 4.1. These values are selected as follows:

- \( W \) is selected to obtain a 50\( \Omega \)-characteristic impedance of the transmission line, \( Z_{c}^{TL} = 50 \Omega \)

- \( L_s \) is directly related to the fundamental frequency \( f_0 \) and is selected such that the third harmonic response is below our maximum simulation frequency of 5GHz (see Section 1.3)

- \( W_s \) has no imposed conditions so we selected a common value for this parameter
4.3. Extraction of the equivalent circuit model

The corresponding electrical parameter values of the literature model in Figure 4.2 are obtained by the equations that are provided in [Calo 04]:

\[ n = \sqrt{\frac{Z_{in}}{Z_L}} \approx \sqrt{\frac{Z_{TL}}{Z_{slot}}} \]  \hspace{1cm} (4.1)

\[ C_0 \approx \frac{\varepsilon_{eff}^{TL} l}{c_0 Z_c^{TL} N_s} \]  \hspace{1cm} (4.2)

\[ L_0 = C_0 (Z_c^{TL})^2 \]  \hspace{1cm} (4.3)

with \( c_0 \) the speed of light in free space, \( \varepsilon_{eff}^{TL} \) the effective permittivity of the transmission line (TL), \( l \) the length of the de-embedded transmission line and \( N_s \) the number of slots. The electrical length of the slotline, \( \theta \), is given by

\[ \theta = \beta L_s = \frac{2\pi f_0 \sqrt{\varepsilon_{eff}^{slot}}}{c_0} L_s \]  \hspace{1cm} (4.4)

Figure 4.5.: \( S_{21} \) of the complete DGS (—) and of an elementary section of the DGS where the delay contribution is removed (—). Top: amplitude response. Bottom: phase response.
4. Filter design using scalable circuit models

Table 4.1.: Selected geometrical parameter values of the DGS.

<table>
<thead>
<tr>
<th>$W$ [mm]</th>
<th>$W_s$ [mm]</th>
<th>$L_s$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>1</td>
<td>80</td>
</tr>
</tbody>
</table>

with $\beta$ the propagation constant of the slotline and $\varepsilon_{eff}^{slot}$ the effective permittivity of the slotline.

The fundamental frequency and its harmonic responses are then given by [Calo 04] as:

$$mf_0 \approx m \frac{c_0}{2L_s \sqrt{\varepsilon_{eff}^{slot}}}$$

with $f_0$ the fundamental frequency and $m$ the resonance mode.

In the above equations, we use the selected geometrical parameter values given in Table 4.1 for the considered example. The corresponding effective permittivity and the characteristic impedance of the microstrip transmission line, $\varepsilon_{eff}^{TL} = 2.82$ and $Z_{c}^{TL} = 50\Omega$ for a Rogers 4003 substrate (Section 1.3), are calculated using the closed-form expressions of [Hamm 75], formalized in [Garg 13]. Note that $W = 3.4\text{mm}$ was selected such that $Z_{c}^{TL} = 50\Omega$. The effective permittivity and the characteristic impedance of the slotline, $\varepsilon_{eff}^{slot} = 1.54$ and $Z_{c}^{slot} = 102\Omega$, are calculated using the closed-form expressions of [Jana 86], formalized in [Garg 13].

Table 4.2.: Electrical parameter values of the literature model in Figure 4.2 corresponding to the selected geometrical parameter values in Table 4.1.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C_0$ [pF]</th>
<th>$L_0$ [nH]</th>
<th>$Z_{c}^{slot}$ [\Omega]</th>
<th>$f_0$ [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.702</td>
<td>0.168</td>
<td>0.420</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>0.710</td>
<td>0.234</td>
<td>0.560</td>
<td>188</td>
</tr>
</tbody>
</table>

Table 4.2 gives the electrical parameter values of the simulated literature model. These values were obtained in two steps. First, they were calculated using Equations (4.1)-(4.5), resulting in the values given in Table 4.2 (Calculated values). However, these values lead to a poor model fit. They need to be tuned or optimized significantly to get a good model fit of the microstrip structure. The
resulting optimized parameter values are given in Table 4.2 (Optimized values). The optimized values were obtained via a Quasi-Newton optimization that is available in Advanced Design System (ADS). The cost function minimizes the error between the complex $S_{11}$- and $S_{21}$-parameters of the model and the EM-simulated microstrip structure.

**Results of the literature model**

Figures 4.6, 4.7 and 4.8 compare the simulation results of the amplitude and phase response of the $S_{11}$- and the $S_{21}$-parameters of the DGS (—) and the literature model (—). The absolute model error $\varepsilon_{abs}$ is shown in (—) (Equation (1.2)) and is expressed in dB.

![Graph showing comparison of S11 parameter](image)

Figure 4.6.: Comparison of the $S_{11}$-parameter of the simulated DGS (—) and the literature model (—). The absolute model error (Equation (1.2)) is shown in (—) and is above the desired error level of $-25\,\text{dB}$ (—) for $f > 1.48\,\text{GHz}$. Top: amplitude response. Bottom: phase response.
4. Filter design using scalable circuit models

The DGS has multiple antiresonances. These appear at the fundamental frequency and at its odd multiples because the feedpoint is positioned at $1/2$ of the slotline, as was explained in Section 1.3.

The absolute RMSE (Equation (1.4)) of $S_{11}$ and $S_{21}$ over the simulated frequency band is $-14.1 \text{ dB}$ and $-18.2 \text{ dB}$ respectively, while the relative RMSE (Equation (1.5)) is only $-5.03 \text{ dB}$ and $-9.61 \text{ dB}$ respectively. This is higher than the maximum absolute RMSE of $-25 \text{ dB}$ we want to achieve. The results of the literature model fit are listed here:

- The magnitude as well as the phase of $S_{11}$ are poorly modeled, especially in-between the two antiresonances $f_0 = 1.48 \text{ GHz}$ and $3f_0 = 4.39 \text{ GHz}$.

- The phase of $S_{21}$ is poorly modeled at the antiresonances $f_0$ and $3f_0$, but is fitted quite good in-between the two antiresonances.

Figure 4.7.: Comparison of the $S_{21}$-parameter of the simulated DGS (—) and the literature model (——). The absolute model error (Equation (1.2)) is shown in (---) and is above the desired error level of $-25 \text{ dB}$ (——) for $f > 1.48 \text{ GHz}$. Top: amplitude response. Bottom: phase response.
4.3. Extraction of the equivalent circuit model

Figure 4.8.: Complete amplitude response of the $S_{21}$-parameter of the simulated DGS (—) and the literature model (--). The insertion loss at $f_0 = 1.48\,\text{GHz}$ is much smaller for the model ($-317\,\text{dB}$) than for the DGS ($-26\,\text{dB}$). The same goes for the insertion loss at $3f_0$.

- The insertion loss at $f_0$ and $3f_0$ is much smaller ($-317\,\text{dB}$, which equals the level of the empirical precision) for the model than for the DGS ($-26\,\text{dB}$) (Figure 4.8). This means that the quality factors or Q-factors of the antiresonances of the literature model are different from the Q-factors of the simulated DGS.

- The second antiresonance of the model ($3f_0 = 4.44\,\text{GHz}$) is shifted with respect to the antiresonance of the DGS ($3f_0 = 4.39\,\text{GHz}$). Hence, the model is not good enough to model a DGS over a wider frequency band.

All these aspects can be explained by the fact that the literature model is lossless while the DGS is not. The accuracy of the model is not good enough to be used in a design context.

The absolute model error, $\varepsilon_{abs}$ (—) (Equation (1.2)), of both $S_{11}$ and $S_{21}$ is low at low frequencies ($< f_0 = 1.48\,\text{GHz}$) but increases significantly for frequencies above $f_0$. This is explained by the fact that the model is lossless while the DGS is clearly lossy. The losses of a two-port are defined as

$$1 - (S_{11}(j\omega_i)S_{11}^*(j\omega_i) + S_{21}(j\omega_i)S_{21}^*(j\omega_i))$$  \hspace{1cm} (4.6)

with, $S_{kl}^*$ the complex conjugate of S-parameter $S_{kl}$. This is based on the properties of a lossless passive two-port network concerning energy conservation [Bele 68]:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$
|\( S_{22} |^2 + |S_{12} |^2 = 1 \)

Figure 4.9 compares the losses, calculated by Equation (4.6), of the DGS (—) and the literature model (—). The losses of the DGS are clearly present above \( f_0 = 1.48 \text{GHz} \), while the losses of the literature model are close to zero over the whole frequency band.

We can summarize the results of the literature model as follows:

- The model proposed in [Calo 04] is basically correct for any lossless DGS. Hence, it can be used in a wide range of applications as long as the losses of the DGS are not considered. However, the model is not accurate enough to be used in the filter design process as we intend to. To solve this, losses need to be included.

- Next to a high accuracy, the model could be simplified by replacing the ideal transmission line by lumped elements to avoid a mixed lumped-distributed design.

- Furthermore, the S-parameters of the DGS are reciprocal \((S_{12} = S_{21})\) and symmetrical \((S_{11} = S_{22})\). However, the model proposed in [Calo 04] proves to be an asymmetrical network \((S_{11} \neq S_{22})\). This is shown in Figure 4.10.

- The equations given in [Calo 04] for the calculation of the electrical parameter values are not accurate. Tuning or even optimization is necessary to achieve a better model fit.
4.3. Extraction of the equivalent circuit model

![Graph showing real and imaginary parts of S_ii](image)

Figure 4.10.: The literature model proves to be an asymmetrical network. The real part (top) and imaginary part (bottom) of the complex reflection parameters $S_{11}$ (—) and $S_{22}$ (—) are shown.

- The model is not suitable to be used over a wide frequency band since the absolute error increases with the frequency and the second antiresonance is shifted with respect to the one of the DGS (see Figures 4.7 and 4.8).

Clearly, the model should be improved to be useful in a design context.

**The improved model**

We will improve the model in a first step by imposing symmetry. This is simply obtained by adding a shunt capacitance and a series inductance on the left side of the transformer in Figure 4.2. To keep the same model response, the values of the capacitance $C_0$ and the inductance $L_0$, given by Equations (4.2) and (4.3), are divided by a factor 2 before adding them at both sides (inductors in series, capacitors in parallel). The final parameters $L$ and $C$ (see Figure 4.11) are then redefined as

$$L = \frac{L_0}{2}$$
Next, the ideal transmission line is replaced by an LC-resonator (Figure 4.11). The ideal transmission line allows the modeling of multiple antiresonances \( f_0 \) and its harmonic responses, while the LC-resonator models only one antiresonance:

\[
f_0 = \frac{1}{2\pi \sqrt{L_{\text{res}}C_{\text{res}}}}
\]

Therefore, this approximation is band-limited around the fundamental frequency, \( f_0 \). Hence, by introducing this approximation, the obtained model becomes band-limited too.

Let us keep in mind we want to use the circuit model for wide-band modeling of the DGS in Chapter 6. Note that the model achieved so far can easily be adjusted to model multiple antiresonances by adding an LC-resonator for each extra antiresonance. The detailed extension to a wide-band model is covered in Chapter 6.

The model can be further simplified by removing the transformer as follows. The central part of the circuit model is a 2-port, as defined in gray in Figure 4.12 on the left. It consists of a cascade of a parallel LC-resonator and a transformer. The impedance matrix of this 2-port can be rewritten as a series impedance, since \( V_1 = 0 \). The ABCD-matrix of this 2-port is calculated as follows:

\[
\begin{pmatrix}
V_2 \\
-I_2
\end{pmatrix} = \begin{pmatrix}
A & B \\
C & D
\end{pmatrix}_n \begin{pmatrix}
A & B \\
C & D
\end{pmatrix}_{LC} \begin{pmatrix}
V_1 \\
I_1
\end{pmatrix}
= \begin{pmatrix}
n & 0 \\
0 & 1/n
\end{pmatrix} \begin{pmatrix}
1 & -j\omega L_{\text{old}}/L_{\text{res}} n \\
0 & \frac{1}{1 - \omega^2 C_{\text{old}} L_{\text{old}}/L_{\text{res}}}
\end{pmatrix} \begin{pmatrix}
V_1 \\
I_1
\end{pmatrix}
\]
4.3. Extraction of the equivalent circuit model

Since \( V_1 = 0 \), we get the following:

\[
\begin{pmatrix}
V_2 \\
-I_2
\end{pmatrix}
= \begin{pmatrix}
n & -j\omega L_{\text{old res}} n \\
0 & 1 - \omega^2 C_{\text{old res}} L_{\text{old res}}
\end{pmatrix}
\begin{pmatrix}
0 \\
I_1
\end{pmatrix}
\]

\[
\Rightarrow \begin{cases}
V_2 = -j\omega L_{\text{res}} n \\
I_2 = \frac{L_{\text{old res}}}{n}
\end{cases}
\]

If we replace \( I_1 = -I_2 n \) we can rewrite the impedance matrix to a series impedance as follows:

\[
V_2 = \frac{j\omega L_{\text{old res}} n^2}{1 - \omega^2 C_{\text{old res}} L_{\text{old res}}} I_2
\]

\[
= \frac{j\omega L_{\text{old res}} n^2}{1 - \omega^2 C_{\text{old res}} L_{\text{old res}} n^2} I_2
\]

\[
= \left( \frac{1}{j\omega L_{\text{new res}} n^2} + j\omega C_{\text{new res}} n^2 \right)^{-1} I_2
\]

The series impedance represents a parallel LC-resonator, as shown in Figure 4.12 on the right hand side. The new element values become:

\[
L_{\text{new res}} = L_{\text{old res}} n^2
\]

\[
C_{\text{new res}} = \frac{C_{\text{old res}} n^2}{n^2}
\]

Note that this lossless representation of a DGS by an LC-resonator can also be found in the literature [Bout 09, Hong 05, Ahn 01].

The final step of the model improvement is to introduce losses into the model to obtain a higher accuracy. The losses of the DGS (---), defined in Equation (4.6), are observed in Figure 4.9. It is clear that they are frequency-dependent. This can be expected because of the presence of the skin-effect for the transmission line and
more importantly because of the radiation losses of the slotline. The dielectric losses seem to be negligible for the substrate used. Both conductor losses [He 93] and radiation losses are frequency-dependent [Posc 08, Yosh 01]. That means that the included lumped losses also need to depend on the frequency to achieve a proper accuracy.

The losses introduced by the skin-effect depend on the frequency in a square-root sense. Therefore, the conductor losses are introduced into the model by a series impedance given by [He 93]

\[
R(f) = R_{\text{cond}} \sqrt{\frac{j(2\pi f)}{2\pi f_{\text{ref}}}} = R_{\text{cond}} \sqrt{\frac{j\omega}{\omega_{\text{ref}}}}
\]

(4.9)

with \(f_{\text{ref}} = 1.5\)GHz, the reference frequency that we add as a normalization.

The dielectric losses, presented in the Telegrapher’s representation by a shunt conductance, \(G\), are negligible for the substrate used.

The radiation losses of the slot are introduced in the model by using a frequency-dependent resistive element. This resistive element, \(R_{\text{res}}(f)\), is connected in parallel with the LC-resonator. The form of \(R_{\text{res}}(f)\) was identified to be

\[
R_{\text{res}}(f) = R_{\text{rad}} \left( \frac{f}{f_{\text{ref}}} \right)^{\alpha}
\]

(4.10)
with \( f_{\text{ref}} = 1.5 \text{GHz} \), representing the same normalization frequency as in Equation (4.9).

\( R_{\text{res}}(f) \) consists of a frequency independent factor \( R_{\text{rad}} \) that is multiplied by the normalized frequency to the power \( \alpha \). Since the dependence in frequency is related to some design parameters (the length and the width of the slot), it can not be fixed when used in filter design. Therefore, the frequency-dependence is modeled by a coefficient \( \alpha \) that needs to be tuned or optimized. Figure 4.13 finally shows the resulting band-limited model.

![Figure 4.13: Extended band-limited geometry-dependent circuit model.](image)

The obtained equivalent circuit model (Figure 4.13) consists of two parts: an RLC-resonator to model the resonating slot of the DGS and a Telegrapher’s representation of the transmission line of the DGS. Note that the RLC-resonator in the equivalent circuit allows to easily explain the DGS characteristics that were described in Section 1.3. The fundamental frequency is defined by the inductance, \( L_{\text{res}} \), and the capacitance, \( C_{\text{res}} \), of the RLC-resonator as

\[
f_0 = \frac{1}{2\pi \sqrt{L_{\text{res}} C_{\text{res}}}} \tag{4.11}
\]

A more detailed explanation about these characteristics and the link to an RLC-resonator is found in the literature [Garg 13].

References exist where radiation losses have been taken into account [Safw 06, Woo 06, Mand 06]. However, the losses in these references are independent of the frequency and they will not properly represent the frequency-dependent radiation losses of the slot of the DGS.
4. Filter design using scalable circuit models

Results of the improved model

Figures 4.14 and 4.15 compare the simulation results of the amplitude and phase response of the simulated $S_{11}$- and the $S_{21}$-parameters of the band-limited model shown in Figure 4.13 (—) and the EM simulation of the DGS (—). The absolute model error $\varepsilon_{abs}$ (Equation (1.2)) is shown in (—) and is expressed in dB.

![Graph showing comparison of $S_{11}$ and $S_{21}$ parameters](image)

Figure 4.14.: Comparison of the $S_{11}$-parameter of the DGS (—) and the band-limited model (—). The absolute model error (Equation (1.2)) is shown in (—) and is below the desired error level of $-25$ dB (—) over $[0.1 - 2.5]$ GHz. Top: amplitude response. Bottom: phase response.

The circuit parameter values of the model are again obtained by the Quasi-Newton optimization in ADS. The cost function minimizes the error of the complex difference between the Em simulated and modeled $S_{11}$- and $S_{21}$-parameters. The obtained values for the circuit parameter are given in Table 4.3. Formulas can be found in the literature for some of these circuit parameters. The parameter values for $C$ and $L$ can be calculated by Equations (4.2) and (4.3) [Calo 04] where $l$ is divided by a factor 2. The parameter values for $C_{res}$ and $L_{res}$ can be obtained from the full-wave simulation of the structure as [Garg 13]
4.3. Extraction of the equivalent circuit model

\[ C_{res} = \frac{\omega_c}{2Z_{0}^{TL}(\omega_0^2 - \omega_c^2)} \]  
(4.12)

\[ L_{res} = \frac{1}{\omega_0^2 C_{res}} \]  
(4.13)

with \( \omega_0 \) the stopband angular frequency, \( \omega_c \) the cut-off angular frequency and \( Z_{0}^{TL} \) the characteristic impedance of the transmission line.

These formulas can be used to find initial values for the circuit parameters \( L \), \( C \), \( L_{res} \) and \( C_{res} \). However, using these do not lead to a very accurate result and besides, no formulas exist (yet) for an initial calculation of the loss parameters \( R_{cond} \), \( R_{rad} \) and \( \alpha \). Hence, optimization is still required to achieve the desired accuracy.

Table 4.3.: Optimized parameter values of the band-limited model in Figure 4.13 corresponding to the geometrical parameter values in Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>[ \text{pF} ]</th>
<th>[ \text{nH} ]</th>
<th>[ \Omega ]</th>
<th>[ \text{pF} ]</th>
<th>[ \text{nH} ]</th>
<th>[ \Omega ]</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0322</td>
<td>1.12</td>
<td>0.883</td>
<td>1.96</td>
<td>5.87</td>
<td>232e1</td>
<td>−5.56</td>
<td></td>
</tr>
</tbody>
</table>

As expected, the model is only accurate over a frequency band that is centered around the fundamental frequency, \( f_0 = 1.48 \text{GHz} \). The accuracy remains high up to the second harmonic response, \( 2f_0 \approx 3 \text{GHz} \).

The absolute RMSE (Equation (1.4)) of \( S_{11} \) and \( S_{21} \) over the frequency band of interest \([0.1 - 2.5]\text{GHz}\) is \(-42.1\text{dB}\) and \(-47.3\text{dB}\) respectively. The relative RMSE (Equation (1.5)) is \(-29.0\text{dB}\) and \(-35.9\text{dB}\) respectively. The absolute RMSE is well below the desired error of \(-25\text{dB}\).

It is clear that introducing radiation losses is necessary to achieve a high model accuracy. This is shown in Figure 4.16, which shows the absolute model error between the DGS simulation response (—) and the completely lossless model (——), the model with only conductor losses (---), the model with only radiation losses (——) and the model of Figure 4.13 with both losses included (——). Without radiation losses (——), we achieve an absolute RMSE of only \(-22.4\text{dB}\) and \(-22.3\text{dB}\) and a relative RMSE of \(-12.4\text{dB}\) and \(-14.3\text{dB}\) for \( S_{11} \) and \( S_{21} \) respectively over the frequency band of \([0.1 - 2.5]\text{GHz}\).

