Analysis of the Baseband Dynamics of Dynamic Power Supply Transmitters Using a Parameter-Varying Framework

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Analysis of the Baseband Dynamics of Dynamic Power Supply Transmitters Using a Parameter-Varying Framework

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Layman’s Introduction

Wireless signals are all around us and make it possible to, for example, browse the internet on your smartphone. Most of these signals are transmitted by antennas installed on radio masts and towers all around us. Due to the abundance of these towers, it is important that these signals are transmitted in an efficient manner; i.e. if you supply the tower with power, you expect the same amount of power to be transmitted by the antennas. Unfortunately, a large part of the supplied power is wasted. The main contributor to bad power efficiency is the so-called Power Amplifier (PA), a device that transforms the small “whispering” signal outputted by a previous stage to a much larger “roaring” output so your smartphone can “hear” the antenna from much farther away.

Many techniques exist that aim to improve power efficiency of the PA, but this work solely focuses on the group of so-called Dynamic Power Supply (DPS) transmitters. These are PAs that have a changing power supply which only provides enough energy for the amplification and, theoretically, none for wasting purposes.

All these PAs work in the assumption that this power supply actually supplies the correct amount of power at the right time instance. If the power supply is a little bit too early or too late and supplies an insufficient amount of power then the communication signal may become corrupted. On the other hand, if too much power is supplied, the power is again wasted. Thus, the relationship between both the power supply’s output value and the communication signal to be amplified needs to be closely monitored.

The main goal of this work is to find a method to identify and rectify any and all issues involving the (mis)alignment of both signals. This is done by taking the power supply’s input and modifying it in such a way (i.e. delaying, advancing, amplifying and/or decreasing the power supply) that the output signal of the power supply arrives at both the perfect time and with the perfect amplitude level, resulting in an overall more energy efficient transmitter.
Radio Frequency (RF) Power Amplifiers (PAs) are the most power-consuming component of any wireless base station. As a result, even the slightest increase in efficiency has a very high impact on lessening a base station’s consumption. One of several possible ways to achieve increased performance is to modulate the PA’s supply voltage in function of the signal’s input envelope. Such a feat is made possible by co-operation of the PA and a, newly introduced, Dynamic Power Supply (DPS). In such an arrangement, maximum performance is only achieved when both components are perfectly matched to each other. However, an immediate problem is that this internal interface, and the exact matching as a result, is seldom observable by the designer.

Classically, these baseband dynamics, as presented at this internal interface, are assumed to solely consist of a linear delay and are consequently treated as such. This has led to the introduction of several time alignment techniques that aim to minimize the negative impact of these dynamics on the total transmitter’s performance metrics. These techniques achieve better performance, but only up to a certain point. For example, non-linear delays nor dynamic ripples can be captured and compensated by these techniques.

In this thesis, an analysis technique for approximating and modelling the signals at this internal interface, and the baseband dynamics as a result, is introduced. Such a technique makes heavy use of the already existing Linear Parameter-Varying (LPV) framework and molds this theoretical technique to the practical properties of a DPS transmitter. The approximated baseband dynamics are then used as a pre-compensation template for increasing the device’s performance beyond what is achievable with a simple time alignment technique. Furthermore, both the static and dynamic dependency of these dynamics are analyzed in function of the transmitter’s average input power, among others.
Outline

Chapter 1: Introduction to Dynamic Power Supply Transmitters
This chapter introduces several important theoretical techniques for the design of DPS transmitters as the exact definitions can be quite ambiguous in the scientific literature. These techniques are then also applied on a simplified transistor model for clarification purposes. Experienced users and/or designers of DPS transmitters should be able to safely skip this introductionary chapter without any problems.

Chapter 2: Simulation & Design of Dynamic Power Supply Transmitters
Here the author hopes to shine some light on the exact nature of the problems encountered when the PA and the DPS are connected. Subsequently, a section is devoted to introducing the Advanced Design System (ADS) Ptolemy cosimulation testbed that is used, for extraction purposes, in the remainder of this work.

Chapter 3: Introduction to the Theoretical Framework
After having a better understanding on the design issues of the supply and/or bias interface, a chapter is now spent on introducing the foundations for a theoretical framework. Such a theoretical discussion is facilitated by simulation examples that aim to keep the discussions grounded and focused on the matter at hand.

Chapter 4: Extraction of the Baseband Dynamics
The introduced identification techniques are now combined into a fully operational identification method that is able to extract the transmitter’s baseband dynamics. Several simulation examples are used in an effort to showcase the method’s versatility.