Table 4.4 gives an overview of the absolute and relative RMSE for the considered lossy and lossless models as well as the for the literature model. The improved,
4. Filter design using scalable circuit models

Figure 4.15: Comparison of the $S_{21}$-parameter of the DGS (—) and the band-limited model (—). The absolute model error (Equation (1.2)) is shown in (—) and is below the desired error level of $-25\text{dB}$ (—) over $[0.1 - 2.5]$ GHz. The insertion loss at $f_0$ is now equal for both model and simulation. Top: amplitude response. Bottom: phase response.

The improved model without radiation losses is almost as bad as the literature model. The conductor losses seem to have a small influence over the frequency band of $[0.1 - 2.5]$ GHz. However, Chapter 6 will prove the necessity of the conductor losses when modeling the structures over a wider frequency band.

Figure 4.17 compares the losses of EM simulated the DGS (—) to the losses from the equivalent circuit model (—) of Figure 4.13. The losses are calculated by Equation (4.6) and are modeled quite well up to a frequency of $3.2\text{GHz}$.

The improved model in Figure 4.13 will be used to design filters with a filter template specified over a frequency range of $[0.1 - 2.5]$ GHz in the remaining of this chapter. In this range, $S_{11}$ as well as $S_{21}$ are modeled well enough to be used in filter design. In Chapter 6, the circuit model is extended to a wide-band model.
4.3. Extraction of the equivalent circuit model

Figure 4.16.: Comparison of the absolute model error between the DGS (---), the completely lossless model (----), the model with only conductor losses (-----), the model with only radiation losses (-----) and the model of Figure 4.13 with both losses included (-----) for the $S_{11}$-parameter (top) and the $S_{21}$-parameter (bottom). The absolute model error is below the desired error level of $-25\,\text{dB}$ (-----) over $[0.1 - 2.5]\,\text{GHz}$ for the model with only radiation losses (-----) and the model with both losses included (-----).

To use the proposed equivalent circuit model in filter design, we need to make the circuit model scalable. Therefore, the circuit parameters of the model need to be related to the design parameters of the DGS by mathematical functions. The generation of these so-called scalable circuit models is described in the next section.
4. Filter design using scalable circuit models

Figure 4.17.: The losses of obtained equivalent circuit model (—) fit the losses of the DGS ($\approx -12.5\, \text{dB}$ above $f_0 = 1.48\, \text{GHz}$, —) quite well up to 3.20GHz.

Table 4.4.: Introducing losses in the model is necessary to achieve a high accuracy. This table gives the absolute and relative RMSE between the simulated DGS and the literature model, the improved model (Figure 4.13) with both losses, with only radiation losses, with only conductor losses and without losses. The RMSE was calculated over the frequency band of [0.1 – 2.5]GHz.

<table>
<thead>
<tr>
<th></th>
<th>$\text{RMSE}_{\text{abs}}$ [dB]</th>
<th>$\text{RMSE}_{\text{rel}}$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{11}$ $S_{21}$</td>
<td>$S_{11}$ $S_{21}$</td>
</tr>
<tr>
<td>Improved model, both losses</td>
<td>–42.1 –47.3</td>
<td>–29.0 –35.9</td>
</tr>
<tr>
<td>Improved model, only radiation losses</td>
<td>–42.1 –47.0</td>
<td>–29.0 –35.8</td>
</tr>
<tr>
<td>Improved model, only conductor losses</td>
<td>–22.4 –22.3</td>
<td>–12.4 –14.3</td>
</tr>
<tr>
<td>Improved model, lossless</td>
<td>–22.3 –22.2</td>
<td>–12.3 –14.3</td>
</tr>
</tbody>
</table>
4.4. Scalability of the circuit model

In this chapter, we opt for a 4-dimensional design space to validate the proposed design by use of a scalable circuit model. We select three geometrical design parameters: \( W, W_s, L_s \) (see Figure 1.3) and one substrate property, the relative permittivity \( \varepsilon_r \).\(^1\) The equivalent circuit model that was extracted in Section 4.3, is composed of 7 circuit parameters: \( L, C, R_{\text{cond}}, R_{\text{rad}}, \alpha, L_{\text{res}} \) and \( C_{\text{res}} \) (see Figure 4.13).

The relation between the design parameters of the microstrip structure and the circuit parameters of the model is not smooth because of its inherent indeterminacy: 7 circuit parameters and only 4 design parameters. Lowering the number of degrees of freedom in the model is necessary to smoothen this relationship. Some circuit parameters can be fixed to lower the degrees of freedom or a relation between the parameters can be imposed. The capacitance \( C \) represents the capacitance between the two conductor layers on the top and bottom of the substrate. Since the thickness of the substrate, \( h \), is not considered as a design parameter, \( C \) can be fixed to 0.0322\( \mu \)F. \( R_{\text{cond}} \) represents the conductor losses. Within the selected design space, the conductor losses do not change significantly. Therefore, \( R_{\text{cond}} \) can be fixed to 0.883\( \Omega \).

Finally, 5 circuit parameters remain: \( L, R_{\text{rad}}, \alpha, L_{\text{res}} \) and \( C_{\text{res}} \). However, when we map 4 design parameters to 5 circuit parameters, there is still one degree of freedom remaining. Fixing another circuit parameter is not possible because this leads to inaccurate results. Although fixing \( C \) and \( R_{\text{cond}} \) led to a decrease in degrees of freedom, the relation between design parameters and circuit parameters is still not very smooth when \( C \) and \( R_{\text{cond}} \) are fixed.

The metamodeling techniques discussed in Section 3.3 are based on global basis functions that cover the complete design space. Combined with a non-smooth mapping, this can lead to multiple local minima. To avoid this problem, we need to use a metamodeling technique based on local basis functions. An associated local sampling method is also required. This section explains the sampling and the metamodeling method based on local basis functions that is used here.

**Design space sampling**

First, the design space should be sampled. Each design space sample corresponds to a set of circuit parameter values. This set is used as a data sample to build

\(^1\)The boundaries of the design space are discussed in the next section (Section 4.5) and are not important to understand the philosophy of this section.
the scalable circuit model. Different sampling techniques exist to select a set of design space samples, for example the Latin Hypercube Sampling (LHS) [Loh 96] or the Quasi Monte Carlo (QMC) sampling methods [Bran 14], which were both discussed in Section 3.3. Sobol and Halton sequences [Bran 14] are famous among the QMC methods. Simple sampling schemes such as a regular tensor-product sampling will immediately result in a high number of design samples, that increase exponentially with the number of dimensions of the design space.

In this work, we have chosen to use the QMC sampling scheme based on Sobol sequences [Bran 14] as this is a local sampling method. As discussed in Section 3.3, Sobol QMC sampling is a Low Discrepancy Sampling (LDS) technique, achieving multidimensional uniformity. The sample distribution is uniform over the whole design space but also for every subdivision of the design space. Hence, Sobol sequences can be evaluated locally. This is necessary for evaluating the local basis functions of the metamodeling technique discussed next.

We note that the QMC sampling based on Sobol sequences should not necessarily be decided on a fixed number of samples. It can be implemented in an iterative way [Wang 07], called adaptive or sequential sampling (Section 3.3), to reach an optimal sample set size.

**Metamodeling**

After sampling of the design space, the extracted data samples (the sets of circuit parameter values belonging to each design space sample) are used to obtain a scalable version of the equivalent circuit model of Section 4.3. By scalable, we mean that there is a link between the design parameters of the microstrip structure and the circuit parameters of the circuit model. The relation between the design parameter values and circuit parameter values is not smooth. To avoid poor modeling, Sobol sequences are used in combination with Delaunay tessellation [Wats 81] to get triangular local basis functions. This technique is explained in more detail next.

The scalable circuit model is based on the use of a tessellation-based linear interpolation. Before performing this multidimensional interpolation process based on the previously described data samples, the design space is divided into cells using simplices. In general, this process is called a tessellation. In this work, we use the Delaunay tessellation [Wats 81] to create the division of the space.

A simplex is the N-dimensional equivalent of a triangle in a 2-dimensional space. An N-dimensional simplex has N + 1 vertices. We define a simplex region of the
4.5. Example: Defected Ground Structure

design space as

\[ \Omega_i \]

with \( i = 1, \ldots, P \) and \( P \) the total number of simplices after the tessellation. The corresponding \( N + 1 \) vertices of the simplex region \( \Omega_i \) are defined as

\[ \vec{g}_{\Omega_i}^k \]

with \( k = 1, \ldots, N + 1 \) and where \( \vec{g} \) denotes the vector of the design parameters.

Once the tessellation of the design space is performed, a tessellation-based linear interpolation is used to build the scalable circuit model. A linear interpolation is carried out inside a simplex using barycentric coordinates [Wats 92] as interpolation functions. A barycentric coordinate system is a system in which each point \( \vec{g} \) inside a simplex \( \Omega_i \) can be written as a unique convex combination of the \( N + 1 \) vertices of the simplex \( \vec{g}_{\Omega_i}^k \) [Wats 92].

A model based on this interpolation scheme for the 4-dimensional design space composed of \( \vec{g} = (W, W_s, L_s, \varepsilon_r) \) can be written as:

\[
\text{Model}(\vec{g}) = \sum_{k=1}^{N+1} \text{Data}(\vec{g}_{\Omega_i}^k) \ell_{\Omega_i}^k(\vec{g})
\]

(4.14)

with \( \Omega_i \) the simplex that contains the general interpolation point \( \vec{g} \), \( \vec{g}_{\Omega_i}^k \) the corresponding vertices of that simplex and \( \ell_{\Omega_i}^k(\vec{g}) \) the corresponding barycentric coordinates for which \( \sum_{k=1}^{N+1} \ell_{\Omega_i}^k(\vec{g}) = 1 \). \( \text{Data}(\vec{g}_{\Omega_i}^k) \) is the known data sample (being the known set of circuit parameter values) belonging to the \( k^{th} \) vertex of simplex \( \Omega_i \). \( \text{Model}(\vec{g}) \) is the interpolated data sample at interpolation point \( \vec{g} \).

We note that when the number of design parameters increases, the curse of dimensionality, that is related to the exponential growth of the size of the sample set in the design space with its dimensionality, needs to be solved. Advanced techniques such as design space reduction [Kozi 14] or segmentation [Loll 09] can be used to counteract this "curse".

4.5. Example: Defected Ground Structure

The design by a scalable circuit model is illustrated on the DGS that is shown in Figure 1.3 (Section 1.3). We consider 5 design cases with different filter specifications for this example structure and discuss the results obtained for each of them. We follow the same design flow as in Section 4.1, Figure 4.1. We assume that an equivalent circuit model is already available for the selected DGS. The model used here is the one that was extracted in Section 4.3 for the example used here.
4. Filter design using scalable circuit models

User settings

The user needs to define the frequency range of interest and the considered design space. The TL structure used in this example is the DGS that was defined in Figure 1.3, Section 1.3.

We simulated the S-parameter data over a frequency range of $[0.1 - 2.5]$ GHz over 241 equidistant frequency samples. Optionally, the number of frequency samples taken in the EM simulation could be reduced to decrease the simulation time. A Vector Fitting (VF) model should then be extracted and evaluated over a denser frequency grid to obtain an accurate model, as was done in Chapter 3. Note that here a local VF model would not be sufficient as the coverage of the whole frequency band of interest is necessary to extract an equivalent circuit model that is accurate over the whole frequency band of interest.

The design parameters can be geometrical parameters as well as material properties. In the numerical example of Chapters 2 and 3, only two geometrical parameters were considered as design parameters: the width, $W_s$, and length of the slotline, $L_s$ (see Figure 1.3). In this chapter, we opt for a more complex 4-dimensional design space, adding the width of the transmission line, $W$ (see Figure 1.3), and the relative permittivity of the substrate, $\varepsilon_r$, along with $W_s$ and $L_s$.

The selected design parameters should be bounded (Table 4.5). Large $W_s$-values lead to high reflections in the passband, suppressing the passband spectrum too much. Small $W_s$-values are not physically realizable. The selected range for $W_s$ is therefore fixed to $[1 - 5]$ mm. Small $L_s$-values lead to $f_0$-values that are too long to be of practical use ($L_s \propto 1/(2f_0)$, Equation (1.6)), making the simulation time become too long. Large $L_s$ increase the size of the finite ground plane beyond practically usable values and results in a simulation time that becomes too large. The selected range for $L_s$ is therefore fixed to $[80 - 100]$ mm. For large $W$-values, the characteristic impedance $Z_0$ no longer decreases significantly. Small $W$-values are not physically realizable. The selected range for $W$ is therefore fixed to $[1.8 - 5.8]$ mm. Concerning $\varepsilon_r$, a range was selected that corresponds to mainstream materials $[1.5 - 6.5]$.

The boundaries of the design parameters indirectly determine the set of the feasible filter specifications or performance quantities ($f_0$, $f_{c1}$, $f_{c2}$ and $A_s$). These boundaries are given in Table 4.5.

The filter specifications of the five considered design cases have to be selected within these boundaries for the designs to be feasible. The selected filter specifications are given in Table 4.6 for the five DGS design examples.
4.5. Example: Defected Ground Structure

Table 4.5.: The design space needs to be limited by the user. The boundaries of the design space indirectly determine the feasible filter specifications.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission line width, $W$ (mm)</td>
<td>1.80</td>
<td>5.80</td>
</tr>
<tr>
<td>Slotline width, $W_s$ (mm)</td>
<td>1.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Slotline length, $L_s$ (mm)</td>
<td>80.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Relative permittivity, $\varepsilon_r$</td>
<td>1.50</td>
<td>6.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feasible filter specifications</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency, $f_0$ (GHz)</td>
<td>1.13</td>
<td>1.71</td>
</tr>
<tr>
<td>Cut-off frequency 1, $f_{c1}$ (GHz)</td>
<td>0.63</td>
<td>1.24</td>
</tr>
<tr>
<td>Cut-off frequency 2, $f_{c2}$ (GHz)</td>
<td>1.46</td>
<td>2.5</td>
</tr>
<tr>
<td>Stopband amplitude, $A_s$ (dB)</td>
<td>$-34$</td>
<td>$-18.7$</td>
</tr>
</tbody>
</table>

Table 4.6.: We used five DGS design cases with different filter specifications to validate the proposed design based on scalable circuit modeling.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$ (GHz)</td>
<td>1.37</td>
<td>1.25</td>
<td>1.50</td>
<td>1.18</td>
<td>1.65</td>
</tr>
<tr>
<td>$f_{c1}$ (GHz)</td>
<td>0.88</td>
<td>0.65</td>
<td>1.00</td>
<td>0.69</td>
<td>1.18</td>
</tr>
<tr>
<td>$f_{c2}$ (GHz)</td>
<td>1.89</td>
<td>1.90</td>
<td>2.10</td>
<td>1.70</td>
<td>2.22</td>
</tr>
<tr>
<td>$A_s$ (dB)</td>
<td>$-24.0$</td>
<td>$-28.0$</td>
<td>$-22.0$</td>
<td>$-24.0$</td>
<td>$-18.0$</td>
</tr>
</tbody>
</table>
Table 4.7.: The boundaries of the circuit parameters can be provided to the optimizer. Circuit parameters $C$ and $R_{\text{cond}}$ are fixed to 0.0322 pF and 0.883 Ω respectively.

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ (nH)</td>
<td>0.01</td>
<td>5</td>
</tr>
<tr>
<td>$R_{\text{rad}}$ (Ω)</td>
<td>100</td>
<td>5000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>$L_{\text{res}}$ (nH)</td>
<td>0.1</td>
<td>10</td>
</tr>
<tr>
<td>$C_{\text{res}}$ (pF)</td>
<td>0.1</td>
<td>10</td>
</tr>
</tbody>
</table>

**Construction of the scalable circuit model**

In this work, we have used 100 sample points within the design space $(W, W_s, L_s, \varepsilon_r)$ chosen by a QMC sampling scheme based on Sobol sequences. Before generating the scalable circuit model, we compute the S-parameters of the filter at 241 frequencies at each of the 100 design space samples using Momentum [Keys 14b] in ADS. Then, the circuit parameters are optimized in order to minimize the error between the FRF of the circuit model (Figure 4.13) and the EM simulation-based FRF of the DGS for each of the 100 design space samples. To this end, the optimization function *lsqnonlin* was used in *Matlab* R2014a.

The function *lsqnonlin* is a nonlinear least-squares solver. It solves curve fitting problems of the form

$$\min_{\vec{x}} \| f(\vec{x}) \|_2^2 = \min_{\vec{x}} (f_1(\vec{x})^2 + f_2(\vec{x})^2 + \ldots + f_n(\vec{x})^2)$$

In our case, $\vec{x}$ is the vector of the circuit parameter values and the functions $f_i$ are the real and imaginary part of $S_{11}$ and $S_{21}$ for all 241 frequency points, hence $n = 964 (=241$ frequency samples times 2 (for real/imaginary part) times 2 (for $S_{11}/S_{21}$)). In this minimization algorithm, the lower and upper bounds on $\vec{x}$ can be provided to the optimization function. These boundaries are given in Table 4.7 and were selected based on ranges that ought to be realistic. The Jacobian matrix is also provided to the optimizer because it can sometimes improve the convergence of poorly scaled problems.

The optimization finds the circuit parameter values corresponding to a specific design space sample. This step provides the sets of circuit parameter values used
as data samples to build the scalable circuit model. The initial values of the circuit parameters for this optimization process were selected randomly within lower and upper boundaries (Table 4.7) given to the optimization function \textit{lsqnonlin}.

The optimized circuit parameter values are used as data samples to build the scalable circuit model using the Delaunay tessellation-based linear interpolation metamodeling method described in Section 4.4.

\textbf{Optimization of the scalable circuit model}

The scalable circuit model that was obtained returns the circuit parameter values, \( L, R_{\text{rad}}, \alpha, L_{\text{res}} \) and \( C_{\text{res}} \), as a function of the design parameters, \( W, W_s, L_s \) and \( \varepsilon_r \). However, in a design context, we need to find the optimal design parameter values leading to a filter response that fulfills a set of desired filter specifications, hence we need to invert the scalable circuit model. This is done in two steps:

1. First, the circuit model is optimized to find the circuit parameters in function of the desired filter specifications.

2. Then, the metamodel (scalable circuit model) is optimized to find the design parameters in function of the set of circuit parameters found in step 1.

In practice, these two optimizations are combined into one optimization. To this end, the optimization function \textit{patternsearch} was used in MATLAB.

The function \textit{patternsearch} finds a local minimum of a given objective function. In our case, it returns the corresponding design parameters. This objective function is the mean of four objective functions (one for each performance quantity \( f_0, f_{c1}, f_{c2} \) and \( A_s \)). The lower and upper bounds of the design parameters (Table 4.5) are provided to the optimizer. The initial values are randomly selected within these boundaries.

We can validate the complete scalable circuit model by calculating the model error between the S-parameters of the scalable circuit model with optimized circuit parameters (found after step 1) and the S-parameters of the simulated DGS with optimized design parameters (found in step 2). This is done for each design case separately.

\textbf{Results}

Table 4.8 gives the initial and optimized design parameter values for the five filter design cases (defined in Table 4.6). The initial design parameter values were
4. Filter design using scalable circuit models

Table 4.8.: The proposed circuit modeling-based design optimization approach provides the desired results. The initial and optimal values of the design parameters are given for the five design cases.

<table>
<thead>
<tr>
<th>Initial</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W ) (mm)</td>
<td>2.59</td>
<td>4.70</td>
<td>5.25</td>
<td>3.82</td>
<td>5.04</td>
</tr>
<tr>
<td>( W_s ) (mm)</td>
<td>4.61</td>
<td>3.40</td>
<td>1.11</td>
<td>4.05</td>
<td>3.99</td>
</tr>
<tr>
<td>( L_s ) (mm)</td>
<td>90.7</td>
<td>170</td>
<td>123</td>
<td>92.6</td>
<td>82.4</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>5.75</td>
<td>5.46</td>
<td>3.53</td>
<td>1.95</td>
<td>4.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optimized</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W ) (mm)</td>
<td>4.67</td>
<td>5.27</td>
<td>5.35</td>
<td>4.08</td>
<td>3.32</td>
</tr>
<tr>
<td>( W_s ) (mm)</td>
<td>1.93</td>
<td>3.44</td>
<td>1.73</td>
<td>2.00</td>
<td>2.73</td>
</tr>
<tr>
<td>( L_s ) (mm)</td>
<td>90.7</td>
<td>93.8</td>
<td>91.3</td>
<td>92.6</td>
<td>82.4</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>3.69</td>
<td>5.47</td>
<td>2.06</td>
<td>6.03</td>
<td>1.99</td>
</tr>
</tbody>
</table>

randomly generated within the 4-dimensional design space. The boundaries of the design space are given in Table 4.5.

Figures 4.18-4.22 show the optimization results for the five design cases. Table 4.9 gives the absolute and relative RMSE (Equation (1.4) and (1.5)) between the optimized scalable circuit model and the optimized DGS for each design case example for the \( S_{21} \)-parameter. These results confirm the high accuracy achieved by the proposed design technique that is based on a physically interpretable and scalable circuit model. The circuit model and the EM response of the optimal designs fulfill the required filter specifications for each of the five cases. In what follows, we provide a detailed discussion concerning the computational resources needed to obtain the results.

All simulations were performed in a Windows environment with 8GB RAM and Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz.

Circuit modeling-based optimization is much faster than EM-based optimization. The average CPU time required for one frequency-domain EM simulation (one EM evaluation at one point in the design space) over 241 frequency samples is 70 s. The evaluation of the FRF (at one point in the design space) of the scalable circuit model over the same number of frequency samples takes only 50 ms on average. Multiplying this CPU time by the number of function evaluations (feval)
4.5. Example: Defected Ground Structure

Figure 4.18.: The S-parameters of the optimized EM design (—) obtained by scalable circuit modeling fulfill the required filter template (•••) for Case 1 (see Table 4.6) starting from the random initial EM design (---). The FRF of the circuit model for the optimized circuit parameter values (—) fits the FRF of the optimized EM design (—) quite well, with an absolute RMSE of $-37.3\,\text{dB}$.

Figure 4.19.: The S-parameters of the optimized EM design (—) by scalable circuit modeling fulfill the required filter template (•••) for Case 2 (see Table 4.6) starting from the random initial EM design (---). The FRF of the circuit model for the optimized circuit parameter values (—) fits the FRF of the optimized EM design (—) quite well, with an absolute RMSE of $-34.4\,\text{dB}$. 
4. Filter design using scalable circuit models

Figure 4.20.: The S-parameters of the optimized EM design (—) by scalable circuit modeling fulfill the required filter template (⋯) for Case 3 (see Table 4.6) starting from the random initial EM design (—). The FRF of the circuit model for the optimized circuit parameter values (—) fits the FRF of the optimized EM design (—) quite well, with an absolute RMSE of $-39.7 \text{dB}$.

Figure 4.21.: The S-parameters of the optimized EM design (—) by scalable circuit modeling fulfill the required filter template (⋯) for Case 4 (see Table 4.6) starting from the random initial EM design (—). The FRF of the circuit model for the optimized circuit parameter values (—) fits the FRF of the optimized EM design (—) quite well, with an absolute RMSE of $-32.2 \text{dB}$.
4.5. Example: Defected Ground Structure

Figure 4.22: The S-parameters of the optimized EM design (—) by scalable circuit modeling fulfill the required filter template (···) for Case 5 (see Table 4.6) starting from the random initial EM design (—). The FRF of the circuit model for the optimized circuit parameter values (—) fits the FRF of the optimized EM design (—) quite well, with an absolute RMSE of −35.1 dB.

Table 4.9: The absolute and relative RMSE of the scalable circuit model error for each optimization case for the $S_{21}$-parameter.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\text{RMSE}_{abs}$ [dB]</th>
<th>$\text{RMSE}_{rel}$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>−37.3</td>
<td>−22.6</td>
</tr>
<tr>
<td>Case 2</td>
<td>−34.4</td>
<td>−19.7</td>
</tr>
<tr>
<td>Case 3</td>
<td>−39.7</td>
<td>−29.6</td>
</tr>
<tr>
<td>Case 4</td>
<td>−32.2</td>
<td>−24.1</td>
</tr>
<tr>
<td>Case 5</td>
<td>−35.1</td>
<td>−23.4</td>
</tr>
</tbody>
</table>
Table 4.10.: Total optimization time using the scalable circuit model when one function evaluation (feval) of the scalable circuit model at one point in the design space takes 50 ms.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>fevals</td>
<td>2073</td>
<td>1046</td>
<td>2319</td>
<td>1352</td>
<td>1410</td>
</tr>
<tr>
<td>CPU time/feval</td>
<td>x 50 ms</td>
<td>x 50 ms</td>
<td>x 50 ms</td>
<td>x 50 ms</td>
<td>x 50 ms</td>
</tr>
<tr>
<td>Total time</td>
<td>104 s</td>
<td>52.3 s</td>
<td>116 s</td>
<td>67.6 s</td>
<td>70.5 s</td>
</tr>
</tbody>
</table>

gives us the results shown in Table 4.10. This confirms the significant amount of computational resources that are saved by using the proposed scalable circuit modeling technique.

Generating the scalable circuit model requires a certain amount of CPU time too, as it requires the acquisition of the 100 EM simulations of 70 s each to generate the S-parameters over 241 frequency samples for all the 100 design space samples. This computational cost is needed to generate data samples needed for the construction of the scalable circuit model. After this data generation, the CPU time needed for the actual construction of the scalable circuit model (or the metamodel based on Delaunay tessellation) is negligible with respect to the CPU time needed to simulate the S-parameter data at these 100 design space samples. Hence, the total CPU time for generating the scalable circuit model is 116.7 minutes (= 70 s times 100 samples).

Once the scalable circuit model is generated, it can be reused in multiple optimization cases, speeding up the filter design tremendously. Table 4.10 gives the total optimization CPU time required in each of the five optimization cases. In all five cases, the required CPU time is much less than the CPU time that EM-based optimization would need considering the high number of function evaluations (2073, 1046, 2319, 1352 and 1410) required by the optimizations. Each function evaluation corresponds to a filter response evaluation at one point in the design space. Hence, the CPU time needed to generate the scalable circuit model (116.7 minutes, 100 design space samples) can be definitely accepted considering the saved computational resources when multiple optimization cases are performed on the same microstrip structure with the same user settings. Chapter 5 gives a better comparison of the EM design and design by scalable circuit model.

Besides, the CPU time that is needed to generate the scalable circuit model can be reduced too. A less dense frequency grid can be used to simulate the DGS at
4.5. **Example: Defected Ground Structure**

the 100 design space samples. The Vector Fitting (VF) method [Gust 99] can then be used to extract a model that allows to evaluate the EM simulated S-parameter data over a much denser frequency grid. This technique was used and described in Chapter 3. A second way to decrease the initial computational effort is to use adaptive frequency sampling when simulating the S-parameters with the EM solver [Keys 14a].

The methodology described in this chapter can be generalized for any microwave filter design. We note that when another filter structure (different from the one used in this work) is designed using the proposed approach, an accurate equivalent circuit model should be at disposal or should be constructed as done in this work.

Defining a new circuit model for a new filter structure requires some engineering work. However, this only needs to be done once for one specific filter structure. This is an effort that is worth paying when we consider the enormous amount of time and physical insight that is gained once the model is obtained.

In general, any electromagnetic response can always approximately be represented by a lumped equivalent circuit model. Specific filter structures will require specific circuit models. The equivalent circuit used in this work to model a DGS is based on the modeling of a resonating structure. Hence, this circuit can be used as a basis model when one wants to model other resonating structures. Obviously, also non-resonating structures can be modeled by equivalent circuits.
This chapter thoroughly compares the three filter design approaches that were discussed in the previous chapters. The previously obtained results are summarized here for the convenience of the reader. Furthermore, a comprehensive qualitative comparison is performed in this chapter.
In the previous chapters we have introduced three design procedures: the currently most popular design that is based on electromagnetic (EM) optimization (Chapter 2, EM approach), and the two newly proposed approaches using metamodeling (Chapter 3, meta approach) and scalable circuit modeling (Chapter 4, circuit approach). The advantages and drawbacks as well as the quantitative results were already discussed in each chapter separately, but a more general overview is given in this chapter for the convenience of the reader.

Figure 5.1 summarizes the methodology that is used in the design approaches that were discussed in the previous chapters. It gives a nice overview of the design flow of the three approaches. The three procedures share the idea that they are all based on an optimization process, that searches within the design space for a feasible solution. It results in optimized design parameters (DP) for the considered microstrip structure that synthesizes a Frequency Response Function (FRF) that fits the desired set of filter specifications (or the desired filter template). The only difference here is the way that is used to obtain the optimized design. The EM approach uses only time-inefficient EM simulations. The meta approach uses metamodels, which directly link the design parameters (DP) to the performance quantities (PQ) (or the filter specifications). The circuit approach uses a scalable circuit model, a circuit model of which the circuit parameters (CP) are linked to the design parameters (DP) by a metamodel. Note that both the metamodels in the meta approach and the scalable circuit model in the circuit approach are reusable in a design context. They need to be constructed only once for a specific microstrip structure and a set of user settings (frequency range and design space).

We will now show the three methods next to each other to clearly show the similarities and differences between them. A thorough absolute quantitative comparison is difficult because there are several differences between the three approaches: they use different design spaces, different sampling schemes, different metamodeling techniques and different optimizers. To allow the reader to compare the approaches anyway, a comprehensive qualitative study is attempted next.

The different timing results obtained for each approach are given in Figure 5.1 and Table 5.1. We have used the following terminology:

- **optim time**
  represents the computation time that is required for one function evaluation (feval) in the optimization process.

- **EM time**
  represents the computation time that is required for one EM simulation of the microstrip structure evaluated over a fixed number of frequency points.
• **VF time**
  represents the computation time that is required for the estimation of one rational model that is obtained by Vector Fitting (VF) [Gust 99] and its evaluation at a fixed number of frequency points.

• **extract PQ time**
  represents the computation time that is required to extract the data samples of the performance quantities (PQ) when starting from the pre-evaluated S-parameter data for one design space sample.

• **extract CP time**
  represents the computation time that is required to extract the data samples of the circuit parameter (CP) values of the circuit model of which the response fits the one of the S-parameter data at one design space sample.

• **meta time**
  represents the computation time that is required to build the metamodels from the extracted data samples (being the performance quantities (PQ) in the case of the meta approach and the circuit parameters (CP) in the case of the circuit approach).

• **circuit time**
  represents the engineering time that is required to construct the equivalent circuit model.

• **total model time**
  represents the total computation time that is required to construct the metamodels. In the meta approach, this time is equal to the sum of the EM time, the VF time and the extract PQ time multiplied by the number of design space samples, plus the meta time (see Table 5.1). In the circuit approach, this time is equal to the sum of the EM time and the extract CP time multiplied by the number of design space samples, plus the meta time (see Table 5.1).

Remember that it also takes some computation time to extract the metamodels used in the new design techniques that are proposed in this work. The total model time, calculated in Table 5.1, that is required in the metamodeling approach is 8.3 minutes. The total model time that is required for constructing the metamodels used in the scalable circuit model approach is 118.8 minutes. Note that this is a single time effort for a specific set of user settings.

The total model time of 118.8 minutes in the circuit approach does not take the circuit time into account. Constructing the circuit model requires some engineering work, which makes it hard to estimate the required circuit time. Note that this
5. **Comparison of the proposed filter design procedures**

time is also required only once for a specific microstrip structure and a set of user settings since the circuit model is reusable.

Table 5.1.: This table gives an overview of the time needed to calculate the results of the examples used to validate the three design approaches of Chapters 2-4. Note that these cannot be compared blindly because the examples use different settings, optimizers, etc. Nevertheless, they give a nice intuitive view on the general improvements of the two techniques proposed in this work.

<table>
<thead>
<tr>
<th></th>
<th>EM approach</th>
<th>Meta approach</th>
<th>Circuit approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM time</td>
<td>49 x 10 s</td>
<td>100 x 70 s</td>
<td></td>
</tr>
<tr>
<td>VF time</td>
<td>+ 49 x 80 ms</td>
<td>+ 100 x 1.3 s</td>
<td></td>
</tr>
<tr>
<td>extract time</td>
<td>+ 49 x 12 ms</td>
<td>+ 100 x 1.3 s</td>
<td></td>
</tr>
<tr>
<td>meta time</td>
<td>+ 0.5 s</td>
<td>+ 0.3 s</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>= 495 s</td>
<td>= 7130 s</td>
<td></td>
</tr>
<tr>
<td>time / feval</td>
<td>70 s</td>
<td>23 µs</td>
<td>50 ms</td>
</tr>
<tr>
<td>average Nº fevals</td>
<td>x 22</td>
<td>x 135</td>
<td>x 1640</td>
</tr>
<tr>
<td>total</td>
<td>= 1540 s</td>
<td>= 3.1 ms</td>
<td>= 82 s</td>
</tr>
</tbody>
</table>

The total model time is much smaller for the meta approach than for the circuit approach (Table 5.1). This is explained as follows:

- A 2-dimensional design space was used in the meta approach example, while a 4-dimensional design space was used in the circuit approach. Hence, more design space samples were required in the circuit approach to cover the whole design space. This first difference leads to a difference of a factor 2 for the EM time and extract time (49 vs. 100 samples).

On the other hand, also two different sampling techniques were used. A regular tensor-product grid was used in the meta approach. If a more efficient sampling technique would have been used there, less samples would have been required for the same amount of information gained. Hence, the number and location of the samples is more optimal for the circuit approach than for the meta approach.

- VF was used in the meta approach to evaluate the simulated S-parameter data over a denser frequency grid. Note that a dense grid was necessary
Figure 5.1: Summary of the three approaches that were discussed in Chapters 2, 3 and 4. It gives an overview of the methodology and the different timing results obtained for each approach. Each step in the design procedures introduces an error. The design errors and model errors for each approach are given in Tables 5.2-5.5. The simulation error is the same for the three approaches.
to accurately extract the performance quantities. Hence, the number of frequency samples used in the EM simulations could be reduced significantly without losing accuracy: 27 frequency samples were used in the meta approach example, while 241 frequency samples were used in the circuit approach example. This leads to a difference of a factor 7 in EM time (10s vs. 70s). Note that the VF technique can also be used in the circuit approach to reduce the EM time there.

- Extracting the circuit parameter (CP) values involves an optimization process, while the performance quantities (PQ) in the meta approach can be extracted directly from the S-parameter data. The optimization process requires more computation time, resulting in a difference of a factor 108 in extract time (1.3s vs. 12ms).

- Different metamodeling techniques were used for both approaches, resulting in a different meta time (0.5s vs. 0.3s).

Once the single effort is put into constructing the models, model based filter design becomes much more time-efficient when compared to EM optimization. The total optim time, which is the total filter design time that is needed when all models are at disposal, is summarized in Table 5.1 for the three approaches. A filter design by EM optimization takes 25.7 minutes on average. Filter design by metamodeling takes only 3.1 ms on average. Filter design by using a scalable circuit model takes 1.4 minutes on average.

Simply comparing these results in an absolute way may lead to biased conclusions since different optimizers were used for the three approaches. The shape of the cost functions is also different for the three approaches, making it hard to compare the number (Nə) of function evaluations (fevals) required for each approach. An average was taken over the different design cases that were used in the examples. The results are discussed in some more detail below:

- There is an enormous speed-up when comparing the meta approach (3.1 ms on average) to the EM approach (25.7 minutes on average). Even if the metamodels are not constructed yet (which takes only 8.3 minutes), the meta approach is faster than the EM approach (25.7 minutes on average). Hence, the time needed to construct the metamodels is definitely acceptable as it saves computational resources when compared to the EM optimization even if the total model time is taken into account.

- There is also a considerable speed-up when comparing the EM approach (25.7 minutes on average) to the circuit approach (1.4 minutes on average), especially considering the fact that the dimensionality of the design space...
is doubled (2D vs. 4D). Even with different design spaces, it takes only five designs to gain speed when using the circuit approach (5 x 25.7 minutes > 118.8 minutes). Hence, the time needed to construct the scalable circuit model (118.8 minutes) is again acceptable considering the saved computational resources when considering multiple designs and is the price to pay to get physical insight in the structure.

• The meta approach is still much faster than the circuit approach. However, the meta approach does not provide the valuable physical insight into the EM working mechanism of the filter that is gained by the equivalent circuit model.

Next to the timing, we should also compare the accuracy of the three approaches. Figure 5.1 illustrates where the different errors, that were introduced in Section 1.2, can occur in the different approaches. In addition to the design error, the two newly proposed model based design approaches also contain a model error, which is the difference between the modeled response and the response that would be obtained by an EM simulation. The model errors for the meta and the circuit approach are summarized in Tables 5.2 and 5.3 respectively.

Note that the model error is different for both techniques. In the meta approach, there are four contributions to the model error (one for each metamodel that was extracted). As this error is a (multidimensional) function in general, we simply use the norm of the residual error as a measure. This error is returned by the least squares estimation that was used to fit the polynomial regression models. In the circuit approach, the model error is the error between the S-parameters of the scalable circuit model with optimized circuit parameters and the S-parameters of the simulated Defected Ground Structure (DGS) with optimized design parameters that correspond to the optimized circuit parameter values. This scalable circuit model error can be calculated for each S-parameter using Equations (1.2)-(1.5). Table 5.3 only shows the absolute and relative Root Mean Square Error (RMSE) of the $S_{21}$-parameter for each design case example separately.

The error that is most important to us is the design error, since this error sets the accuracy of the filter that is designed. A design error is calculated for each filter specification using Equation (1.1). The design errors that are obtained for the design cases are given in Table 5.4 for the EM and meta approach and in Table 5.5 for the circuit approach.

Note that the design errors are of the same order of magnitude for the EM approach and the meta approach. This means that we do not loose accuracy by using the meta approach when compared to the EM approach. The design errors of the circuit approach are slightly higher, although this strongly depends on the design.
Table 5.2: This table gives the model errors of the metamodels extracted in the meta approach.

<table>
<thead>
<tr>
<th>Relative model errors of the metamodel approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0(W_s, L_s) )</td>
</tr>
<tr>
<td>( f_{c1}(W_s, L_s) )</td>
</tr>
<tr>
<td>( f_{c2}(W_s, L_s) )</td>
</tr>
<tr>
<td>( A_s(W_s, L_s) )</td>
</tr>
</tbody>
</table>

Table 5.3: This table gives the absolute and relative RMSE of the scalable circuit model error for each optimization case in the circuit approach for the \( S_{21} \)-parameter.

<table>
<thead>
<tr>
<th>RMSE(_{abs})</th>
<th>RMSE(_{rel})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.0136</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.0193</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.0104</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.0245</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.0176</td>
</tr>
</tbody>
</table>

case: the results of Case 1 of the circuit approach (Table 5.5) are of the same order of magnitude as those of Case 1 of the EM and meta approach (Table 5.4). The slightly higher design errors in the circuit approach are probably due to the non smooth relation between the design parameters of the DGS and the circuit parameters of the circuit model as discussed in Chapter 4 and the fact that we are optimizing a 4-dimensional design space instead of a 2-dimensional one.

The obtained design errors should be interpreted with care. It might be the case that the filter template (as shown in Figure 1.2) is fully satisfied, even though one or more design errors are nonzero. This is for example the case for the filter specification \( A_s \), the stopband amplitude. Even though this specification was well satisfied for all design cases of all three approaches (see Figures 2.3-2.5, 3.6-3.8 and 4.18-4.22), the design error, \( E_{A_s} \), remains quite high (Tables 5.4 and 5.5). This is due to the definition of the design error (Equation (1.1)). A more correct way would be to take inequalities into account in the definition of those design
errors, but this leads to the counter intuitive result that the error is zero when the boundary condition of the corresponding filter specification is met.

It is worth comparing the accuracy of the design results of the EM approach and the meta approach in more detail since the same design cases were studied with the same user settings for these two approaches. The optimized design parameter values are therefore also given in Table 5.4. A difference is noticeable between the optimized design parameters achieved by the two methods. This can be explained by the fact that different optimizers were used and the cost functions were defined slightly different in Advanced Design System (ADS) (for the EM approach) and in MATLAB (for the meta approach). Together with the fact that the definition of the design errors does not take inequalities into account, this explains the possible differences in optimized design parameter values.