Chapter 5: Measurements & Synchronization Issues
After testing the identification procedure on several simulation examples, it is time to bring the proposed technique into the real world. Unfortunately, properly synchronizing the measurement instruments, both in frequency and phase, was a bigger challenge than was initially expected and, as such, is the subject of the first part of this chapter. After passing this hurdle, results obtained by measurement of a DPS transmitter are discussed in depth.
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1. Introduction to Dynamic Power Supply Transmitters

This chapter aims to introduce the necessary concepts for understanding the current state-of-the-art design philosophies behind Dynamic Power Supply (DPS) transmitters. To facilitate such an endeavour, a simplified transistor model is first introduced that serves as the foundation for all further simulation examples. Afterwards, the concepts and philosophy behind waveform engineering, an important design methodology for Power Amplifiers (PAs), is discussed in detail. This leads to the definition of important, so-called, Figures of Merit (FOMs), such as efficiency and output power, that are defined and, subsequently, derived from the device’s in- and output ports. Armed with this information, later sections try to tackle the classic PA classes, compare them relatively to each other and discuss their inherent shortcomings. From there onwards the chapter gradually builds up to its own namesake, discussing the ideas behind efficiency enhancement techniques such as envelope tracking, polar modulation and their many possible hybrid combinations.

Most of the necessary mathematics are referred to in the literature as to keep this chapter focused on the goals at hand. Almost everything found in this chapter is based on the works of both Steve Cripps [Crip 06] and Earl McCune [McCu 15].
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.1.: Symbol for a generic Field-Effect Transistor (FET) symbol on which the relevant quantities are denoted.

1.1. A Simplified Transistor Model

For demonstration’s purposes, a simplified transistor model is to be introduced on which to test and validate all relevant amplifier classes and concepts. Such a model aims to capture the effects of the knee voltage, as discussed later, but does not concern itself with modelling the output conductance, the effects of parasitic capacitors nor more advanced and complex phenomena such as trapping, thermal behaviour and other non-idealities.

As this work mainly concerns itself with transistor devices belonging to the so-called FET variety, conventions and nomenclature belonging to these kind of devices is respected. Although it should be stated that most of the conceptual explanations, as discussed in later sections, are also applicable to devices belonging to the Bipolar Junction Transistor (BJT) category with some minor alterations. A symbol for a generic FET is shown in Figure 1.1 with annotation of the relevant device ports and physical quantities. On the level of the transistor, black box behaviour is consistently assumed throughout this work, meaning that one does not concern oneself with the actual physical inner workings, but describes the behaviour of the device by looking solely at the quantities at the device’s ports (GATE, DRAIN and SOURCE in this case). The device is thus assumed to be something akin to a magic black box.

For brevity’s sake the relationship between these terminals can easily be summarized in a few sentences. The voltage difference between the gate and source of the transistor ($v_{gs}$) induces (or in some cases impedes) the creation of a conductive channel between the drain and source terminals. The resulting current flowing through this channel ($i_{ds}$) is a function of this channel’s characteristics as well as the voltage between drain and source ($v_{ds}$) whose contributions are by no means mutually exclusive.
The drain-source current $i_{ds}$ of this simplified transistor model, as proposed by [Crip 06], is given by the following semi-continuous function:

$$i_{ds}(t) = \begin{cases} 0 & v_{gs}(t) < V_t \\ I_{max} (1 - e^{-v_{ds}(t)/V_{knee}}) & V_t < v_{gs}(t) < V_{gs,\text{max}} \\ I_{max} (1 - e^{-v_{ds}(t)/V_{knee}}) & v_{gs}(t) > V_{gs,\text{max}} \end{cases}$$  \hspace{1cm} (1.1)$$

in which:

- $i_{ds}(t)$, $v_{ds}(t)$, and $v_{gs}(t)$ are the device’s quantities as defined in Figure 1.1.
- $I_{max}$ is the maximum current that can be conducted by the drain-source channel.
- $V_t$ is the device’s gate threshold voltage below which the drain-source channel is unable to conduct any current. In other words, the conductive channel is reminiscent of an open circuit when working below this threshold.
- $V_{gs,\text{max}}$ is the gate voltage beyond which no further increase in drain current $i_{ds}(t)$ is observed. Increasing the gate voltage of the device beyond this point only results in a higher energy bill and not in any useful signal amplification.
- $V_{knee}$ is the device’s knee voltage, generally defined as the point at which the transistor is conducting 63% of its maximum current [Crip 06], and is a measure for how fast the device’s current becomes independent from the drain voltage. For drain-source voltages higher than this point, the transistor might be considered a perfect voltage-controlled current source.

Values for these parameters were chosen with the intent of being as close as possible to a more realistic transistor device (Cree’s CGH40006P), of which the model is used in subsequent chapters. The proposed model, given above, is implemented in Keysight’s Advanced Design System (ADS) and the simulation setup for extracting the IV-curves, alongside the choice of parameters, is shown in Figure 1.2.

One unusual setting, as evident in this schematic, is the choice of a negative threshold voltage $V_t$ as, from a conventional standpoint, the threshold voltage is mostly assumed positive instead. Reason being that Cree’s CGH40006P is a so-called ‘depletion’ mode transistor [Cree 15] in which the drain-source channel is naturally on, and thus conducts current, at zero gate-source voltage. Lastly, the knee voltage $V_{knee}$ is chosen a tad bit lower (about $\sim 1$ V) than for Cree’s model, mainly for demonstration purposes, as seen in Section 1.4, as this allows for achieving both higher power efficiency and gain that are closer to the theoretical maxima (as achievable by an ideal device without knee voltage). The resulting IV-curves of this theoretical model are plotted in Figure 1.3 to demonstrate the behaviour of the proposed transistor model.
I[1,0] = \frac{v1}{50} \\
I[2,0] = ids(v1, v2) \\
Vdc = VGS \\
I_Probe \\
V_DC \\
V_DRAIN \\
V_GS \\
SDD2P

Figure 1.2: Simulation setup for the two-dimensional DC-sweep of the simplified transistor model.
One of the more important decisions for power amplifier design is on the choice of the boundary between the so-called triode and saturation regions [McCullum 15]. The saturation region being the region in which the drain current \( i_{ds} \) is strictly independent from the drain voltage \( v_{ds} \), while the triode region defines the zone in which the current is function of both the drain and gate voltages. Depending on the designer’s intended application, the device should either behave as a perfect voltage-controlled current source, as achieved in the saturation region, or should behave as an ohmic contact with drain current and voltage being linearly dependent on each other, as is accomplished in the triode region. Both cases require the definition of some kind of boundary curve that allows the designer to set constraints on the allowable excursions of the device’s time-domain waveforms.

An important, and immediate, observation is that this boundary doesn’t actually exist; even for this simplified transistor model. The contribution of the drain voltage to the current drops exponentially but will never truly be zero as seen in Equation 1.1. The result is a trade-off between either going for:

- better power efficiency by allowing a certain measure of intrusion into the triode region and accept the resulting non-linear clipping of the peak drain current.
- better input-output linearity by denying as much intrusion into the triode region as possible, but pay the price in power efficiency due to the excessively large voltage offset required.

Taking the partial derivative of the drain current with respect to the drain voltage, results in a metric that aims to describe this particular boundary known as the Power Supply Sensitivity (PSS):

\[
PSS_{\text{linear}} = \frac{\partial i_{ds}(v_{ds}, v_{gs})}{\partial v_{ds}}
\]

or more conveniently, by working with a logarithmic scale:

\[
PSS_{\text{dB}} = 20 \log_{10}(PSS_{\text{linear}})
\]

This metric can then be exploited to define an approximative boundary curve between triode and saturation regions, quite simply by calculating the PSS metric at each gate-source voltage and selecting the resulting derivatives, which are function of the gate voltage, that correspond to identical PSS metrics. Shown in Figure 1.4 are PSS boundary curves corresponding to different values of this metric, namely \(-30\) dB, \(-50\) dB, and \(-70\) dB. As can be seen, enforcing a lower PSS metric into the design results in higher drain-source voltage offsets that are most likely only worth the loss in power efficiency in applications in which the highest kind of linearity is desired.
1. Introduction to Dynamic Power Supply Transmitters

(a) Drain-source current $i_{ds}$ in function of the drain-source voltage $v_{ds}$ for increasing levels of the gate-source voltage. Possible boundaries between the perceived triode and saturation regions, corresponding to PSS with values of respectively $-30\,\text{dB}$ ($\square$), $-50\,\text{dB}$ ($\blacksquare$) and $-70\,\text{dB}$ ($\blacklozenge$), are also shown.

(b) Drain-source current $i_{ds}$ in function of the gate-source voltage $v_{gs}$ for increasing levels of the drain-source voltage.

Figure 1.3: IV-curves of the simplified transistor model acquired using the simulation setup shown in Figure 1.2.
Figure 1.4: Zoomed-in version of the IV-curves and PSS boundary curves previously shown in Figure 1.3. Depicted boundary curves correspond to PSS metrics with values of $-30\text{dB}$ (■), $-50\text{dB}$ (■) and $-70\text{dB}$ (■).

In subsequent sections the PSS is chosen equal to a value of $-30\text{dB}$ to demonstrate some degree of clipping of the current at highest input powers. In an actual design, the choice of PSS would depend on tight constraints set by the eventual application on the permissible level of generated non-linear spectral content. Also, in practical applications, to account for differences in transistor yield, some leeway has to be taken in the form of an additional overhead voltage $V_{OH}$ that is added to the PSS boundary curve as a safeguard [McCu 15].