Table 5.4.: This table gives the optimized design parameters for the three design cases used to validate design by EM optimization (Chapter 2) and design by metamodels (Chapter 3). The relative design errors (defined in Equation (1.1)) are given for each of the performance quantities. Although the same design cases were used for both techniques, some difference occur in optimized design parameters and design errors.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EM</td>
<td>Meta</td>
<td>EM</td>
</tr>
<tr>
<td>( W_s ) [mm]</td>
<td>0.81</td>
<td>0.85</td>
<td>2.74</td>
</tr>
<tr>
<td>( L_s ) [mm]</td>
<td>99.2</td>
<td>99.9</td>
<td>94.7</td>
</tr>
<tr>
<td>( E_{f_0} )</td>
<td>0.0008</td>
<td>0.0040</td>
<td>0.0037</td>
</tr>
<tr>
<td>( E_{f_{c_1}} )</td>
<td>0.0011</td>
<td>0.0022</td>
<td>0.0056</td>
</tr>
<tr>
<td>( E_{f_{c_2}} )</td>
<td>0.0013</td>
<td>0.0019</td>
<td>0.0044</td>
</tr>
<tr>
<td>( E_{A_f} )</td>
<td>0.3350</td>
<td>0.3476</td>
<td>0.0878</td>
</tr>
</tbody>
</table>

In all design approaches, the simulation error should also be taken into account. This error is independent of the design approach and was already discussed in Section 1.3. It will not be considered here.

**Conclusion**

Table 5.6 summarizes the advantages and disadvantages of the three filter design approaches. Design by EM optimization is very time-consuming. This drawback
Table 5.5.: The relative design errors (defined in Equation (1.1)) are given for each of the performance quantities, for the five design cases used to validate design by scalable circuit modeling (Chapter 4).

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{f_0}$</td>
<td>0.0066</td>
<td>0.0160</td>
<td>0.0020</td>
<td>0.0110</td>
<td>0.0091</td>
</tr>
<tr>
<td>$E_{f_{c1}}$</td>
<td>0.0034</td>
<td>0.0185</td>
<td>0.0110</td>
<td>0.0754</td>
<td>0.0144</td>
</tr>
<tr>
<td>$E_{f_{c2}}$</td>
<td>0.0053</td>
<td>0.0174</td>
<td>0.0033</td>
<td>0.0367</td>
<td>0.0167</td>
</tr>
<tr>
<td>$E_{A_s}$</td>
<td>0.4477</td>
<td>0.3338</td>
<td>0.4696</td>
<td>0.6524</td>
<td>0.2926</td>
</tr>
</tbody>
</table>

can be resolved by using models in the filter design procedure. Both model based design approaches are much more time-efficient than the EM approach. The meta approach obtains the best results concerning time efficiency.

Table 5.6.: Summary of the advantages and disadvantages of the three filter design approaches.

<table>
<thead>
<tr>
<th></th>
<th>EM approach</th>
<th>Meta approach</th>
<th>Circuit approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-efficient</td>
<td>✗</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Reusable</td>
<td>✗</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Automatable</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Accurate</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Physical insight</td>
<td>✗</td>
<td>✔️</td>
<td>✔️</td>
</tr>
</tbody>
</table>

Besides being time-consuming, the EM approach has another drawback: no information about the structure can be reused in future designs. The design process must be repeated completely when the filter specifications change. This drawback is also resolved by model based filter design. The models are reusable for future applications with different filter specifications sharing the same set of user settings: the same filter structure, the same design space and the same frequency range. Hence, model based design allows to recycle the time and effort that is needed to construct these models.

The models are preferably extracted for a set of user settings that are as broad as possible in order to exploit the advantage of being reusable in design as much as possible. To show that this is indeed feasible, the circuit model (Figure 4.13) is
extended in the next chapter to a wide-band circuit model so that it can also be used in wide-band applications.

The generation of the metamodels in both the meta and the circuit approach can be automated up to the same level as the design by EM optimization. Every filter design approach, even the EM approach, requires some user interaction to select the filter template, the microstrip structure, the design space and the frequency range of interest corresponding to the application at hand. Only the construction of the equivalent circuit model in the circuit approach requires extra engineering work that can not be automated completely. However, note again that this engineering effort is only required once for a specific set of user settings since the models are reusable in successive filter designs.

An advantage of the EM approach is the high accuracy that is guaranteed. In order to keep this advantage in model based filter design, the constructed models should also be as accurate as the EM simulator. To achieve a high accuracy of the metamodels, enough design space samples should be selected. More design space samples requires more time-consuming EM simulations to extract the data samples for the metamodels, which will make the model time to increase. Hence, there is a trade-off between accuracy and model time. Note again that this model time is only required once for a specific set of user settings since the models are reusable. To achieve a high accuracy of the equivalent circuit model, losses were taken into account in the model (see Section 4.3). In this work, the models used in the design examples are accurate enough to guarantee the accuracy of the proposed design techniques.

Another important disadvantage of the EM approach is the lack of physical insight. An equivalent circuit model provides physical insight that is very precious to a designer. It allows a designer to better understand the working mechanism of the structure under study and to predict the characteristics and the behavior of the structure without the need to evaluate it through an expensive EM optimization. Once an equivalent circuit model has been constructed for a particular microstrip structure, it can be used in a circuital simulator to study its behavior under different conditions. Hence, it empowers the designer to efficiently analyze and understand a complex structure that would otherwise take too much time to simulate.

An equivalent circuit also helps the designer to make proper design choices even before starting a design. For example, using the scalable circuit model the designer can limit the design space in a smart way knowing that an optimized design that fulfills the filter template can be found within this design space. On the contrary, this is not guaranteed when using the EM approach, in which case
the time-consuming design process should be repeated with other design space settings.

To conclude, model based filter design is much more time-efficient than design by EM optimization without significant loss of accuracy. Constructing the models requires a one time effort that is acceptable considering the fact that the models are reusable in future filter designs with the same user settings. Constructing the metamodels can be automated. However, constructing the equivalent circuit model requires some engineering effort. Although design by using a scalable circuit model is less time-efficient than design by metamodeling, it provides much more physical insight in the working mechanism of the filter, which is so useful to a designer.

In general, model based design has many advantages over design by EM optimization. It is up to the designer to make a trade-off between time-efficiency and physical insight to decide which model based design approach is best suited for a particular application.
6. Extensions of the circuit model

The results of this chapter are an extension of the model that was constructed in Chapter 4. The work in this chapter is published in [Van 19].

In RF filter design it is important to be aware of the spurious frequency bands that are present at the wide-band frequency response of the filter. Designers would benefit from having knowledge about the shape of these spurious bands.

Section 6.1 extends the band-limited equivalent circuit model for a Defected Ground Structure (DGS) that was obtained in Section 4.3 to a wide-band model that captures these spurious frequency bands in the model response. Section 6.2 uses this wide-band model to model a DGS with different slot positions. Section 6.3 takes a first step towards the extension of this wide-band model for a DGS with multiple slots.
6. Extensions of the circuit model

6.1. Wide-band equivalent circuit model

Spurious frequency bands are multiple pass bands that are present in the wide-band frequency response of a microwave filter structure. These spurious frequency bands occur around multiples of the fundamental frequency, $f_0$, because they are harmonically related. Therefore, they are called harmonic responses from now on. They are often unwanted since they can interfere with potentially unwanted signals that are transmitted in other frequency bands.

A designer can really benefit from acquiring information about the shape and the attenuation of the spurious harmonic responses. The harmonic responses that are relevant and need to be taken into account vary according to the application at hand. To maximally exploit the reusability in design, the band-limited equivalent circuit model that was obtained in Section 4.3 is extended here to obtain a wide-band model that takes these harmonic responses into account.

An accurate equivalent circuit model was proposed in Chapter 4 to model the fundamental response of a DGS around $f_0$. This DGS consists of a single rectangular slot that is positioned perpendicularly to and symmetrically around the transmission line. Here, we extend the band-limited equivalent circuit model to a wide-band model that can accurately represent the higher-order harmonic responses that are present next to the fundamental frequency response around $f_0$. This wide-band model gives the designer all the insight needed to properly design a filter in the first stage of the design procedure without the need to iterate over the design process multiple times.

Figure 6.1 repeats the band-limited equivalent circuit model that was proposed in Chapter 4. This model consists of two parts. An RLC-resonator models the resonating slot of the DGS, where $L_{res}$ and $C_{res}$ define the fundamental frequency and the susceptance slope of the parallel LC-resonator. $R_{res}(f)$ represents the radiation losses of the slot. The signal transmission line of the DGS is modeled by one elementary section of the Telegrapher’s representation: $L$, $C$ and $R(f)$, where $R(f)$ represents the conductor losses. The dielectric losses are negligible for the substrate used.

Modeling an extra harmonic response requires an extra RLC-resonator in general. Cascading RLC-tanks to model multiple antiresonances has been proposed in the literature [Safw 06, Woo 06]. However, in our model, the resistor representing the slot losses is frequency-dependent. Hence, this one resistor is sufficient to model the losses over a wide frequency band. Our model therefore only requires an extra LC-tank per extra harmonic response to be modeled. The wide-band equivalent circuit model that we obtain is shown in Figure 6.2 and can model $K$
6.1. Wide-band equivalent circuit model

Figure 6.1.: Extended band-limited geometry-dependent circuit model.

antiresonances. Note that the wide-band model can be reduced to the band-limited model of Figure 6.1 if $K = 1$.

Figure 6.2.: Wide-band circuit model that models antiresonances $k = 1 : K$ of a DGS.

The number of circuit elements in our model is very parsimonious. We only need 2 extra circuit parameters ($L_{\text{res}}$ and $C_{\text{res}}$) to model an extra harmonic response, while the models proposed in the literature [Safw 06, Woo 06] require 3 extra parameters ($L$, $C$ and $R$) for each response.

**Results of the wide-band model**

The wide-band equivalent circuit model, shown in Figure 6.2, is validated using microwave measurements of a DGS. A DGS design with a symmetrical feedpoint is shown in Figure 6.3. It is designed on a Rogers 4003 substrate (Section 1.3: Substrate settings). Table 6.1 gives the dimensions of the DGS design. The S-
parameters of the DGS are measured with a Vector Network Analyzer (VNA) over a frequency band of \([0.1 – 5]\) GHz.

![Image of a DGS design with a symmetric feedpoint and geometrical parameters of Table 6.1. Left: top view showing the signal transmission line. Right: bottom view showing the slotline.]

**Figure 6.3.** A DGS design with a symmetric feedpoint and geometrical parameters of Table 6.1. Left: top view showing the signal transmission line. Right: bottom view showing the slotline.

**Figure 6.4.** The slot of a DGS with a symmetric feedpoint is only excited at the odd modes. Hence, only the harmonic responses corresponding to the even modes will be present in the FRF of the DGS.

To validate the accuracy of the proposed wide-band model, we consider an example up to the third harmonic response (measured up to 5GHz). To obtain an
6.1. **Wide-band equivalent circuit model**

Table 6.1.: Selected geometrical parameter values of the DGS.

<table>
<thead>
<tr>
<th>$W$ [mm]</th>
<th>$W_s$ [mm]</th>
<th>$L_s$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>1</td>
<td>80</td>
</tr>
</tbody>
</table>

An accurate circuit model up to the third harmonic response, the model needs to include two antiresonances: one at $f_0$ and one at $3f_0$. Note that the slot is not excited at $2f_0$, as no energy coupling can occur between the transmission line and the slotline at the corresponding mode. This is shown in Figure 6.4. Hence, the energy around $2f_0$ is not captured in the slot and is therefore present in the FRF.

The wide-band model is used to model two antiresonances, hence $K = 2$ in Figure 6.2. The values of the circuit parameters are optimized to minimize the error between the S-parameters of the wide-band model and the S-parameters of the measured DGS as obtained after a de-embedding step that is similar to the one explained in Section 4.3. The optimization function *Quasi-Newton* was used in ADS [Keys 14a] to estimate the model parameters (as in Section 2.3). Table 6.2 gives the resulting circuit parameter values.

Table 6.2.: The optimized parameter values of the wide-band model in Figure 6.2 of which the response fits the response of the DGS design with geometrical parameter values that are given in Table 6.1.

<table>
<thead>
<tr>
<th>Microstrip line of the DGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ [pF]</td>
</tr>
<tr>
<td>0.471</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slotline of the DGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation losses</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>LC-resonator @ $f_0$</td>
</tr>
<tr>
<td>LC-resonator @ $3f_0$</td>
</tr>
</tbody>
</table>

Figures 6.5-6.6 compare the amplitude and the phase response of the $S_{11}$- and $S_{21}$-parameters of the measured DGS (___) and of the simulated wide-band circuit model (—). The absolute model error (Equation (1.2)) is shown in (— —) and is...
6. Extensions of the circuit model

Figure 6.5.: Comparison of the $S_{11}$-parameter of the measured DGS (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (− −) and fulfills the desired error level of $-25\text{ dB}$ (—) quite well. The absolute RMSE is $-28.9\text{ dB}$. Top: amplitude response. Bottom: phase response.

The absolute model error (− −) is below the desired error level of $-25\text{ dB}$ (—) over most of the considered frequency range. At frequencies where this desired error is not achieved, the relative model error is small enough. Hence, the circuit model response fits the measured response of the DGS quite well.

The absolute Root Mean Square Error (RMSE) (Equation (1.4)) of $S_{11}$ and $S_{21}$ over the frequency band of interest of $[0.1 - 5] \text{ GHz}$ is equal to $-28.9\text{ dB}$ and $-28.9\text{ dB}$, respectively. The relative RMSE (Equation (1.5)) is equal to $-21.6\text{ dB}$ and $-21.9\text{ dB}$, respectively. The absolute RMSE is clearly below the desired error of $-25\text{ dB}$.

In Chapter 4, Figure 4.16 already proved the importance of a proper inclusion of the radiation losses in the model to achieve a high accuracy. When modeling over a broader frequency band, the conductor losses also play an important role in attaining a high accuracy. This is shown in Figure 6.7 where the absolute model...
error is shown between the measured DGS (—) and the completely lossless model (—), the model with only conductor losses (—), the model with only radiation losses (—) and the model of Figure 6.2 with both losses (—) for the $S_{11}$-parameter (top) and the $S_{21}$-parameter (bottom).

With the proposed wide-band model (—) we achieve an absolute RMSE of $-28.9\, \text{dB}$ and $-28.9\, \text{dB}$ for $S_{11}$ and $S_{21}$, respectively. With only the radiation losses (—) we lose about $9\, \text{dB}$ of accuracy. With only the conductor losses (—) we lose about $5\, \text{dB}$ of accuracy. With a completely lossless model (—) we lose about $11\, \text{dB}$ of accuracy.

Table 6.3 gives an overview of the absolute and relative RMSE for the considered lossy and lossless models. The conductor losses now have a bigger influence than in the case of the band-limited model (Section 4.3). Figure 6.7 shows that the
6. **Extensions of the circuit model**

Conductor losses and the radiation losses work in complementary frequency bands: the radiation losses are mainly important around the antiresonance frequencies, while the conductor losses have the most effect in the transition band between two antiresonance. The conductor losses become more important with increasing frequency, as could be expected because of the skin-effect.

In conclusion, we proposed a wide-band equivalent circuit model for symmetrical DGS structures with a single slot. This model can be used to describe multiple antiresonances of the DGS response. The number of circuit elements in our model is very parsimonious. This makes the model efficient and keeps the identification of the equivalent circuit parameters simple.

Table 6.3.: Introducing losses in the wide-band model is necessary to achieve a high accuracy. This table gives the absolute and relative RMSE between the measured DGS design in Figure 6.3 and the wide-band model (Figure 6.2) with both losses, with only radiation losses, with only conductor losses and without losses. The RMSE was calculated over the frequency band of [0.1 – 5] GHz.

<table>
<thead>
<tr>
<th></th>
<th>absolute RMSE</th>
<th>relative RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{11}$</td>
<td>$S_{21}$</td>
</tr>
<tr>
<td>Wide-band model, both losses included</td>
<td>$-28.9$</td>
<td>$-28.9$</td>
</tr>
<tr>
<td>Wide-band model, only radiation losses</td>
<td>$-20.9$</td>
<td>$-19.5$</td>
</tr>
<tr>
<td>Wide-band model, only conductor losses</td>
<td>$-23.5$</td>
<td>$-23.5$</td>
</tr>
<tr>
<td>Wide-band model, lossless</td>
<td>$-18.4$</td>
<td>$-17.6$</td>
</tr>
</tbody>
</table>
6.2. Defected Ground Structure with asymmetric slot positions

In this section, we show that the wide-band equivalent circuit model that was obtained in Section 6.1 is also valid for the modeling of a DGS with a rectangular slot that is positioned asymmetrically with respect to the transmission line (in other words, an asymmetrical feedpoint of the slot). The position of the feedpoint selects which harmonic responses are captured in the slot and which ones are not. The harmonic responses that are captured in the slot will be absent in the FRF.

Figure 6.8 shows the first four modes for a DGS with a feedpoint at $1/3$ of the slot (right) and for a DGS with a feedpoint at $1/4$ of the slot (left). In general, when the transmission line is positioned at $1/\rho$ of the slotline (right), the electric field at the
6. Extensions of the circuit model

Figure 6.8: The slot of a DGS with an asymmetric feedpoint at $1/p$ resonates at the modes, $m$, that are not a multiple of $p$: $m \neq p.k$ with $k = 1 : \infty$. The harmonic responses corresponding to these modes will be absent in the FRF.

feedpoint is zero for the modes with modal indices, $m$, that are a multiple of $p$:

$$m = p.k$$

(6.1)

with $k = 1 : \infty$. Hence, the slot will not resonate at these modes and the harmonic responses corresponding to these modes will be present in the FRF.

Results for the asymmetrical DGS

The wide-band circuit model in Figure 6.2 can be used to model a DGS with a slot that is positioned asymmetrically with respect to the transmission line. The model is validated on measurements for three different design cases, shown in Figure 6.9. A DGS with a feedpoint of the slot at $1/3$ (Case 1), $1/4$ (Case 2) and $1/5$ (Case 3) of its length is examined. The dimensions of the DGS ($W$, $W_s$ and $L_s$) are identical for the three design cases, only the position of the feedpoint changes. The identical ground plane with slot is shown on the left in Figure 6.9. The dimensions are given in Table 6.1.

The electric field is zero at the feedpoint of the modes that are a multiple of 3 ($m = 3k$) in Case 1, 4 ($m = 4k$) in Case 2 and 5 ($m = 5k$) in Case 3 with $k = 1 : \infty$. The corresponding harmonic responses will be present in the FRF since they are not captured in the slot. Within the selected frequency range $[0.1 - 5]$ GHz for Case
Figure 6.9.: The measured DGS with feedpoint at 1/3, 1/4 and 1/5 of the slotline. Left: the slotted ground plane is identical for the three cases.

1 and 2, respectively the first 2 and 3 antiresonances are measured and taken into account in our wide-band model ($K = 2$ and $K = 3$ in Figure 6.2). Within the selected frequency range [0.1 – 6] GHz for Case 3, 4 antiresonances are measured and taken into account in our model ($K = 4$ in Figure 6.2).

Table 6.4.: Optimized parameter values of the wide-band model in Figure 6.2 that was fit on the DGS design with feedpoint at 1/3.

<table>
<thead>
<tr>
<th>Microstrip line of the DGS</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ [pF]</td>
<td>$L$ [nH]</td>
<td>$R_{cond}$ [Ω]</td>
</tr>
<tr>
<td>0.533</td>
<td>1.50</td>
<td>2.35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slotline of the DGS</th>
<th>Radiation losses</th>
<th>LC-resonator @ $f_0$</th>
<th>LC-resonator @ $2f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{rad}$ [Ω]</td>
<td>$\alpha$</td>
<td>$C_{res,1}$ [pF]</td>
<td>$L_{res,1}$ [nH]</td>
</tr>
<tr>
<td>119e1</td>
<td>-1.54</td>
<td>2.73</td>
<td>5.48</td>
</tr>
</tbody>
</table>

The circuit parameters of the proposed wide-band model (Figure 6.2) are extracted from the measured data of the DGSs response using the Quasi-Newton optimizer in ADS [Keys 14a] for the three design cases. This optimizer minimizes the difference between the S-parameter response of the circuit model and the corresponding
Figure 6.10.: Comparison of the $S_{11}$-parameter of the measured DGS with a feed-point at $1/3$ (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (--) and fulfills the desired error level of $-25$ dB (--) quite well. The absolute RMSE is $-27.3$ dB. Top: amplitude response. Bottom: phase response.

measured S-parameter response of the DGS. A Rogers 4003 substrate was again used (Section 1.3: Substrate settings).

Figures 6.10-6.15 compare the amplitude and phase response of the $S_{11}$- and $S_{21}$-parameters of the measured DGS designs (—) and of the corresponding simulated wide-band models (—) for the 3 cases of interest. The absolute model error (Equation (1.2)) is shown in (--) and is expressed in dB. The optimized circuit parameter values can be found in Tables 6.4-6.6.

Table 6.7 gives the absolute and relative RMSE values (Equations (1.4)-(1.5)) for the considered design cases. For Cases 1 and 2, the absolute model error is below the desired error of $-25$ dB. For Case 3 this is not the case. However, in Case 3 we consider a wider frequency band, which makes it harder to achieve the minimum absolute model error over the whole band.
6.2. Defected Ground Structure with asymmetric slot positions

Figure 6.11.: Comparison of the $S_{21}$-parameter of the measured DGS with a feed-point at $1/3$ (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (—) and fulfills the desired error level of $-25\text{dB}$ (—) quite well. The absolute RMSE is $-28.9\text{dB}$. Top: amplitude response. Bottom: phase response.

In general, the model error, shown in (—), increases with the frequency for all three cases. The magnitude as well as the phase of the first two antiresonances is modeled quite well (see Figures 6.11, 6.13 and 6.15). From the third antiresonance on, the model error increases slightly. This leads to an absolute RMSE that is slightly higher than the desired error of $-25\text{dB}$ for Case 3.

In the design context we handled so far, modeling the fundamental antiresonance is the most important because this is the part we want to control to fulfill the filter template. Knowledge about the shape and bandstop behavior of the unwanted higher order harmonic responses is in first instance just a nice added value for the designer. It will not influence how well the filter specifications at the fundamental frequency are fulfilled.
To conclude, the fundamental frequency is modeled quite well. The harmonic responses are modeled well enough to give necessary information to the designer. These results are satisfying and prove that the model flexibility is sufficient to take the position of the slotline into account.

Figure 6.12.: Comparison of the $S_{11}$-parameter of the measured DGS with a feedpoint at $1/4$ (---) and the simulated wide-band circuit model (—). The absolute model error is shown in (--) and fulfills the desired error level of $-25\text{dB}$ (---) quite well. The absolute RMSE is $-27.1\text{dB}$. Top: amplitude response. Bottom: phase response.
6.2. Defected Ground Structure with asymmetric slot positions

Figure 6.13.: Comparison of the $S_{21}$-parameter of the measured DGS with a feed-point at $1/4$ (---) and the simulated wide-band circuit model (--). The absolute model error is shown in (-----) and fulfills the desired error level of $-25\,\text{dB}$ (-----) quite well. The absolute RMSE is $-25.5\,\text{dB}$. Top: amplitude response. Bottom: phase response.
6. Extensions of the circuit model

Table 6.5: Optimized parameter values of the wide-band model in Figure 6.2 that was fit on the DGS design with feedpoint at $1/4$.

<table>
<thead>
<tr>
<th>Microstrip line of the DGS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ [pF]</td>
<td>0.500</td>
<td>1.23</td>
<td>2.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slotline of the DGS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation losses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{rad}$ [Ω]</td>
<td>786</td>
<td>−0.846</td>
<td>3.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{res,1}$ [pF]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{res,1}$ [nH]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{res,2}$ [pF]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{res,2}$ [nH]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{res,3}$ [pF]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{res,3}$ [nH]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: Optimized parameter values of the wide-band model in Figure 6.2 that was fit on the DGS design with feedpoint at $1/5$.

<table>
<thead>
<tr>
<th>Microstrip line of the DGS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ [pF]</td>
<td>0.416</td>
<td>1.10</td>
<td>2.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slotline of the DGS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation losses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{rad}$ [Ω]</td>
<td>625</td>
<td>−0.483</td>
<td>5.50</td>
</tr>
</tbody>
</table>
6.2. Defected Ground Structure with asymmetric slot positions

Figure 6.14.: Comparison of the $S_{11}$-parameter of the measured DGS with a feed-point at $1/5$ (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (– —) and fulfills the desired error level of $-25\text{dB}$ (—) quite well at most frequencies. The absolute RMSE is $-23.3\text{dB}$. Top: amplitude response. Bottom: phase response.
Figure 6.15.: Comparison of the $S_{21}$-parameter of the measured DGS with a feed-point at $1/5$ (---) and the simulated wide-band circuit model (---). The absolute model error is shown in (——) and fulfills the desired error level of $-25$ dB (---) quite well at most frequencies. The absolute RMSE is $-22.8$ dB. Top: amplitude response. Bottom: phase response.
Table 6.7.: The wide-band model is also valid to model multiple antiresonances of an asymmetrical DGS. This table gives the absolute and relative model errors for the wide-band model (Figure 6.2) that was fit on measurements of the DGS designs in Figure 6.9 over a frequency band of [0.1 – 6] GHz for the DGS with feedpoint at 1/5 and over a frequency band of [0.1 – 5] GHz for the other two examples.

<table>
<thead>
<tr>
<th>Feedpoint at 1/3</th>
<th>absolute RMSE</th>
<th>relative RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{11}$</td>
<td>$S_{21}$</td>
</tr>
<tr>
<td>Feedpoint at 1/3</td>
<td>−27.3</td>
<td>−28.9</td>
</tr>
<tr>
<td>Feedpoint at 1/4</td>
<td>−27.1</td>
<td>−25.5</td>
</tr>
<tr>
<td>Feedpoint at 1/5</td>
<td>−23.3</td>
<td>−22.8</td>
</tr>
</tbody>
</table>
6.3. DGS with multiple slots

So far we studied a DGS with a single rectangular slot. In many applications, however, a DGS with multiple slots, called a periodic DGS (see Section 1.3), might be more suitable [Kim 00, Lim 02].

We want to keep the design procedure based on scalable circuit models, that was proposed in Chapter 4, and make it applicable to a periodic DGS. Therefore, we should improve the achieved wide-band equivalent circuit model from Section 6.1 (Figure 6.2) to model multiple slots. This is not straightforward since these slots interact with each other through coupling if they are close enough to each other [Garg 13].

We start by validating our wide-band model for a DGS with two identical slots for simplicity. A spacing, $p$, between two de-embedded elementary slot sections is selected that is large enough to avoid or at least keep the coupling effect sufficiently low to avoid strong coupling. Figure 6.16 shows such a DGS. The de-embedded slot sections are marked in (—). The spacing between these two elementary sections is a microstrip line with length $p$ in the DGS microstrip structure (Figure 6.16 (c)).

![Figure 6.16](image)

Figure 6.16: A DGS with two identical slots is shown with a spacing, $p$, between the two de-embedded slot sections (—). (a) cross-section. (b) and (c) top view.
A de-embedded section of a DGS with a single rectangular slot (—) can be modeled by the wide-band model shown in Figure 6.17 (left). In this section, we represent this model by the box shown on the right in Figure 6.17.

Figure 6.17.: Wide-band circuit model that models antiresonances \( k = 1:K \) of a DGS and its equivalent representation.

In a first attempt, we use two wide-band models to model a DGS with two slots: one for each elementary slot section (as was shown in Figure 6.16 (—)). As a first attempt, the spacing between two slot sections is modeled by a simple lossless delay-line. This delay-line has an electrical length that corresponds to the spacing, \( p \). The characteristic impedance of this delay-line equals 50Ω if we assume that the spacing, \( p \), is large enough to neglect the coupling between the slots. If \( p \) is small enough for coupling to occur, the characteristic impedance of the delay-line between the slots increases [Garg 13].

Note that a DGS is often used for high-impedance implementation of transmission lines. A DGS allows to create lines with a higher characteristic impedance without the need to make the transmission line width too small.

Figure 6.18.: Cascading two wide-band circuit models with a delay-line in between models a DGS with two slots without taking coupling into account.
6. Extensions of the circuit model

We use this model for the modeling of two DGS examples: one with a large spacing of \( p = 100 \text{ mm} \) to avoid coupling, and one with a small spacing of \( p = 2 \text{ mm} \), where a strong coupling will be present between the two slots. Both examples use two identical slots with the same dimensions as in the previous sections (Table 6.1).

Figures 6.19 and 6.20 compare the amplitude and phase response of the \( S_{11} \)- and \( S_{21} \)-parameters of the simulated DGS (shown in Figure 6.16) (—) and of the corresponding cascade of wide-band models (—). The spacing between the slots is \( p = 100 \text{ mm} \). The absolute model error (Equation (1.2)) is shown in (—) and is expressed in dB. An absolute RMSE (Equation (1.4)) of \(-18.9\text{dB}\) and \(-20.5\text{dB}\) is obtained for \( S_{11} \) and \( S_{21} \) respectively. Note that this does not meet the desired error of \(-25\text{dB}\). This means that the coupling between two slots cannot be ignored completely, even for a large spacing.

![Figure 6.19](image-url)  
Figure 6.19.: Comparison of the \( S_{11} \)-parameter of the simulated DGS with 2 identical slots with a spacing, \( p = 100\text{mm} \), (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (—) and is above the desired error level of \(-25\text{dB} \) (—). The absolute RMSE is \(-18.9\text{dB}\). Top: amplitude response. Bottom: phase response.
Figure 6.20: Comparison of the $S_{21}$-parameter of the simulated DGS with 2 identical slots with a spacing, $p = 100\text{mm}$, (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (— —) and is above the desired error level of $-25\text{dB}$ (— —). The absolute RMSE is $-20.5\text{dB}$. Top: amplitude response. Bottom: phase response.

Figures 6.21 and 6.22 compare the amplitude and phase response of the $S_{11}$- and $S_{21}$-parameters of the simulated DGS (shown in Figure 6.16) with a smaller spacing of $p = 2\text{mm}$ (—) and of the corresponding cascade of wide-band models (—). The absolute model error (Equation (1.2)) is shown in (— —) and is expressed in dB. An absolute RMSE (Equation (1.4)) of $-15.0\text{dB}$ and $-16.7\text{dB}$ is obtained for $S_{11}$ and $S_{21}$ respectively.

Since two identical slots of length $L_s = 80\text{mm}$ are used, we expect a first antiresonance around the fundamental frequency, $f_0 = 1.5\text{GHz}$, and a second antiresonance, around $3f_0 \approx 4.4\text{GHz}$ as in the previous sections of this chapter. However, the coupling expresses itself in the S-parameters by a double antiresonance. We now see two antiresonances around $f_0$, at $1.3\text{GHz}$ and $1.5\text{GHz}$, and two around $3f_0$, at $3.8\text{GHz}$ and $4.8\text{GHz}$, as shown in Figure 6.22, where we would expect only one
for each. Hence, by decreasing the spacing between the two slots, the coupling is more clearly pronounced. The model in Figure 6.18 cannot model the coupling well enough.

![Graph showing |S11| and \( \angle S_{11} \)](image)

Figure 6.21.: Comparison of the \( S_{11} \)-parameter of the simulated DGS with 2 identical slots with a spacing, \( p = 2 \text{mm} \), (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (— —) and is above the desired error level of \(-25\text{dB}\) (— —). The absolute RMSE is \(-15.0\text{dB}\). Top: amplitude response. Bottom: phase response.

Small spacings between slots, as in the second example where \( p = 2 \text{mm} \), are often desirable in practice. A small spacing keeps the physical dimensions of the DGS small. Furthermore, the coupling effect that then occurs enhances the DGS characteristics, which is exploited in several applications [Kim 00, Lim 02].

If the coupling between slots is strong, the coupling effect should be modeled in order to achieve a high accuracy of the equivalent circuit model. In a first attempt, capacitors can be used in case of an electric coupling or inductors in case of a magnetic coupling [Hong 01]. A mixed coupling should be used if neither the electric coupling nor the magnetic coupling can be ignored. A detailed
6.3. Defected Ground Structure with multiple slots

Figure 6.22: Comparison of the $S_{21}$-parameter of the simulated DGS with 2 identical slots with a spacing, $p = 2\text{mm}$, (—) and the simulated wide-band circuit model (—). The absolute model error is shown in (—) and is above the desired error level of $-25\text{dB}$ (—). The absolute RMSE is $-16.7\text{dB}$. Top: amplitude response. Bottom: phase response.

analysis of the coupling is out of the scope of this work. Nevertheless, recent research [Zeng 19] shows that the electric coupling between resonating slots is much smaller than the magnetic coupling and hence a coupling through mutual inductances should be sufficient to model the coupling effect.

As first attempt to model the coupling, we used a magnetic coupling between the inductors of the LC-resonators of the two cascaded wide-band models. The resulting model structure is shown in Figure 6.23. Note that the values of the components of the LC-resonators are not equal in the two cascaded models because four antiresonances should be taken into account. The other component values remain the same in both cascaded models.
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Note also that the delay-line between the two models, as is shown in Figure 6.18, is not necessary anymore. If the spacing, \( p \), is small, the microstrip line of length \( p \) in the DGS can be accurately modeled by the Telegraphers’ representation in the cascaded models.

![Wide-band model diagram](image)

Figure 6.23: Wide-band equivalent circuit model for a DGS with 2 slots that models 2 antiresonances.

Figures 6.24 and 6.25 compare the amplitude and phase response of the \( S_{11} \)- and the \( S_{21} \)-parameters of the simulated DGS with 2 identical symmetrical slots and those of the corresponding cascade of two wide-band models with a magnetic coupling between the inductors of the LC-resonators. We achieve an RMSE of −21.5 dB and −21.9 dB for \( S_{11} \) and \( S_{21} \), respectively. The coupling between the 2 slots is modeled quite well, although an additional improvement could be realized by also introducing an electric coupling in order to achieve the desired RMSE of −25 dB. In order to further improve the modeling of the coupling effect between slots, the interpretation of coupled slotlines could be used [Garg 13].

We can conclude that the wide-band equivalent circuit model that was extracted in this work, can be used in a wide variety of applications due to its reusability. A first attempt was made to extend the model for a periodic DGS, where coupling occurs between slots. The potential of the existing wide-band model for one slot was shown. It proved its solidity when used as a basic building block when modeling other resonating structures.
6.3. Defected Ground Structure with multiple slots

Figure 6.24.: Comparison of the $S_{11}$-parameter of the simulated DGS with 2 identical slots with a spacing, $p = 2\text{mm}$, (−) and the simulated cascade of wide-band circuit models with a magnetic coupling between the inductors of the LC-resonators (−−). The absolute model error is shown in (−−−) and is above the desired error level of $-25\text{dB}$ (−−). The absolute RMSE is $-21.5\text{dB}$. Introducing a magnetic coupling improves the model fit. Top: amplitude response. Bottom: phase response.
Figure 6.25.: Comparison of the $S_{21}$-parameter of the simulated DGS with 2 identical slots with a spacing, $p = 2\text{mm}$, (—) and the simulated cascade of wide-band circuit models with a magnetic coupling between the inductors of the LC-resonators (—). The absolute model error is shown in (—) and is above the desired error level of $-25\text{dB}$ (—). The absolute RMSE is $-21.9\text{dB}$. Introducing a magnetic coupling improves the model fit. Top: amplitude response. Bottom: phase response.
Part II.

Figures of merit for characterizing nonlinear devices
7. Assess the nonlinearity at the system level

The results of this chapter are published in [Rola 17, Van 18a]. The work in this chapter is based on the Best Linear Approximation framework that has been developed at the ELEC department of the VUB [Pint 12].

When designing a telecommunication system, the designer needs to limit the signal distortion that is present in any signal output of every device to meet the spectral masks that are imposed by the communication standard. A number of widely used figures of merit exist in the literature and are used to characterize the (nonlinear) behavior of a (modulated) device. This chapter proposes a novel technique to extract multiple measurement-based figures of merit using a single measurement that is taken from one single measurement setup. The method obtains separate estimates of the linear contribution, the noise contribution and the in-band and out-of-band nonlinear contribution. These estimates are used next to calculate the signal-to-noise (and distortion) ratio, the noise power ratio, the adjacent channel power ratio, and other figures of merit. The estimates are extracted in mean squares sense for a class of modulated excitation signals resembling the real communication signals like Long Term Evolution (LTE).

Section 7.1 gives the state of the art. Section 7.2 explains the identification method that we use here, called the Best Linear Approximation (BLA). The BLA splits the linear dynamics, the nonlinear distortion and the measurement noise. This allows to calculate improved figures of merit that are closer to their actual definitions than those obtained with the classical measurement setups. Section 7.3 defines the considered figures of merit in the classic way and in within the BLA-framework. Section 7.4 compares the new measurement approach to the classical one using experimental results.
7. **Assess the nonlinearity at the system level**

## 7.1. State of the art

In a telecommunication system, every linear or nonlinear component that is present in the Radio Frequency (RF) chain should be designed with care. The nonlinear behavior of a device has a significant impact on the overall performances of the telecommunication system whether it is a required effect, such as in a mixer, or a perturbation, such as the nonlinear distortion of an amplifier. A number of widely used figures of merit exist to characterize the nonlinear distortion of a modulated device [Walk 11]. Each distortion type comes with its own figures of merit and associated measurement setup. For example, the in-band nonlinear distortion is measured by the Noise Power Ratio (NPR) and the Error Vector Magnitude (EVM), while the out-of-band (nonlinear) distortion is characterized by the Signal-to-Noise Ratio (SNR), the Signal-to-Noise and Distortion ratio (SINAD) and the Adjacent Channel Power Ratio (ACPR).

For some specific components like power amplifiers, additional figures of merit exist, e.g. the 1-dB compression point and the Third Order Intercept point (TOI). For multi-carrier signals, the EVM, NPR and ACPR are widely used quantities to characterize the nonlinearity of active devices that are intended to be linear in systems operating under modulation [Walk 11]. The modulated transmission signal used during these tests needs to mimic the application or the standard signals, e.g. Wireless Local Area Network (WLAN), Long Term Evolution (LTE) [Rumn 13], for these figures of merit to be repeatable and reliable.

Traditionally, the above mentioned figures of merit are extracted using different measurement setups and require different measurements using different excitation signals. Hence, the testing time has the tendency to grow proportionally to the number of figures of merit to be determined. This becomes more and more problematic in the context of 5G, where agile receivers/transmitters will receive even more tests while the allotted testing time will certainly not increase [Walk 19]. The technique proposed here uses 1 single measurement setup and 1 measurement of 1 excitation signal to extract all these figures of merit at once.

Traditional figures of merit try to quantify how close the non-ideal output signal is to its ideal counterpart. However, they do not give any additional information about the source of the non-ideality. The non-ideality can be caused by spurious linear dynamics, nonlinear distortion or both. The approach used in this chapter separates the linear dynamics from the nonlinear distortion and the measurement noise for every spectral line (excited and non-excited), giving the designer the insight that is needed to help making proper design choices.
Another advantage is that the proposed method estimates the nonlinear distortion at the excited frequency bins, while these are often not known in the traditional methods. Taking the nonlinearity at excited frequency bins into account improves the accuracy of the measured figures of merit, bringing them closer to their original definition.

In this chapter, we use the Best Linear Approximation (BLA) method [Pint 12] to extract a set of measurement-based figures to assess the nonlinearity of a power amplifier. The new measurement approach uses a single special excitation signal to extract all figures of merit at once. The BLA method is shortly introduced in Section 7.2 for the convenience of the reader, together with the special excitation signal that is needed. Section 7.3 gives an overview of the different figures of merit that are extracted using the BLA method. Finally, Section 7.4 compares the results of the new measurement approach with the classical measurement approaches for each figure of merit.

7.2. The Best Linear Approximation

Most real-life systems show both a linear and a nonlinear behavior. To take the linear and nonlinear behavior of a system or a device into account, models are required. Nonlinear models have become increasingly important for measurement engineers. However, identifying a nonlinear device is not straightforward as many categories of nonlinearities exist and one nonlinear model cannot catch all of them. Hence, most models are ad hoc selections of a single reconfiguration of a device and excitation signal only. Besides, even if a good nonlinear model can be identified, the simple properties of linear systems are no longer valid for these nonlinear models. This hampers the intuitive understanding of the systems’ operation when using these nonlinear models.

The Best Linear Approximation (BLA) models a nonlinear system using a local linear model that depends on the nonlinear operating point of the device. It is only valid for a class of signals with a given probability density function (pdf) and a given Power Spectral Density (PSD) and for a single system. The BLA consists of an Linear Time-Invariant (LTI) system and two output noise sources, one for the nonlinear distortion and one for the measurement noise. The LTI system approximates the linear behavior of this system in mean-square sense. Using the BLA allows to extend, with some care, the concepts of linear system theory to nonlinear systems. The class of nonlinear systems that is considered in this work is restricted to Periodic-In Same Period-Out (PISPO) systems. These are systems
7. Assess the nonlinearity at the system level

whose steady-state response to a periodic input is a periodic signal with the same period as the input [Pint 12].