### 1.2. Design Methodologies

A considerable part of the design of Radio Frequency (RF) PAs amounts to making the correct choice of the device’s impedance environment. As seen here and in later sections, presenting the correct impedances at the device's in- and output ports is crucial for bringing out the device’s full capabilities. Over the decades since the first transmitter’s inception, a number of design methodologies have been invented for this very purpose, of which the most prominent one is the load-pull technique. Due to more recent developments on the accuracy and availability of non-linear modelling and de-embedding techniques, the waveform engineering design methodology has also gained considerable prominence. The load-pull and the waveform engineering technique work on vastly different principles and a comparison on their benefits and disadvantages is appropriate.
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.5.: In reality, only access to the device’s external ports is possible, while the transistor’s internal terminals are encapsulated by different layers of parasitic behaviour. Quantities denoted at the external ports correspond to those depicted in Figure 1.1.

Load-pull

This particular technique gets the job done by presenting a grid of different impedances to the device’s ports and, subsequently, measuring and/or simulating all the relevant device quantities at each and every one of these impedance candidates at a single frequency of excitation. At that moment the design simplifies to finding the best candidate quantities at which the device obeys the wanted design prerequisites. This method’s simplicity assures that it can be implemented on every possible transistor technology without much modification. As a result, a mature and accurate simulation model is not required and this is indeed this technique’s biggest strength. One of the main problems is that scanning all these candidates could potentially take a very long time. Another potential disadvantage is that the designer doesn’t really know what is happening in the device itself as the load-pull technique doesn’t implicitly take into account the internal current and voltage waveforms. (For simplicity’s sake, the most basic implementation of the load-pull technique is assumed here as discussed in [Crip 06].) As a result, the device could be operating in a region in which the impedances presented at the device’s internal nodes are not enabling the transistor to achieve its full potential.

As can be seen in Figure 1.5, the device’s internal terminals, at which Equation 1.1 is assumed valid, are hidden inside several layers of parasitics and cannot be measured in reality. Even without the availability of a complex de-embedding model or technique, used to recover these internal nodes from deep within the device, the load-pull technique is still perfectly able to produce designs that adhere to the design prerequisites.
1.2. Design Methodologies

**Advantages:**

- Load-pull does not require access to the device’s internal quantities nor requires the availability of any non-linear de-embedding techniques.
- Only requires the presence of relatively simple measurement equipment regardless of the transistor’s technology.

**Disadvantages:**

- Depending on the size and accuracy of the wanted impedance design space, a combinatorial explosion of all possible complex valued impedances at all harmonic frequencies takes place that vastly increases the necessary amount of measurements or simulations.
- No clear indication is given on the transistor’s internal states, potentially restricting the device to work in a non-ideal situation.

**Waveform Engineering**

The concept of waveform engineering equates to presenting a specific impedance environment to the device’s internal current source such that the wanted, pre-specified current and voltage waveforms are realized at this internal node. Obviously, this actually requires that these internal quantities can in some way or form be approximated or simulated. In recent years, considerable effort has been made to introduce advanced non-linear de-embedding models that work in both simulation and measurement setups [Jang 14].

In the case of simulations, the transistor’s manufacturer already did considerable effort to transform their device’s measurement data to a non-linear model useable in a simulated design environment. Instead of spending the time and effort to extract a brand-new de-embedding model, the manufacturer’s model parameters and equations could, potentially, be used instead. In most cases, the internal components and equations of this non-linear simulation model are not accessible to the designer as it constitutes a large part of the foundry’s Intellectual Property (IP). In recent years, most transistor manufacturers have slightly relented their strict model secrecy and have started giving designers access to their model’s internal nodes, greatly increasing the useability of the waveform engineering technique in practical designs [Peng 14].
1. Introduction to Dynamic Power Supply Transmitters

**Advantages:**

- The designer knows the device’s internal voltage and current waveforms and, thus, has assurance that the transistor device is working optimally and in the exact PA class (see later) required for the application at hand.

**Disadvantages:**

Waveform engineering requires that the designer has access to (or can at least approximate in some way) the following properties:

- **Access to the device’s internal ports**
  Knowing what impedances the internal voltage and current waveforms experience is a fundamental aspect of the waveform engineering technique. Preferably these properties are made available by the transistor’s foundry, but they can also be extracted by transforming the extrinsic impedance environment to an approximate version of their intrinsic counterparts by using an appropriate non-linear de-embedding technique \[\text{Jang 14}\].

Additionally, these internal impedances still have to be transformed to the device’s extrinsic ports as those are the only places at which the designer has complete and direct control on the impedance environment. As such:

- **Access to de-embedding techniques or models**
  De-embedding techniques allow the designer to take the external port quantities and transform them to an approximate version of the internal model waveforms. Extraction of such a model requires considerable knowledge on the subject matter and is still the subject of ongoing research \[\text{Jang 14}\]. Having such a model allows for the internally defined impedance environment to be directly transformed to the device’s output terminals.