The Best Linear Approximation (BLA) of a nonlinear system depends on the input signal. The BLA is the same for a class of input signals sharing the same pdf and the same PSD [Pint 12]. Hence, the BLA model and all figures of merit that are extracted by using the BLA are valid for all excitation signals within a selected class of signals.

Extracting the BLA in the frequency domain is easy when using a periodic excitation signal. However, actual communication signals, e.g. LTE signals, are not periodic. This means that we should carefully select a periodic excitation signal that belongs to the class of signals sharing the same pdf and the same PSD as the actual communication signals. This is done such that the results obtained with this periodic excitation signal represent the results that would be obtained when using actual communication signals.

The selected excitation signal is a random phase multisine. This is a periodic signal that belongs to the class of random excitation signals with a Gaussian pdf. This class of signals also contains Gaussian noise and most actual communication signals. The PSD of the random phase multisine can easily be shaped to resemble the PSD of a communication signal. Hence, the results extracted using a random phase multisine match those that are obtained in real applications using actual communication signals. Figure 7.1 shows a random phase multisine (left) in the time domain and compares this signal to an LTE signal (right). The two signals clearly look alike.

A multisine, $u(t)$, is a sum of harmonically related sines [Pint 12]:

$$u(t) = \sum_{k=1}^{F} A_k \cos (2\pi f_0 kt + \phi_k)$$

(7.1)

with $F$ the number of tones, $T_0 = 1/f_0$ the period of the multisine, $A_k$ the amplitude of the $k^{th}$ tone and $\phi_k$ its phase. For the pdf of multisine to be Gaussian, the phases $\phi_k$ should be uniformly random distributed in $[0, 2\pi]$ and, in theory, the number of tones should be infinite, $F = \infty$. In practice, $F > 20$ proves to work already very well for $A_k = A \forall k$, which will be the case here. Otherwise much more might be needed.

A multisine with a random phase distribution is called a random phase multisine. Two realizations of a random phase multisine are shown in Figure 7.2 in the frequency domain. They have the same amplitude, but a different random phase realization.
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Figure 7.1.: The random phase multisine resembles actual communication signals, e.g. an LTE signal. This figure shows the amplitude of a random phase multisine (left) and of an LTE signal (right) in the time domain.

Figure 7.2.: Two random phase multisines have the same amplitude in the frequency domain, but a different random phase.
When applying a random phase multisine to a PISPO system, the measured output of the nonlinear system, $B_m(\omega)$, can be split into a linear dynamic output, $B_r(\omega)$, a noise source representing the stochastic nonlinear distortion at the output, $N_{NL}(\omega, P_{in})$, and a noise source representing the measurement noise at the output of the system, $N_n(\omega)$ [Pint 12]:

$$B_m(\omega) = G_{BLA}(\omega)A_r(\omega) + N_{NL}(\omega, P_{in}) + N_n(\omega)$$

$$= B_r(\omega) + N_{NL}(\omega, P_{in}) + N_n(\omega) \quad (7.2)$$

with

- $B_m(\omega, P_{in})$ the measured output signal,
- $B_r(\omega)$ the dynamic output signal,
- $G_{BLA}(\omega)$ the Best Linear Approximation (BLA) of the nonlinear system in mean squares sense for the considered class of excitation signals,
- $A_r(\omega, P_{in})$ the noiseless periodic multisine excitation signal,
- $N_{NL}(\omega, P_{in})$ the stochastic nonlinear contribution to the output signal, which is a colored noise source with zero mean and standard deviation $\sigma_{NL}(\omega, P_{in})$ and which can be measured at all measurement frequency bins (excited and non-excited),
- $N_n(\omega)$ the measurement noise, which is a colored noise source with zero mean and standard deviation $\sigma_n(\omega)$ and which can be measured at all measurement frequency bins.

The BLA of the nonlinear system consists of two contributions:

$$G_{BLA}(\omega) = G_0(\omega) + G_B(\omega) \quad (7.3)$$

with $G_0(\omega)$ the dynamic gain of the true underlying linear system (if it exists) and $G_B(\omega)$ the systematic nonlinear contribution to the Frequency Response Function (FRF) (these nonlinearities are independent of the random phase of the multisine). Extra references and a more detailed and theoretical description of the Best Linear Approximation method can be found in [Pint 12].

A robust method to measure the stochastic nonlinear distortion $N_{NL}(\omega, P_{in})$ uses $M$ different random phase multisine realizations. Figure 7.2 shows two realizations of a random phase multisine in the frequency domain. The amplitude response (top figure) are the same, but the random phases of the excited frequency differ. Note that the amplitude as well as the phase at non-excited frequency bins is zero. Each realization of the signal is measured over a number of periods, $P$. Averaging
7.2. The Best Linear Approximation

the measured responses over the $P$ periods gives an estimation for the mean value of each realization and of the noise variance $\sigma_n^2(\omega)$. Averaging the measured responses over the $M$ different random phase realizations gives an estimation for the total variance $\sigma_{\text{total}}^2(\omega, P_{\text{in}}) = \sigma_n^2(\omega) + \sigma_{\text{NL}}^2(\omega, P_{\text{in}})$. An estimation for the variance of the nonlinear contributions, $\sigma_{\text{NL}}^2(\omega, P_{\text{in}})$, can be calculated from this formula. For a more detailed explanation of the robust method described here, we refer to [Pint 12]. At least two steady-state periods are needed to calculate the noise variance ($P \geq 2$) and at least seven realizations are needed to ensure that the properties of the estimator used in the parametric modeling step hold ($M \geq 7$) [Scho 12].

If measurement time is critical, a single experiment (1 realization) is sufficient to obtain the 3 quantities in Equation (7.2) at the cost of a lower accuracy when compared to multiple realizations. This is called the fast method. Again, at least two steady-state periods are needed to be able to calculate the noise variance ($P \geq 2$). Averaging over only $P = 2$ periods results in a higher variability of the measurement. A single experiment does not allow to separate the linear contribution $G_{\text{BLA}}(\omega)A_r(\omega)$ from the nonlinear noise contribution $N_{\text{NL}}(\omega, P_{\text{in}})$ on the excited lines by measurement. It requires an interpolation to do so instead.

To be able to estimate the nonlinear noise contribution $N_{\text{NL}}(\omega, P_{\text{in}})$ on the excited lines, a special random phase multisine is required where some of the in-band harmonics are randomly omitted. The omitted in-band harmonics are called detection lines. This special multisine is called a random phase multisine with random harmonic grid. Using this type of multisine allows an estimation of $\sigma_{\text{NL}}(\omega, P_{\text{in}})$ at the excited lines based on an interpolation of the contributions at adjacent non-excited lines or detection lines. One can proof that this introduces an error in the estimation of the standard deviation that can become as large as 3 dB [Scho 08].

In Section 7.4, we will use a random phase multisine excitation with random harmonic grid where one randomly selected spectral line out of each group of three successive lines is omitted. Figure 7.3 shows the response of such an excitation signal when applied to a power amplifier. Note that the robust method can also be applied when using different realizations of a random phase multisine with random harmonic grid. Since the robust method provides more accurate results than the fast method, the robust method will be used in the next two sections whenever the BLA-framework is used to estimate multiple figures of merit.

Figure 7.3 shows the measured response of a random phase multisine with random harmonic grid (with a power level of $P_{\text{in}} = 5$ dBm) ($\bullet$) when applied to a power amplifier (the measurement setup is discussed in Section 7.4). As discussed above (Equation (7.2)), the output spectrum $B_{\text{in}}(\omega)$ can be split into a dynamic output
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Figure 7.3.: Output spectrum for $P_{in} = 5$ dBm for a random phase multisine excitation with a random harmonic grid ($\bullet$). Following Equation (7.2), the measured output $B_m(\omega)$ is split into the dynamic output signal $B_r(\omega)$ ($\bullet$), the measurement noise with standard deviation $\sigma_n(\omega)$ ($\bullet$) and the nonlinear contribution with standard deviation $\sigma_{NL}(\omega, P_{in})$ ($\bullet$). The total distortion standard deviation $\sigma_{total}$ is shown in ($\bullet$).

signal $B_r(\omega)$ ($\bullet$), the measurement noise $N_n(\omega)$ with standard deviation $\sigma_n(\omega)$ ($\bullet$) and the nonlinear contribution $N_{NL}(\omega, P_{in})$ with standard deviation $\sigma_{NL}(\omega, P_{in})$ ($\bullet$). The total distortion standard deviation $\sigma_{total}$ is shown in ($\bullet$). Note that the nonlinearities ($\bullet$) are present at out-of-band non-excited lines (spectral regrowth) and at in-band non-excited lines (detection lines).

Figures 7.4-7.6 show the dynamic output signal $B_r(\omega)$ with the standard deviation of the measurement noise $\sigma_n(\omega)$ (Figure 7.4), the standard deviation of the nonlinear distortion $\sigma_{NL}(\omega, P_{in})$ (Figure 7.5) and the total standard deviation $\sigma_{total}$ (Figure 7.6) as a function of the input power level, $P_{in}$.

The level of the measurement noise increases slightly with the input power, $P_{in}$ at the excited frequency bins (Figure 7.4). This can be expected as the noise that is present at the excited bins of the input signal is amplified together with the signal power at these bins.

We expect the nonlinear distortion contribution to be zero for infinitely small input powers, $P_{in}$, and to gradually start increasing and spreading over a wider frequency band with an increasing input power. From Figure 7.5, we see that the nonlinear distortion, $\sigma_{NL}(\omega, P_{in})$, is not decreasing any further when the input power decreases below a frequency dependent limit. Besides that, the variability of
Figure 7.4.: The dynamic output signal $B_r(\omega)$ (top, in red) and $\sigma_n(\omega)$ (bottom, in blue), both expressed in dBV, are shown as a function of the input power level, $P_{in}$. 
Figure 7.5.: The dynamic output signal $B_r(\omega)$ (top) and $\sigma_{NL}(\omega,P_{in})$ (bottom), both expressed in dBV, are shown as a function of the input power level, $P_{in}$. 
Figure 7.6.: The dynamic output signal $B_r(\omega)$ (top) and $\sigma_{\text{total}}$ (bottom), both expressed in dBV, are shown as a function of the power level of the input signal, $P_{in}$. 
7. Assess the nonlinearity at the system level

the nonlinear distortion increases at the frequency and power bins where we would expect the nonlinear distortion to reach a level that is comparable to the level of the measurement noise, $\sigma_n(\omega)$. The reason is that when the measurement noise dominates, it becomes difficult to estimate the level of nonlinear contributions and measurement noise accurately. The variability of the nonlinear distortion is very high at frequency bins where the nonlinear distortion is expected to become of the same magnitude as the measurement noise. Therefore, $\sigma_{NL}(\omega, P_{in})$ is not to be trusted at these bins. Stochastic significance testing can be used as a selection criterion to decide whether only measurement noise or a nonlinear contribution with noise is present at a certain frequency bin. This is done in [Van 02], where a $t$-test is used to test the signal presence hypothesis.

The information that is contained in the measured frequency and power dependent total variance $\sigma^2_{\text{total}}(\omega, P_{in}) = \sigma^2_n(\omega) + \sigma^2_{NL}(\omega, P_{in})$ is summarized in Figure 7.6. Note that the distortion increases gradually with the input power level and spreads over a wider bandwidth, spanning 3 times the excited bandwidth from $P_{in} \approx -10\text{dBm}$ on and 5 times that bandwidth from $P_{in} \approx 0\text{dBm}$ on. Note also the presence of the measurement noise floor at the frequency bins that are further away from the excited frequency band.

The BLA is and remains a small signal model of the power amplifier under test around a nonlinear operating point. A large signal model can be extracted based on the BLA using a metamodeling technique, as was introduced in Part I of this work. This model is then based on a set of small signal BLA models at well-chosen operating points. However, extracting this large signal model is out of the scope of this work.

7.3. Best-Linear-Approximation-based extraction of figures of merit

Splitting the measured output of a nonlinear system in a linear dynamic term, a nonlinear distortion term and the measurement noise via the BLA allows to extract different figures of merit starting from the same measurement. These BLA-based extracted figures of merit estimate the classic figures of merit very well. On top of that, using the BLA allows to extract new models of the figures of merit in such a way that they lie closer to their actual definition.

To illustrate this, we consider three commonly used out-of-band nonlinear distortion measures: the Signal-to-Noise Ratio (SNR), the Signal-to-Noise And Distortion ratio (SINAD) and the Adjacent Channel Power Ratio (ACPR), and three
commonly used in-band nonlinear distortion measures: the Error Vector Magnitude (EVM), the Noise Power Ratio (NPR) and the 1-dB compression point. First, the classic definition of these figures of merit is given. Next, the new definitions based on the BLA framework are introduced. Both definitions are compared in the Section 7.4 and the similarities and differences are shown through measurement examples.

**SNR**

The Signal-to-Noise Ratio (SNR) measures the signal strength relative to the measurement noise. It is defined as the ratio between the expected value ($\mathbb{E}$) of the signal power and the expected value of the noise power [Walk 11]. It is useful for extracting e.g. Bit Error Rate (BER) curves as a function of the SNR [Ghar 04].

In the classical case, the SNR is measured using a sine wave excitation and a spectral measurement over a certain bandwidth. This is called $\text{SNR}_{\text{Classic}}$ from now on. The BLA uses a multisine excitation instead. The SNR defined within the BLA-framework is called $\text{SNR}_{\text{BLA}}$. It is calculated as:

$$\text{SNR}_{\text{BLA}}(P_m) \triangleq \frac{\mathbb{E}_{k \in \{\text{Exc}\}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{\text{All}\}} \{\sigma_n^2(l)\}} \quad (7.4)$$

with $\{\text{Exc}\}$ the set of excited frequency bins and $\{\text{All}\} = \{\text{Nonexc}\} \cup \{\text{Exc}\}$ the set of all excited and non-excited frequency bins within the complete measurement bandwidth to be considered. Hence, the SNR is defined for a specific integration bandwidth, which is the measured frequency band in our case. Note that in case of a sine excitation, $\text{SNR}_{\text{Classic}}$ is calculated in the same way, but $\{\text{Exc}\}$ then consists of only one excited frequency. The results of both approaches are compared in Section 7.4.

**SINAD**

The Signal-to-Noise And Distortion ratio (SINAD) is defined as the ratio between the expected value of the signal power and the sum of the expected value of the noise and the expected value of the nonlinear distortion power [Walk 11].

In the classical case, the SINAD is measured by a sine wave excitation measurement ($\text{SINAD}_{\text{Classic}}$). In this case, the nonlinear distortion obviously only includes the harmonics of the excitation tone. However, real communication signals always have an excitation bandwidth, causing spectral regrowth to appear in the output
Assess the nonlinearity at the system level

Spectrum close to and inside the excitation bandwidth (Figure 7.3). This cannot be measured with the classical sine wave excitation signal.

An alternative measurement is to use a 2-tone excitation signal. In that case, spectral regrowth will occur and this will lead to a more correct measure of the SINAD. However, actual communication signals contain more than two tones. Within the BLA-framework, a multisine excitation is used that resembles a real communication signal as much as possible. The SINAD calculated within the BLA-framework, $\text{SINAD}_{\text{BLA}}$, is defined as

$$\text{SINAD}_{\text{BLA}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{Exc}\}} \{ |B_m(k)|^2 \}}{\mathbb{E}_{l \in \{\text{Nonexc}\}} \{ \sigma_n^2(l) + \sigma_{\text{NL}}^2(l, P_{\text{in}}) \}}$$

(7.5)

with $\{\text{Exc}\}$ the set of excited frequency bins and $\{\text{Nonexc}\}$ the set of non-excited frequency bins within the complete measurement band (in-band and out-of-band). Hence, the SINAD is also defined for a specific integration bandwidth, which is the measured frequency band in our case.

Besides the contributions at the in-band and out-of-band non-excited frequency lines, there will also always be nonlinearities present at the excited frequency bins. The BLA-method estimates the nonlinear distortion at these excited frequency bins as well (as explained in Section 7.2). This allows us to introduce a new definition of the SINAD, $\text{SINAD}_{\text{New}}$:

$$\text{SINAD}_{\text{New}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{Exc}\}} \{ |B_m(k)|^2 \}}{\mathbb{E}_{l \in \{\text{All}\}} \{ \sigma_n^2(l) + \sigma_{\text{NL}}^2(l, P_{\text{in}}) \}}$$

(7.6)

with $\{\text{Exc}\}$ the set of excited frequency bins and $\{\text{All}\} = \{\text{Nonexc}\} \cup \{\text{Exc}\}$ the set of all excited and non-excited frequency bins within the complete measurement band (in-band and out-of-band).

This new model is closer to the actual idea behind the definition of the SINAD. Although the nonlinear contributions are averaged over the selected frequency bins, the denominator then becomes larger because the nonlinear contributions are higher at the in-band frequency bins. Including the excited frequency bins in the expected value therefore provides a higher and a more correct weight to the nonlinear distortion that is present at the in-band excited frequency bins. The results are compared in Section 7.4.
7.3. **Best-Linear-Approximation-based extraction of figures of merit**

ACPR

The Adjacent Channel Power Ratio (ACPR) or the Adjacent Channel Leakage Ratio (ACLR) characterizes the spectral regrowth that is caused by the nonlinearity of a device in a communication system [Raab 02, Cola 09].

The total ACPR is mostly defined as the ratio between the expected value of the in-band signal power and the expected value of the out-of-band power [Cola 09, Pedr 02]. Sometimes it is also defined as the ratio between the expected value of the out-of-band power and the expected value of the in-band signal power [Walk 11]. We will use the first definition here:

$$\text{ACPR}_{BLA}(P_{in}) = \frac{\mathbb{E}_{k \in \{|IB\}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{|OB\}} \{\sigma_n^2(l) + \sigma_{NL}^2(l, P_{in})\}}$$  \hspace{1cm} (7.7)

with $\{|IB\}$ the set of all excited $\{|Exc\}$ and all in-band non-excited frequency bins (=detection lines) $\{|Det\}$ (hence, $\{|IB\} = \{|Exc\} \cup \{|Det\}$) and $\{|OB\}$ the set of all the out-of-band non-excited frequency bins.

Note that the $\text{ACPR}_{BLA}(P_{in})$ is defined in the same way as the classical one, $\text{ACPR}_{BLA}(P_{in})$. Unfortunately, these definitions of the ACPR include both nonlinear distortion as well as distortion due to the measurement noise. The BLA-method allows to split the nonlinear distortion contribution from the measurement noise. Therefore, we can define a new ACPR that only depends on the nonlinear distortion and which lies closer to its actual definition:

$$\text{ACPR}_{NoNoise}(P_{in}) = \frac{\mathbb{E}_{k \in \{|IB\}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{|OB\}} \{\sigma_{NL}^2(l, P_{in})\}}$$  \hspace{1cm} (7.8)

The ACPR can also defined for the two sidebands (upper and lower channels) of the excited frequency band. The BLA can also be used to calculate these variants. However, this has not been calculated in this work.

Note that multitone signals are already used for ACPR classic measurements [Pedr 99].

EVM

The Error Vector Magnitude (EVM) is widely used to assess the nonlinearity of a system, a component, or a device operated under modulated excitation. A
production-level EVM test excites the Device Under Test (DUT) with binary data signals obeying the communication standard at hand. The extracted EVM is then called a digital EVM [Helf 05].

In this work, we focus on the measurement and modeling of the analog EVM [McKi 04]. The excitation then is a continuous time signal rather than a bit sequence [Heut 97]. The continuous time definition of the EVM is the ratio of the powers of the error signal and the reference signal

\[
E_{\text{VM}}_{\text{continuous}} \triangleq \lim_{T \to \infty} \frac{\frac{1}{T} \int_{-T}^{+T} b^2_{\text{err}}(t) \, dt}{\lim_{T \to \infty} \frac{1}{T} \int_{-T}^{+T} b^2_{r}(t) \, dt}
\]

(7.9)

with \( b_r(t) = \tilde{g} a_r(t) \) the power wave of the output reference signal, \( b_{\text{err}}(t) = b_m(t) - \tilde{g} a_r(t) \) the error signal for a static in-band gain of the DUT, \( b_m(t) \) the measured output wave of the DUT, \( a_r(t) \) the excitation reference signal, and \( \tilde{g} \) the static gain attributed to the DUT.