In this work, the waveform engineering design methodology is selected as the design methodology of choice. Reason being that full control is desired on the internal device’s time-domain waveforms such that different PA classes, requiring specific impedance environments, can be easily demonstrated. Furthermore, the more realistic transistor model used later (Cree’s CGH40006P) allows access to the internal drain-source current and voltage. Techniques described in subsequent sections, and demonstrated on the simplified transistor model, are thus readily adapted to this more complex device.

In the case of the simplified transistor model, as defined in Equation 1.1, direct access to the device’s internal current source is available since no parasitics were added. The internally defined device quantities are thus already available without need for any de-embedding techniques.
1.3. A Simulation Testing Environment

Simulation and demonstration of different possible amplifiers requires a sort of waveform engineering testbed, in which the previously introduced simplified transistor model’s properties can be exploited by an adequate choice of operating point and load impedance environment. A simplified schematic of the eventual testbed implemented in Keysight’s ADS is shown in Figure 1.6 and all relevant components are now to be discussed in due detail.

**Input Power**

The purpose of an RF transmitter is to amplify an RF input signal $v_{in}(t)$ to the device’s output. Such an input signal will most likely contain some form of modulation, either by dynamically changing the amplitude, phase or both. However, for current purposes the input signal is, for simplicity’s sake and as already depicted in Figure 1.6, always assumed to be of a purely sinusoidal nature, as such:

$$v_{in}(t) = V_{in} \cos(\omega_0 t)$$
in which:
• $V_{\text{in}}$ is the time-independent amplitude of the sinusoidal input signal $v_{\text{in}}(t)$.
• $\omega_0 (= 2\pi f_0)$ is the fundamental frequency at which the input signal is placed.

As this is a power amplifier and not a voltage amplifier, the actual input metric that is to be used should be the input power $P_{\text{in}}$. In the case of the proposed testing environment the gate of the transistor device is chosen to present an ideal 50Ω impedance, denoted as $Z_0$, to the ideal voltage source $v_{\text{in}}$. Thus the assumption is made that the input impedance has already been perfectly matched by an ideal frequency-independent matching network. In the simulation setup this is achieved by forcing the gate’s input current to $v_{\text{in}}/Z_0$ as was depicted in Figure 1.2. As a result, the input power $P_{\text{in}}$ is found to obey following formula:

$$P_{\text{in}} = \frac{1}{2} \Re \{ V_{\text{in}} I_{\text{in}}^* \} = \frac{|V_{\text{in}}|^2}{2Z_0}$$

Or when converted to a logarithmic scale (in dBm), as is convention:

$$P_{\text{in,dBm}} = 10\log_{10}(\frac{P_{\text{in}}}{\text{1mW}})$$

where:
• $|\square|$ denotes the modulus (absolute value) operator.
• $\square^*$ denotes the conjugate operator, which is redundant if $Z_0$ is real valued as there’s no phase difference between $V_{\text{in}}$ and $I_{\text{in}}$, but is added for the sake of correctness/completeness.
• $\Re \{\square\}$ is the operator that takes the real part from a complex number. Only the active (=real) power, which is the power that is actually being dissipated or generated by the device’s components, is useful for analysis purposes while the reactive (=imaginary) power, periodically exchanged between device and periphery, does not contribute directly to the power amplification.
• the factor $\frac{1}{2}$ is added to justify comparison of DC and RF powers, by working with root mean square (rms) quantities instead of absolute ones.

### Output Power

At the device output, the chosen load impedance $Z_{\text{load}}$ dictates the actual output power that is available to next stage after the PA. This load impedance is directly derived from the wanted time-domain waveforms of the device’s internal quantities, being $v_{\text{ds}}(t)$ and $I_{\text{ds}}(t)$ as seen in Figure 1.6, using the waveform engineering design methodology.
1.3. A Simulation Testing Environment

As is seen later in section 1.4, the wanted output voltage waveform is generally chosen to be perfectly sinusoidal. This requires that all higher-order non-linear harmonics of the output voltage are shorted, which is in this case achieved by using an ideal parallel LC-resonator, which shorts all spectral content at other frequencies than \( f_0 \). The result is that a perfectly amplified version of the input voltage appears over the load impedance:

\[
v_{\text{out}}(t) = v_{\text{ds}}(t) = V_{\text{out}} \cos(\omega_0 t + \varphi)
\]

with \( \varphi \) being equal to zero for the ideal transistor. Comparably as for the input power, this gives following formula for the device’s output power:

\[
P_{\text{out}, \text{dBm}} = 10 \log_{10} \left( \frac{1}{2} \Re \left\{ V_{\text{out}} I_{\text{out}}^* \right\} \right) \quad \text{mW}
\]

**Supply & Bias Power**

To avoid any ambiguity on the DC-operating point, as is occasionally present in scientific literature, the naming conventions for the DC operating points as found in [McCu 15] are embraced in this text:

**Bias** The DC-operating point at the device’s gate is denoted as the bias and constitutes the bias voltage \( V_{\text{bias}} \) and current \( I_{\text{bias}} \).

Since this is a device of the FET variety, the bias current \( I_{\text{bias}} \) will always be negligible and should not be taken into account. However, this is not the case in the schematics as shown in Figure 1.6 and 1.7 since their respective inputs are matched at 50\( \Omega \) across all frequencies for simplicity’s sake. In these cases, the resulting DC bias current is simply not taken into account.

**Supply** The DC-operating point at the device’s drain is denoted as the supply and consists of the supply voltage \( V_{\text{supply}} \) and current \( I_{\text{supply}} \).

This avoids situations in which there’s any ambiguity whatsoever; e.g. is a bias modulated device supposed to modulate its bias or supply voltage? Both options are represented in literature [Bacq 08, Chen 05] and equally valid due to differing naming conventions.

The supply power delivered by the power supply is given by following formula:

\[
P_{\text{supply}, \text{dBm}} = 10 \log_{10} \left( \frac{V_{\text{supply}} I_{\text{supply}}}{1 \text{ mW}} \right)
\]

and does not require a factor \( \frac{1}{2} \) as there’s no difference between an absolute and an rms DC-value. Similarly, the conjugate nor the real value operator are required for this power metric as only real power exists at DC.
Full Simulation Setup

The complete simulation testbed for waveform engineering purposes is shown in Figure 1.7 and is implemented in Keysight’s ADS. This simulation makes use of the Harmonic Balance (HB) simulator as is discussed in [Kund 95]. The total (=summed) voltage arriving at the transistor’s drain is a sinusoidal with DC-offset, as such:

\[ v_{ds}(t) = V_{\text{supply}} + V_{\text{out}}\cos(\omega_0 t) \]

while the restrictions on the drain-source current \( i_{ds}(t) \) are:

\[ i_{ds}(t) = I_{\text{supply}} + I_{\text{out}}\cos(\omega_0 t) + \sum_{k=2}^{\infty} I_k \cos(k\omega_0 t) \]

Spectral content at or around these harmonic frequencies \((2\omega_0, 3\omega_0, \ldots)\), originating from the transistor’s non-linear behaviour, is not part of the useful energy content. However, it has its uses in realizing the correct time-domain waveforms at the device’s drain as mandated by the chosen PA operational class as discussed later.

Lastly, some additional components are required to correctly separate the RF and DC–signal paths. DC-current is prohibited to flow to the load impedance and, similarly, RF spectral content has no business being at the ideal supply voltage source (which behaves as a short circuit at RF frequencies). In this simulation testbed (and the simplified schematic previously depicted in Figure 1.6) this separation is achieved using idealized coils and capacitors with following properties:

- RF choke (\( DC_{\text{Feed}} \))
  
  Fictitious coil that has a frequency-dependent impedance as such:

  \[ Z_{RF\,\text{choke}}(\omega) = \begin{cases} 
  0 & \omega = 0 \\
  \infty & \omega \neq 0 
  \end{cases} \]

- DC blocking capacitor (\( DC_{\text{Block}} \))

  Fictitious capacitor that has a frequency-dependent impedance as such:

  \[ Z_{DC\,\text{block}}(\omega) = \begin{cases} 
  \infty & \omega = 0 \\
  0 & \omega \neq 0 
  \end{cases} \]

In reality these ideal components have to replaced by actual coils and capacitors that are carefully chosen to block only their respective part of the total signal spectral content which evidently becomes more difficult once the signals-under-test cease being perfectly sinusoidal.
Figure 1.7.: Simulation setup for the waveform engineering testing environment using the HB simulator of ADS, shown with requisite impedances and bias voltage required for Class-A behaviour.
1. Introduction to Dynamic Power Supply Transmitters

Figures Of Merit

In following sections, the amplifier classes and topologies under investigation are mainly compared with respect to each other using two important Figures of Merit (FOMs). A whole array of other figures, describing various other device properties, could be defined and compared as well, but this would bring this introductionary discussion much too far. Interestingly, the introduced figures have some dependency on each other and, as will be evident soon enough, sacrificing one for the other can be a valuable trade-off which should be exploited.

Power Gain

The power gain describes the amplification of the input excitation to the device’s output, on a logarithmic scale for scaling purposes. In most cases this metric is preferably both as high as possible and independent from the device’s input power $P_{in}$, which is described as having a flat gain. Division (or substraction when working with a logarithmic scale) of out- and input power gives the device’s input-output power gain $G_{in→out}$:

$$G_{in→out}(\text{dB}) = 10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = P_{out,\text{dBm}} - P_{in,\text{dBm}}$$ (1.2)

Efficiency

The transmitter’s efficiency is defined as the percentage of supply power that is transformed to RF frequencies and is presented as available output power at the device’s load impedance. In it’s most simple form, the efficiency can thus be calculated as:

$$\text{Efficiency (\%)} = 100 \frac{P_{out}}{P_{supply}}$$ (1.3)