To our knowledge, the classical EVM measurement proceeds in two steps. First, it estimates the static gain \( \tilde{g} \) of the output reference signal \( b_r(t) \) using only a part of the measured input \( a_m(t) \) and the corresponding output \( b_m(t) \). A (different) part of the signal \( a_m(t) \) is attributed to the symbol \( a_r(t) \) and is used next to obtain the error \( b_{\text{err}}(t) \). These signals are random data sequences, hence they produce aperiodic signals. As the transformation to the frequency domain is error-prone for aperiodic signals, this transformation is avoided in prior art. All processing is performed in the time domain instead.

This two step scheme shows some weaknesses, especially when small EVM values have to be determined accurately. As the DUT is a nonlinear and dynamic system, the linearized gain becomes a dynamic frequency response that depends on the PSD and the pdf of the input signal \( a_r(t) \). An accurate value for this response can only be obtained if the dynamic part of the gain is properly incorporated. This requires that the properties of the excitation signal that is used to determine the (dynamic) gain and the excitation signal that is used to measure the EVM are equal. If the signal properties of the two parts of the input signal differ, the estimated gain and the actual dynamic frequency response will differ too. This results in an EVM estimate that combines the linear dynamic error and the nonlinear distortion of the DUT.

To get around these limitations, we take a closer look at the class of random input signals with a pre-specified pdf and PSD, matching that of the targeted telecommunication signal. A careful analysis shows that this class also contains...
some periodic signals, the random phase multisine signals. Selecting the periodic random multisine signal as an excitation for the DUT allows to transform the EVM to the frequency domain without time-to-frequency transformation errors, to obtain [McKi 04]:

$$EVM_{\text{classic}} = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{+T} |B_m(\omega) - G_{\text{BLA}}(\omega)A_r(\omega)|^2 d\omega$$

(7.10)

where uppercase letters refer to the Fourier spectra of the corresponding signals. $B_m(\omega)$ is the measured output wave (the response of the DUT) and $B_r(\omega)$ the dynamic output wave corresponding to the periodic input wave $A_r(\omega)$. $G_{\text{BLA}}(\omega)$ represents the (quasi-)static (LTI) gain attributed to the DUT.

The classical EVM measurement assumes a purely static gain $G_{\text{BLA}}(\omega) = \tilde{G}_0$ in Equation (7.10). Practically speaking, this assumes that the frequency response is constant over the modulation bandwidth. In practice, however, a linear dynamic distortion can and often will be present. Besides, to the knowledge of the authors, it is not clear that the signal, used in the classical methods to measure the static gain, shares its pdf and PSD with actual communication signals that is used to measure the EVM. As a consequence, the classical measured EVM sums the linear dynamic and the nonlinear distortion: it becomes dependent both on the specific excitation signal and the linear system dynamics.

The method proposed here estimates the dynamic output signal $B_r(\omega)$ and the error simultaneously using the BLA, hereby avoiding a separate gain estimation. The BLA approximates the response of a nonlinear period-conserving dynamic system excited by a signal of fixed pdf and PSD in mean square sense.

Starting from the BLA, the EVM can now be obtained in a single step. The numerator of Equation (7.10) is easily obtained as the sum of the variances of the nonlinear and measurement noise sources over the bandwidth. The denominator boils down to the sum of the power of the averaged spectral lines of $B_r(\omega)$, taken at the excited frequency bins only. The EVM then becomes

$$EVM_{\text{BLA}}(P_{in}) \triangleq \frac{\mathbb{E}_{k \in \{\text{Det}\}} \{\sigma_n^2(k) + \sigma_{NL}^2(k, P_{in})\}}{\mathbb{E}_{l \in \{\text{Exc}\}} \{|B_r(l)|^2\}}$$

(7.11)

with $\{\text{Exc}\}$ the set of excited frequency bins and $\{\text{Det}\}$ the set of all the in-band spectral lines that carries nonlinear distortion.
Using the BLA-framework allows to calculate the nonlinear distortion at the excited frequency bins \(\{\text{Exc}\}\) as well as the detection lines \(\{\text{Det}\}\), or \(\{\text{IB}\} = \{\text{Exc}\} \cup \{\text{Det}\}\). Note that this is also true when the EVM is calculated in the time domain. Within the BLA-framework, this leads to a more correct calculation of the EVM that lies closer to its actual definition:

\[
EVM_{\text{New}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{IB}\}} \{\sigma_n^2(k) + \sigma_{NL}^2(k, P_{\text{in}})\}}{\mathbb{E}_{l \in \{\text{Exc}\}} \{|B_r(l)|^2\}}
\] (7.12)

Note that the presence of the measurement noise in this expression has an influence on the obtained EVM, while intuitively one expects it to contain the nonlinear distortion only. As the BLA framework provides a separate estimation of the total and the measured variance, it becomes possible to obtain a compensation for the influence of the measurement noise on the measured EVM. The EVM without noise contribution is then calculated by

\[
EVM_{\text{NoNoise}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{IB}\}} \{\sigma_{NL}^2(k, P_{\text{in}})\}}{\mathbb{E}_{l \in \{\text{Exc}\}} \{|B_r(l)|^2\}}
\] (7.13)

A comparison of EVM measurements is given in Section 7.4.

**NPR**

The Noise Power Ratio (NPR) measures the in-band distortion for a modulated signal. It is defined as the ratio between the expected value of the signal power and the expected value of the in-band nonlinear distortion power [Cola 09, Somb 11, Walk 11, Pedr 02]. The NPR is defined in the same way as SINAD, except for the frequency band over which the noise and distortion power are averaged: the SINAD is a measure for the distortion in the complete measurement band, while the NPR is a measure for the in-band distortion only.

Classically, the NPR is measured by exciting the system with band-limited Gaussian noise with a small notch in the excited frequency band, which we will label as \(\{\text{Notch}\}\) [Reve 00]. The \(\text{NPR}_{\text{Classic}}\) is then calculated by

\[
\text{NPR}_{\text{Classic}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{Exc}\}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{\text{Notch}\}} \{\sigma_n^2(l) + \sigma_{NL}^2(l, P_{\text{in}})\}}
\] (7.14)
This way of measuring the NPR leads to an underestimation of the distortion because the nonlinear distortion is usually smaller in the notch than at the excited frequency bins [Pedr 99]. This is because the notch eliminates many intermodulation products that would otherwise fall on these exact frequency bins. However, it was proven that not only an underestimation but also an overestimation of the NPR is possible, depending on the ratio of the number of non-excited bins inside the notch and the number of excited bins.

Figures 7.7 and 7.8 (top) show two excitation multitone signals (band-limited Gaussian noise) of 10 and 40 excited bins respectively, each with a notch of 3 bins in the middle, $A$ (—). These excitation signals are applied to a simulated DUT, which is a power amplifier with a mathematical model $y = a_1x + a_2x^2 + a_3x^3$. Figures 7.7 and 7.8 (bottom) show the simulated response of this DUT to such an excitation signal. This response is split into the linearized output signal, $B = G_{BLA}A$ (—), the nonlinear distortion (—) and the measurement noise (—). Note that these figures are meant as an illustration. In this illustration, we consider only second and third order nonlinearities of the DUT. For modulated signals, the even order nonlinearities fall outside the excitation frequency band. Hence, in this case only the third order nonlinear contributions are important.

Note that this example is merely an illustration for the number of nonlinear contributions per bin. Hence, quantitative results concerning the amplitude should not be compared to real-life examples. The amplitude of the nonlinear distortion bins (—) is purely related to the number of stochastic nonlinear contributions that fall on each bin. In real-life examples, there will be stochastic as well as coherent contributions and these contributions are all complex. The coherent contributions are not considered in our illustration, neither is the phase of the stochastic contributions.

In the case where $A$ consists of 10 excited bins, the average number of nonlinear contributions that fall on the excited bins equals 27.4, while the average number of nonlinear contributions that fall on the non-excited bins equals 20.7. By using the classic NPR calculation (expected value of the signal power at the excited bins divided by the expected value of the distortion in the notch), the nonlinear contribution is underestimated (80%), leading to an overestimation of the NPR. Usually, the number of excited bins is much larger than the number of bins in the notch. In the case where $A$ consists of 40 excited bins, the average number of nonlinear contributions that fall on the excited bins equals 490.7, while the average number of nonlinear contributions that fall on the non-excited bins equals 523.3. By using the classic NPR calculation, the nonlinear contribution is overestimated (106%), leading to an underestimation of the NPR.
7. **Assess the nonlinearity at the system level**

![Figure 7.7](image)

Figure 7.7.: Calculating the NPR by using the response (bottom) of the DUT to a multitone signal with $F = 10$ excited bins and a gap of 3 bins in the middle (---) (top) leads to an overestimation of the NPR. The response is split into $B = G_{BLA}A$ (---), the nonlinear distortion (---) and the measurement noise (---).

The position of the notch within the excited frequency band also has an influence on the NPR. The size of the notch (the number of non-excited bins in the notch) is very important to obtain a good estimation of the NPR using this method.

The multisine excitation signal with random harmonic grid used in the BLA-method has many detection lines that can replace the use of the notch in the excitation signal used in the classical measurements. The position of the detection lines is spread throughout the excitation band, eliminating the drawback of the influence of the position of the notch when compared to the classical NPR measurement. This is important because the level of nonlinear distortion is not uniformly distributed at the excited frequency bins.

Figure 7.9 (top) shows a random phase multisine excitation with $F = 40$ excited bins and a random harmonic grid where one out of three bins is omitted, $A$ (---).
7.3. Best-Linear-Approximation-based extraction of figures of merit

Figure 7.8.: Calculating the NPR by using the response (bottom) of the DUT to a multitone signal with \( F = 40 \) excited bins and a gap of 3 bins in the middle (---) (top) leads to an underestimation of the NPR. The response is split into \( B = G_{BLA} \) (---), the nonlinear distortion (---) and the measurement noise (---).

Figure 7.9 (bottom) shows the response to such an excitation of a DUT. This response is split into the output signal, \( B \) (---), the nonlinear distortion (---) and the measurement noise (---). The BLA-based NPR is calculated by dividing the expected value of the signal power at the excited bins by the expected value of the nonlinear distortion at the detection lines and the excited bins. No under- or overestimation is done here since the nonlinear distortion is known at both excited and non-excited bins.

If only one realization \( M = 1 \) is used (fast method, Section 7.2), the NPR is calculated by dividing the expected value of the signal power at the excited bins by the expected value of the nonlinear distortion at the detection lines. The ratio of the number of stochastic nonlinear contributions at the detection lines and the sum of the number of stochastic nonlinear contributions at the excited bins converges
Assess the nonlinearity at the system level

Figure 7.9.: Calculating the NPR by using the response (bottom) of the DUT to a random phase multisine with \( F = 40 \) excited bins with a random harmonic grid where 1 out 3 bins is omitted (—) (top) leads to a more accurate estimation of the NPR. The response is split into \( B = G_{\text{BLA}}A \) (—), the nonlinear distortion (—) and the measurement noise (—).

to 100\% for an infinite number of excited bins \( F \to \infty \). Even for a limited number of bins as low as \( F = 100 \), a ratio of 98\% is already obtained. This shows that the stochastic convergence is fast.

Using a random phase multisine with random harmonic grid gives a more consistent estimation of NPR (for \( F \to \infty \)) because the detection lines are randomly selected and spread over the whole excited frequency band. Hence, it takes the shape of the nonlinear distortion in the measurement band into account. We define the NPR within the BLA-framework as:

\[
NPR_{\text{BLA}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{\text{Exc}\}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{\text{Det}\}} \{\sigma_n^2(l) + \sigma_{\text{NL}}^2(l, P_{\text{in}})\}} \tag{7.15}
\]
Using the BLA-framework allows to calculate the nonlinear distortion at the excited frequency bins as well as the detection lines \( \{ \text{Exc} \} \cup \{ \text{Det} \} \). This leads to a more correct measurement of the NPR:

\[
\text{NPR}_{\text{New}}(P_{\text{in}}) \triangleq \frac{\mathbb{E}_{k \in \{ \text{Exc} \}} \{|B_m(k)|^2\}}{\mathbb{E}_{l \in \{ \text{IB} \}} \{ \sigma_n^2(l) + \sigma_{NL}^2(l, P_{\text{in}}) \}}
\] (7.16)

The results of the three approaches are compared in Section 7.4.

### 1-dB compression point

The 1-dB compression point is defined as the input power level \( P_{\text{in}} \) at which the gain of the power amplifier has decreased by 1\,\text{dB} with respect to the estimated gain of the underlying linear system, \( G_0(\omega) \).

Since we apply a multisine excitation instead of a sine, we will have a band of excited frequency bins. We can calculate the gain of the DUT for each power level of the input at each excited frequency bin: \( G_{\text{BLA}}(\omega, P_{\text{in}}) \). From this data, \( G_0(\omega) \) and a 1-dB compression point can be extracted for each of the excited frequency bins when using the BLA-method. This is worth the effort since it allows us to track the influence of the dynamics of the DUT in \( G_0(\omega) \) which is frequency-dependent.

We can also calculate the average of the 1-dB compression point over the excited frequencies, which we will label the 1-dB compression point for a multisine excitation.

### 7.4. Experimental results

As a test-case, we performed modulated measurements on a Mini-circuits ZFL11-AD+ power amplifier. We excited the amplifier with an IQ-modulated signal: a random phase multisine with random grid consisting of 4096 samples and an excitation bandwidth of 100 frequency bins of which 66 are excited (1 out of 3 is omitted). The sampling frequency is 128\,\text{MHz}, the center frequency is 1.4\,\text{GHz} and the frequency resolution is 62.5\,\text{kHz}. The input power was swept from \(-20\,\text{dBm}\) to 5\,\text{dBm} with steps of 1\,\text{dB}. 16 different random phase realizations of the random phase multisine were generated by a PXI-4656R Vector Signal Generator (VSG). We measured 10 consecutive periods of each excitation for each input power. The incident and reflected waves were measured through a wave coupler by 4
synchronous PXI-4656R Signal Vector Analyzers (VSA). The measurement setup is shown in Figure 7.10. Figure 7.3 shows part of the frequency spectrum of the measured output.

In what follows, we will discuss the measurement results of the figures of merit that were discussed above and that are extracted within the BLA-framework. These figures of merit are extracted using the output spectrum. Nevertheless, the BLA-method can also be used to assess the nonlinearity at the input spectrum.

The BLA-extracted SNR and NPR are compared with the classical models for these figures of merit defined in the previous section. We used a sine wave excitation signal for the classical SNR calculation and a band-limited Gaussian noise excitation with a small notch in the excited frequency band for the classical NPR calculation. The SNR and SINAD are defined here for an integration bandwidth of 256MHz.

We used different realizations of a random phase multisine with random harmonic grid for all BLA-based models of the figures of merit. All excitation signals were designed to have the same Root Mean Square (RMS)-value to be able to compare the results obtained by the classic excitation signals to the results obtained by the multisine excitation in case of the SNR and NPR.
7.4. Experimental results

**SNR**

Figure 7.11 (top) compares the classic SNR calculation, \(\text{SNR}_{\text{Classic}}\) (\(\times\)), and the BLA-based SNR, \(\text{SNR}_{\text{BLA}}\) (\(\circ\)), of the system as a function of the input power level, \(P_{\text{in}}\). The calculation of \(\text{SNR}_{\text{Classic}}\) consists of two steps: first the RMS-value of the noise alone is measured, then the DUT is excited with a sine and the RMS-value of the signal is measured. \(\text{SNR}_{\text{Classic}}\) is the ratio of the obtained RMS-value of the signal and the RMS-value of the noise. \(\text{SNR}_{\text{BLA}}\) was calculated using Equation (7.4). The quantities in this equation are extracted using the robust BLA-method that was described in Section 7.2.

The SNR \((-\) increases with the input power as expected. \(\text{SNR}_{\text{Classic}}\) and \(\text{SNR}_{\text{BLA}}\) are approximately equal over the whole range of input powers. This is because the noise floor as well as the RMS-value of the two signals are the same. Hence, the BLA-based SNR model proves to be a good estimate of the classical SNR model.

**SINAD**

Figure 7.11 (top) compares the BLA-based SINAD, \(\text{SINAD}_{\text{BLA}}\) (\(\circ\)) calculated by Equation (7.5), and the new SINAD model, \(\text{SINAD}_{\text{New}}\) (\(\triangle\)) calculated by Equation (7.6), of the system as a function of the input power, \(P_{\text{in}}\). Figure 7.11 (bottom) shows the standard deviation of the nonlinear distortion \(\sigma_{\text{NL}}(\omega, P_{\text{in}})\) \((-\)) and of the measurement noise \(\sigma_{n}(\omega)\) \((-\)) , averaged over \(\{\text{All}\}\) as a function of \(P_{\text{in}}\).

The SINAD \((-\) is approximately equal to the SNR up to \(P_{\text{in}} \approx -6\text{dBm}\) because at low input powers, the measurement noise dominates and the SINAD boils down to the SNR (see Equation (7.4)-(7.6)). From this point on, the nonlinearities slightly start to take over the measurement noise and the SINAD starts to drop. This is also visible in Figure 7.11 (bottom), where \(E_{\{\text{All}\}}\{\sigma_{\text{NL}}\}\) \((-\)) and \(E_{\{\text{All}\}}\{\sigma_{n}\}\) \((-\)) are shown as a function of the input power.

\(\text{SINAD}_{\text{BLA}}\) (\(\circ\)) is slightly higher than \(\text{SINAD}_{\text{New}}\) (\(\triangle\)). The denominator of \(\text{SINAD}_{\text{New}}\) is averaged not only over the non-excited frequency bins \(\{\text{Nonexc}\}\), but also over the excited frequency bins \(\{\text{Exc}\}\), \(\{\text{All}\} = \{\text{Nonexc}\} \cup \{\text{Exc}\}\) and therefore becomes larger as was predicted earlier in Section 7.3.

**ACPR**

Figure 7.12 (top) shows the BLA-based ACPR (\(\times\)) of the system as a function of the input power, \(P_{\text{in}}\). This ACPR is calculated using Equation (7.7). Figure 7.12
7. Assess the nonlinearity at the system level

Figure 7.11: Top: Two different models of the SNR: $\text{SNR}_{\text{Classic}}$ (●) and $\text{SNR}_{\text{BLA}}$ (○) and two different models of the SINAD: $\text{SINAD}_{\text{BLA}}$ (○), $\text{SINAD}_{\text{New}}$ (▲) are shown as a function of the input power. Bottom: The standard deviation of the noise $\sigma_n(\omega)$ (—) and of the nonlinear contribution $\sigma_{\text{NL}}(\omega, P_{in})$ (——) is shown as a function of the input power, averaged over \{All\} as used in the calculation of the SINAD. The SINAD starts to drop when the level of nonlinear distortion takes over the measurement noise.
7.4. Experimental results

Figure 7.12.: The top figure shows the BLA-based ACPR (•) of the system as a function of the input power, $P_{in}$. This ACPR is calculated using Equation (7.7). Bottom: The standard deviation of the noise $\sigma_n(\omega)$ (—) and of the nonlinear contribution $\sigma_{NL}(\omega, P_{in})$ (—) is shown as a function of the input power, averaged over $\{IB\}$ as used in the calculation of the ACPR. The ACPR starts to increase when the level of nonlinear distortion takes over the measurement noise.

When the measurement noise, $\sigma_n(\omega)$ (—), is dominant, the ACPR increases with an increased input power level, $P_{in}$. This is expected as the numerator of Equation (7.7) increases, while the denominator remains unchanged. As soon as the nonlinear distortion, $\sigma_{NL}(\omega, P_{in})$ (—), takes over the measurement noise, $\sigma_n(\omega)$ (—), the ACPR starts to drop as the denominator slowly starts to increase with respect to the numerator: the spectral regrowth increases faster than the output power at the excited frequency bins.
Assess the nonlinearity at the system level

Figure 7.13.: Top: Two different models of the EVM: $EVM_{\text{New}}$ (—) and $EVM_{\text{NoNoise}}$ (—) are shown as a function of the input power, $P_{\text{in}}$. Bottom: The standard deviation of the noise $\sigma_n(\omega)$ (—) and of the nonlinear contribution $\sigma_{\text{NL}}(\omega, P_{\text{in}})$ (—) is shown as a function of the input power, averaged over $\{\text{IB}\}$ as used in the calculation of the EVM. The EVM starts to increase when the level of nonlinear distortion takes over the measurement noise.