However, this definition does not take into account that a, sometimes significant, amount of input power is required to actually drive the transistor’s gate into the desired PA mode. Several other definitions exist that aim to take into account this input power as part of the consumed energy. The most common efficiency metric used for PAs is the so-called Power-Added Efficiency (PAE):

$$\text{PAE (\%)} = 100 \frac{P_{out} - P_{in}}{P_{supply}}$$ (1.4)
1.4. Classic Power Amplification

However, such a definition has problems when the input power becomes larger than the actual output power as the PAE drops to negative efficiencies at these instances and loses any physical meaning. To prevent cases like these, the Total-Added Efficiency (TAE) can be defined as a possible alternative efficiency metric [McCu 16]:

\[
\text{TAE} (%) = 100 \frac{P_{\text{out}}}{P_{\text{supply}} + P_{\text{in}}} \quad (1.5)
\]

In most cases, there is not really any significant difference between these three efficiency metrics and, as a result, only the efficiency defined in Equation 1.3 will be employed as the efficiency FOM of choice in subsequent sections and chapters.

1.4. Classic Power Amplification

This section aims to introduce the relevant classic power amplification techniques, PA modes which were invented with the intent of achieving peak power efficiency at a single input power. These techniques are classic in the sense that most of them exist for well over a century, but also in that they have some major shortcomings for dealing with modern communication signals on the aspects of efficiency.

As per waveform engineering requirements, each of these techniques is defined by a specific impedance environment and strict DC-operating conditions. An easy way to verify proper time-domain behaviour is the usage of the loadline, which is defined as the time-domain trajectory along which the drain-source voltage waveform \(v_{\text{ds}}^{\text{int}}\) moves in function of the device’s drain current waveform \(i_{\text{ds}}^{\text{int}}\). In most cases, the device’s intrinsic loadline is also an easy indicator for spectral purity, efficiency and linearity.

Two additional properties are introduced for convenience’s sake:

- The so-called conduction angle \(\theta\) gives the angle during which the drain-source channel is conductive in a single period. This angle can either be normalized between 0 and 1 (like will be done here), or can be expressed in radians. Having a smaller conduction angle generally results in higher efficiency, but decreases the input-output gain significantly.

- Closely linked to the conduction angle is the quiescent current \(I_q\), which is defined as the current that flows through the channel when an RF input signal with zero amplitude is applied. As this current can be easily measured, it gives an approximative indication of the conduction angle value without actually having access to the drain-source channel.
1. Introduction to Dynamic Power Supply Transmitters

Over the years, useful waveform configurations have been documented, and continue to be added, to the state-of-the-art in the form of so-called PA classes. Only the following subset of this ever-growing series of PA classes is assumed of any importance here:

- **Class-A**
  This class possesses a conduction angle equal to 1.0 and is thus constantly conducting current, even without any input signal. Such an arrangement results in low efficiency, but has the highest linearity and gain of all PA classes under discussion.

- **Class-B**
  Class-B only conducts current half of the time \((\theta = 0.5)\) which results in much higher efficiency capabilities. Unfortunately, both gain and linearity suffer and are lower than for a Class-A PA.

- **Class-AB & Class-C**
  A continuum of classes lies in-between, and even beyond, Class-A and -B when choosing the conduction angle somewhere in the closed interval \([0, 1]\). The conduction angle of a Class-AB PA lies anywhere in-between Class-A and Class-B, while Class-C reduces the angle to values below those necessary for Class-B operation \((0 < \theta < 0.5)\). Both classes have their respective uses in the state-of-the-art, but are of lesser importance for the matter at hand. For completeness sake, such possible applications can be found for Class-C in the usage as the peak amplifier in a so-called Doherty amplifier, while deep Class-AB \((I_q \leq 0.1I_{\text{max}})\) is especially useful when the transistor’s transconductance exhibits square law characteristics (versus the linear characteristics as shown in Figure 1.3) [Crip 06].

Verifying the behaviour of these classes becomes straightforward by using the waveform engineering testing environment introduced in previous section and depicted in Figure 1.7.
1.4. Classic Power Amplification

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>( v_{ds} (V_{\text{rms}}) )</th>
<th>( i_{ds} (A_{\text{rms}}) )</th>
<th>( Z_{\text{load}} (\Omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( V_{\text{supply}} )</td>
<td>( \frac{I_{\text{max}}}{2} )</td>
<td>/</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>( \frac{V_{\text{supply}}}{\sqrt{2}} )</td>
<td>( \frac{I_{\text{max}}}{2\sqrt{2}} )</td>
<td>( \frac{2V_{\text{supply}}}{I_{\text{max}}} )</td>
</tr>
<tr>
<td>( kf_0 (\forall k \geq 2) )</td>
<td>0</td>
<td>0</td>
<td>( \in \mathbb{U} )</td>
</tr>
</tbody>
</table>

Table 1.1.: Theoretically ideal values for voltage and current at the device’s drain for a Class-A amplifier. Both current and voltage are 0 for \( k \geq 2 \) (with \( k \) belonging to \( \mathbb{Z} \), the set of all integer numbers), confirming that this amplifier class is in fact as linear as can be. For higher \( k \) values the impedance is unconstrained, i.e can be any value in the universal set \( \mathbb{U} \).