EVM

Figure 7.13 (top) compares $EVM_{\text{New}}$ (—) and the noise-compensated EVM, $EVM_{\text{NoNoise}}$ (—), of the system as a function of the input power, $P_{\text{in}}$. Figure 7.13 (bottom) shows the standard deviation of the nonlinear contribution $\sigma_{\text{NL}}(\omega, P_{\text{in}})$ (—) and of the measurement noise $\sigma_n(\omega)$ (—), averaged over $\{\text{IB}\}$ as a function of $P_{\text{in}}$.

The negative slope in the $EVM_{\text{New}}$ (—) for the lower signal levels is caused by the presence of the measurement noise floor. As $\sigma_{\text{total}}^2(\omega, P_{\text{in}}) \approx \sigma_n^2(\omega)$, the $EVM_{\text{New}}$ (—) will decrease proportionally with the input power, $P_{\text{in}}$, as can be seen in Figure 7.13 (top). When the nonlinear contribution starts to dominate the measurement
noise \( P_{in} \approx -8\text{dBm} \), shown in Figure 7.13 (bottom)), the EVM will increase with the input power as can be intuitively expected.

As the BLA-framework provides a separate estimation of the total \( \sigma^2_{\text{total}}(\omega, P_{in}) \) and the noise variance \( \sigma^2_n(\omega) \), it becomes possible to obtain a compensation for the influence of the measurement noise on the measured EVM: \( \text{EVM}_{\text{NoNoise}} \). The negative slope in the \( \text{EVM}_{\text{New}} \) for the low input powers has almost completely disappeared in \( \text{EVM}_{\text{NoNoise}} \). The reason why a smaller negative slope is still visible can be explained by the fact that the standard deviation of the nonlinear contribution \( \sigma_{\text{NL}}(\omega, P_{in}) \) shows a minimum around \( P_{in} \approx -11\text{dBm} \) (see Figure 7.13 (bottom)). This coincides with the minimum in \( \text{EVM}_{\text{NoNoise}} \). This minimum in \( \sigma_{\text{NL}}(\omega, P_{in}) \) is explained in Section 7.2, Figure 7.5: the nonlinear contribution can only be estimated well at the frequency bins where the measurement noise is not dominant. Hence, below \( P_{in} = -8\text{dBm} \), the estimation of \( \sigma_{\text{NL}}(\omega, P_{in}) \) is not accurate. Normally, we would expect \( \sigma_{\text{NL}}(\omega, P_{in}) \) to go to zero for decreasing input powers, \( P_{in} \). If that would be the case, the noise-compensated EVM, \( \text{EVM}_{\text{NoNoise}} \), would not show a negative slope for low \( P_{in} \).

The BLA-framework also allows to calculate the uncertainty on the measured EVM variants. For a system level specification, this is an important advantage of the BLA-based EVM measurement.

**NPR**

Figure 7.14 (top) compares the classical NPR model, \( \text{NPR}_{\text{Classic}} \), the BLA-based NPR, \( \text{NPR}_{\text{BLA}} \), and the new NPR model, \( \text{NPR}_{\text{New}} \), of the system as a function of the input power, \( P_{in} \). Figure 7.14 (bottom) shows the standard deviation of the nonlinear contribution \( \sigma_{\text{NL}}(\omega, P_{in}) \) and of the measurement noise \( \sigma_n(\omega) \), averaged over \( \{\text{IB}\} \) as a function of \( P_{in} \). The \( \text{NPR}_{\text{Classic}} \), \( \text{NPR}_{\text{BLA}} \) and \( \text{NPR}_{\text{New}} \) are calculated using Equations (7.14), (7.15) and (7.16) respectively.

\( \text{NPR}_{\text{Classic}} \), \( \text{NPR}_{\text{BLA}} \) and \( \text{NPR}_{\text{New}} \) are approximately equal for low input power levels \( P_{in} \). From \( P_{in} \approx -9\text{dBm} \) on, the in-band nonlinear behavior, starts to dominate over the measurement noise. This is also visible in Figure 7.14 (bottom), where \( \sigma_{\text{NL}}(\omega, P_{in}) \) and \( \sigma_n(\omega) \) are shown as a function of the input power, averaged over \( \{\text{IB}\} \).

The difference between \( \text{NPR}_{\text{Classic}} \) and \( \text{NPR}_{\text{BLA}} \) from \( P_{in} = -16\text{dBm} \) on is expected because the two excitation signals are different. Whether there will be a nonlinear contribution on a certain bin and how large this contribution is, depends on which bins are excited. When two signals have different spectral excitation lines, a nonlinear device will create different intermodulation products.
Figure 7.14.: Top: Three different models of the NPR: \( \text{NPR}_{\text{Classic}} \) (\( \times \)), \( \text{NPR}_{\text{BLA}} \) (\( \circ \)) and \( \text{NPR}_{\text{New}} \) (\( \triangle \)) are shown as a function of the input power, \( P_{\text{in}} \).

Bottom: The standard deviation of the noise \( \sigma_n(\omega) \) (\( \cdash \)) and of the nonlinear contribution \( \sigma_{\text{NL}}(\omega, P_{\text{in}}) \) (\( \cdash\cdash \)) is shown as a function of the input power, averaged over \( \{\text{IB}\} \) as used in the calculation of the NPR. The BLA- and new NPR model start to drop when the level of nonlinear distortion takes over the measurement noise.
Therefore, the nonlinear contributions do not fall on the same frequency bins although NPR\textsubscript{Classic} (\(\times\)) and NPR\textsubscript{BLA} (\(\circ\)) were extracted by measurements with the same RMS-value.

The advantage of the multisine excitation is that it belongs to the same class of excitation signals as a real communication signal. Therefore, the NPR that is measured with a multisine excitation provides a more realistic measure than the NPR measured using the classical excitation signal.

Below \(P_{in} = -16\text{dBm}\) we would expect NPR\textsubscript{Classic} (\(\times\)) to be approximately equal to NPR\textsubscript{BLA} (\(\circ\)) and NPR\textsubscript{New} (\(\Delta\)). However, NPR\textsubscript{Classic} (\(\times\)) shows a higher variability here due to the fact that the measurement noise at low input powers is only averaged over a small number of frequency bins (in the notch). Averaging out over different realizations \(M\) and/or over a higher number of consecutive periods \(P\) solves this and this is clearly visible in NPR\textsubscript{BLA} (\(\circ\)) and NPR\textsubscript{New} (\(\Delta\)).

NPR\textsubscript{BLA} (\(\circ\)) is slightly lower than NPR\textsubscript{New} (\(\Delta\)) because the nonlinear contribution extracted by the BLA (\(N_{NL}(\omega, P_{in})\) in Equation (7.2)) is in general smaller at the excited bins than at the non-excited in-band frequency bins. This is due to the coherent nonlinearities that are present at the excited frequency bins, which are not included in \(N_{NL}(\omega, P_{in})\). Instead, they are hidden in \(G_B(\omega)\) (Equation (7.3)). Therefore, the denominator becomes smaller when averaged out over \{\text{IB}\} = \{\text{Exc}\} \cup \{\text{Det}\}\) (in case of NPR\textsubscript{New}) than when averaged over \{\text{Det}\}\) (in case of NPR\textsubscript{BLA}) only.

Figure 7.15 (top) compares the SNR\textsubscript{BLA} (\(\circ\)), SINAD\textsubscript{BLA} (\(\circ\)), SINAD\textsubscript{New} (\(\Delta\)), NPR\textsubscript{BLA} (\(\circ\)) and NPR\textsubscript{New} (\(\Delta\)). Figure 7.15 (bottom) shows the standard deviation of the nonlinear contribution \(\sigma_{NL}(\omega, P_{in})\) (\(\rightarrow\)) and of the measurement noise \(\sigma_n(\omega)\) (\(\leftarrow\)), averaged over \{\text{IB}\} and the standard deviation of the nonlinear contribution \(\sigma_{NL}(\omega, P_{in})\) (\(\rightarrow\)) and of the measurement noise \(\sigma_n(\omega)\) (\(\leftarrow\)), averaged over \{\text{All}\} as a function of \(P_{in}\).

For small input powers the nonlinear behavior of the power amplifier is negligible. Hence, the NPR (\(\leftarrow\)) is more or less equal to the SNR (\(\rightarrow\)) and the SINAD (\(\leftarrow\)) for small input power levels, \(P_{in}\). The nonlinear contribution is higher in-band (\(\rightarrow\)) than out-of-band (\(\leftarrow\)). The denominator of the NPR, where only in-band nonlinearities are taken into account, will therefore be larger than the denominator of the SINAD, where in-band and out-of-band nonlinearities are taken into account. Therefore, the NPR starts to decrease faster than the SINAD as a function of the input power.

The classical SINAD and NPR estimate the expected value of the nonlinear distortion over the frequency bins. The BLA-method allows to know the nonlinear
7. **Assess the nonlinearity at the system level**

![Graph showing SNR/SINAD/NPR vs. input power](image)

Figure 7.15.: This figure compares the $\text{SNR}_{BLA}$ (○), $\text{SINAD}_{BLA}$ (○), $\text{SINAD}_{New}$ (▲), $\text{NPR}_{BLA}$ (○) and $\text{NPR}_{New}$ (▲) that were discussed above. Bottom: The standard deviation of the noise $\sigma_n(\omega)$ (---) and of the nonlinear contribution $\sigma_{NL}(\omega, P_{in})$ (—) averaged over $\{IB\}$ (—) and over $\{All\}$ (—).

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distortion as a function of the frequency (see Figure 7.3). Hence, the SINAD and NPR could also be defined as a function of the frequency within the BLA-framework. This would be an extra advantage for the designer.

### 1-dB compression point

The 1-dB compression point, $P_{1dB}$, is the input power level, $P_{in}$, at which the gain of the power amplifier, $G_{BLA}$, has decreased by 1 dB with respect to the estimated gain of the underlying linear system, $G_0$. As $G_0$ changes in frequency, a 1-dB compression point can be extracted for each excited frequency bin.

Figure 7.16 shows amplifier gain, $G_{BLA}$, at 1 excited frequency bin as a function of the input power (○). The estimated gain of the underlying linear system, $G_0$, is
7.4. Experimental results

Figure 7.16.: This figure shows $G_{\text{BLA}}$ at 1 excited frequency bin as a function of the input power ($\circ$). The estimated gain of the underlying linear system, $G_0$, is shown in (-----). The 1-dB compression point at this excited frequency bin (◆) is $P_{\text{1dB}} = -2\, \text{dBm}$.

shown in (-----) for this excited frequency bin. The 1-dB compression point at this excited frequency bin is shown in (◆) and equals $P_{\text{1dB}} = -2\, \text{dBm}$.

$G_{\text{BLA}}(\omega)$ can be calculated for each excited frequency bin. Figure 7.17 (top) shows the 3-dimensional $G_{\text{BLA}}$ as a function of the input power and the excited frequency bins. Figure 7.17 (bottom) shows the top view of this 3D-plot with the corresponding 1-dB compression point for every excited frequency bin (white • and white ◆). Figure 7.18 shows the gain of the true underlying linear system, $G_0(\omega)$, as a function of the excited frequency bins. It is clear that this gain is dynamic.

The 1-dB compression point for a multisine excitation signal can be defined within the BLA-framework as the mean value of the 1-dB compression points over the excited frequency bins. This 1-dB compression point for this example is $\mathbb{E}\{P_{\text{1dB}}\} = -1.79\, \text{dBm}$.

The 1-dB compression point can also be extracted from an AM-AM characterization [Pedr 02]. An AM-AM and AM-PM characterization could also be extracted using the BLA-method.
7. Assess the nonlinearity at the system level

Figure 7.17.: The top figure shows the 3D-BLA, $G_{BLA}$, as a function of the input power, $P_{in}$, and the excited frequency bins. The bottom figure shows the top view of the 3D-plot. The corresponding 1-dB compression point is shown for every excited frequency bin (top: white •, bottom: white •).
7.4. Experimental results

Figure 7.18.: This figure shows the dynamic gain of the true underlying linear system $G_0(\omega)$ as a function of the excited frequency bins. This gain is almost constant over the excited frequency band, so the dynamics are very small.

Conclusion

The BLA-framework allows to estimate different measurement-based figures of merit using just a single measurement setup and a single type of excitation signal: the random phase multisine.

This method allows to split the measured output spectrum in three contributions: the linear dynamics present at the excited frequency bins, the nonlinear distortion present at both excited and non-excited in-band and out-of-band frequency bins and the measurement noise present at all excited and non-excited frequency bins.

Being able to estimate the nonlinear contribution at the excited frequency bins allows an improved accuracy of the estimated measurement-based figures of merit, bringing them closer to their actual definition.

The figures of merit discussed in this chapter were extracted at a specific operating point, hence the models for the figures of merit are small signal models, as is the BLA model. They can be extended to large signal models by using metamodeling techniques as used in Part I of this work. This technique is applied in [Ferr 17] where a novel state-space interpolation metamodel technique is used to estimate a large signal model (LPV model) based on a set of local (small signal) LTI models that are identified at different fixed operating points. This paper proves the potential of finding a large signal BLA model using metamodeling, however, this is out of the scope of this work.
Other figures of merit than the ones described in this section can be extracted using the BLA-framework: the AM-AM and AM-PM characterization, the inter-modulation distortion (IMD), the harmonic distortion (HMD), the third-order intercept point (TOI), etc.
8. Conclusions and future work

In this final chapter, we first summarize the contributions made in both parts of the thesis: Part I: Model driven filter design and Part II: Figures of merit for characterizing nonlinear devices. Then, we propose some possible further improvements to extend this work in future research.
8. Conclusions and future work

Conclusions

The goal of this work is to facilitate the design of a complete telecommunication system by helping designers to get more insight into the realizations of the complicated design of each component in this system. This goal is achieved by introducing models into the design procedure of linear microwave filters on the one hand (Part I) and into the characterization of nonlinear devices on the other hand (Part II). The new model-based filter design procedures are validated on a Defected Ground Structure (DGS) realization. The new measurement-based figures of merit for characterizing nonlinear devices are validated by measurements on a power amplifier. The main contributions are summarized and a take-home message is formulated for each part separately.

Part I: Model driven filter design

- Metamodels are constructed to link the design parameters of a complete microstrip filter structure directly to the filter characteristics (or the filter specifications). The metamodels are used in the optimization process instead of the computationally expensive electromagnetic (EM) simulations, resulting in an accurate and time-efficient optimization directly on the structure of interest.

- A scalable equivalent circuit model is constructed to adequately and accurately describe the microstrip filter structure of interest. The scalable circuit model is used in the optimization process instead of the computationally expensive EM simulations, resulting in an accurate, time-efficient optimization. On top of that, the circuit model provides physical insight into the designer concerning the EM operation mechanism of the filter.

- The two newly proposed filter design procedures are successfully applied on a number of DGS examples. A thorough comparison of these two techniques and the existing filter design by EM optimization is given.

- The extracted equivalent circuit model is extended to a wide-band model that can capture the spurious frequency bands that are present in the wide-band DGS response. This wide-band model is valid for a more general DGS with different slot positions. A proof of concept shows that this model can also be used to model a DGS with multiple slots.

Message: Introducing models into the design procedures of linear components helps the designer to gain more physical insight in the EM operation mechanism
of the structure and to speed up the design process while keeping a high accuracy level.

**Part II: Figures of merit for characterizing nonlinear devices**

- Using the Best Linear Approximation (BLA), different measurement-based figures of merit were extracted using just a single measurement setup and a single type of excitation signal: the random phase multisine. The results extracted using this excitation signal match those that are obtained in real applications using actual communication signals.
- New measures for the figures of merit were defined that lie closer to their actual definition.
- The measurement-based figures of merit have been successfully used to characterize a power amplifier and were compared to the classical-measurement-based ones.

**Message:** Introducing models into the characterization process of nonlinear components allows to perform multiple characterization tests at once and to reduce the testing time significantly.

**Future work**

As research is an ever ongoing activity, it is clear that both parts of this work can be further extended or improved in future research. For some extensions, a first attempt was already made that shows their potential. The extensions are categorized for each part separately.

**Part I: Model driven filter design**

- In Chapter 3, we used metamodels for optimization purposes. Metamodels can also be used to perform more time-efficient variability analyses by using design space exploration where a small variation around one variable is considered. Of course, the extracted metamodels should then be accurate enough, because the design space exploration is performed very locally. Many studies can already be found concerning the use of metamodels for global design space exploration purposes or for variability analyses (local design space exploration purposes) [Wang 07, Klei 08, Oste 18].
8. Conclusions and future work

• In Chapter 4, we extracted a band-limited scalable circuit model for a symmetric DGS with a single rectangular slot. In Chapter 6, we generalized this model to a wide-band model that can model multiple harmonic responses of an asymmetric DGS. This wide-band equivalent circuit model should still be made scalable in order to be useful in a design context.

• In Chapter 6, we made a first attempt to model a DGS with multiple slots. It was shown that the extracted wide-band equivalent circuit model can be used to model a DGS with multiple slots if the spacing between the slots is large enough to avoid a strong coupling between the slots. In practice, however, a strong coupling is often desired because it enhances the DGS characteristics. Hence, it would be useful to model this coupling effect. Recent research [Zeng 19] shows that the electric coupling between resonating slots is much smaller than the magnetic coupling. Hence, a coupling through mutual inductances is probably sufficient to model the coupling effect between multiple slots.

• To maximally exploit the reusability in design, the wide-band model could be even further extended by making it applicable to a DGS of which the slots take different shapes, e.g. a rectangular dumbbell or a circular dumbbell. This would possibly lead to other or more antiresonances in the Frequency Response Function (FRF). The wide-band model should be able to capture these without increasing the amount of parameters too much.

Part II: Figures of merit for characterizing nonlinear devices

• The figures of merit in Chapter 7 were extracted at a specific operating point. Hence, the models for the figures of merit are small signal models, as is the BLA model. They can be extended to large signal models by using metamodeling techniques that were introduced in Part I of this work. A proof-of-concept can be found in [Ferr 17], where a novel state-space interpolation metamodel technique is used to estimate a large signal model (LPV model) based on a set of local (small signal) Linear Time-Invariant (LTI) models that are identified at different fixed operating points. This paper proves the potential of finding a large signal BLA model using metamodeling.

• The figures of merit in Chapter 7 were extracted using the output spectrum. The BLA-method that was applied to extract these figures of merit can also be used to assess the nonlinearity at the input spectrum. Hence, the BLA can be used to assess and compensate for the nonlinearity of the generator.
In cooperation with National Instruments in Belgium (NI Belgium), we are currently working on an experimental validation of the work in Chapter 7 on real application data. First results show that the Orthogonal Frequency Division Multiplexing (OFDM) signals that are contained in the excitation signal of a WLAN AX standard resemble a random phase multisine. We attempt to insert our multisine signals into these OFDM blocks in order to be able to test the work in Chapter 7 on real measurement systems.

In Chapter 7 we used the Single-Input Single-Output (SISO) BLA to assess the nonlinearity at the system level. This work can be extended to assess the nonlinearity of a complete communication link by using the Multiple-Input Multiple-Output (MIMO) BLA. A similar approach was conducted in [Coom 18].
List of Publications

Journal publication


Journal publication under review


Conference publications


List of Publications


Scientific honors

Second best paper award in student paper competition at IEEE MTT-S International Microwave and RF Conference (IMaRC2017).
### Bibliography

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In the development of the 5G communication technology, there is an exponential increase in applications and devices. The 5G standard brings higher data rates, a lower latency and a higher reliability to the end users, resulting in the need for more bandwidth. As a result, designers face many challenges to cope with all these demanding specifications. The design of linear as well as nonlinear devices will become more complicated and there is a need for new approaches towards design techniques as well as measurements. We propose a new model-based framework to facilitate the design and characterization of linear as well as nonlinear components.

The first part of this work uses models to improve the design procedure of microwave filters. The time-consuming electromagnetic optimization used in the current design process is replaced in two different ways. First, it is replaced by metamodels, allowing a significant speed-up of the design process while keeping a high accuracy level. In a next stage, an equivalent circuit model is used to provide physical insight into the electromagnetic operation of the structure, which helps a designer to ensure robust and reliable designs.

The second part of this work introduces a measurement procedure based on the Best Linear Approximation model-framework to characterize the nonlinear behavior of a power amplifier. Separate estimates of the linear term, the noise term and the in-band and out-of-band nonlinear distortion allow to extract multiple measurement-based figures of merit with a single measurement taken from a single measurement setup. This approach simplifies the required characterization tests and reduces the testing time significantly.

The proposed modeling techniques can be generalized to the design and characterization of other microwave components. The use of models proves to be a promising way to deal with the challenges that await a designer.