**Class-A**

Class-A is the most logical step-up from a small-signal amplifier, in which the bias and supply voltages are chosen right in the middle of the supply rails to allow for maximal linear excursion. As a result such an amplifier will be extremely linear and possesses the highest power gain of all the classic power amplification techniques, which might be a necessity when working at frequencies too close to the transistor’s \( f_{\text{max}} \) frequency threshold [Zhao 13]. The unfortunate downside of working with such a perfectly linear device is an unimpressive efficiency metric of only 50% and this at the highest allowed input amplitude. When working at high powers such a huge waste of energy is unacceptable and would require excessive cooling infrastructure to prevent overheating of the device’s pheriphery.

For such an amplifier, the ideal would be a current and voltage that swing maximally, in sinusoidal fashion, between the available supply rails. The resulting desired voltage and current’s spectral values, for all harmonics, are summarized in Table 1.1. Thus, for an ideal device without any knee voltage, the necessary load impedance can simply be calculated by dividing the fundamental voltage and current, as per Ohm’s law, resulting in:

\[
Z_{\text{load}} (\omega_0) = \frac{2V_{\text{supply}}}{I_{\text{max}}} \approx 37.3 \Omega
\]
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.8: Typical loadline of a Class-A PA (■) depicted alongside the triode boundary belonging to a PSS of −30dB.

When taking the actual knee region into account this load impedance should be modified to make sure that the loadline never crosses into triode territory. To demonstrate some degree of current compression and, more importantly, to not decrease the low efficiency even more, the PSS boundary region is chosen equal to −30dB. This gives the following modified formula for the fundamental load impedance [Crip 06]:

\[ Z_{\text{load}}(\omega_0) = \frac{2(V_{\text{supply}} - V_{\text{PSS}})}{I_{\text{PSS}}} \approx 32.9 \Omega \]

in which \( V_{\text{PSS}} \) and \( I_{\text{PSS}} \) are, respectively, the boundary voltage and current values corresponding to the chosen PSS boundary region that allow for maximum waveform excursion. The resulting PA loadline, employing the impedance environment of Table 1.1, is depicted in Figure 1.8. Some current compression is evident at low drain-source voltages due to a slight intrusion into the triode region, but other than that the loadline does not exhibit any non-linearities whatsoever. Indeed, the time-domain voltage and current waveforms belonging to this loadline, as shown in Figure 1.9a, are quite clearly of a purely sinusoidal nature. Of further interest is the fact that voltage and current are 180° out-of-phase which is due to the fact that the current is defined as being positive when flowing into the drain-source channel. Evidently, fundamental \((=\omega_0)\) power is generated at the transistor side and only consumed at the load impedance; at which the voltage and current waveforms are indeed defined in-phase.
1.4. Classic Power Amplification

For showcasing the class’ dependency on input power, the $RF_{\text{amplitude}}$ variable, as found in Figure 1.7 is swept from 0.1 to 2V. With 2V being the amplitude of a sinusoid that fits perfectly between the transistor’s input boundaries $V_t$ and $V_{gs,\text{max}}$. As is traditional the device’s FOMs are generally only strictly calculated at the device’s maximum input power and this results in following metrics for both ideal and non-zero knee cases:

**Power Gain ($@ RF_{\text{MAX}}$ amplitude)**

$$G_{\text{in}\rightarrow\text{out}}(\text{dB}) = 10\log_{10}(0.5\frac{(V_{\text{supply}})^2}{37.3\,\Omega}) - 10\log_{10}(0.5\frac{(RF_{\text{MAX}}\text{amplitude})^2}{50\,\Omega}) = 24.2\,\text{dB}$$

$$G_{\text{in}\rightarrow\text{out}}(\text{dB}) = 10\log_{10}(0.5\frac{(V_{\text{supply}} - V_{\text{PSS}})^2}{32.9\,\Omega}) - 10\log_{10}(0.5\frac{(RF_{\text{MAX}}\text{amplitude})^2}{50\,\Omega}) = 23.5\,\text{dB}$$

**Efficiency ($@ RF_{\text{MAX}}$ amplitude)**

$$\text{Efficiency}^{\text{ideal}}(\%) = 100\frac{P_{\text{out}}^{\text{ideal}}}{P_{\text{supply}}^{\text{ideal}}} = 100\frac{V_{\text{supply}}^{l_{\text{max}}/2}}{2V_{\text{supply}}^{l_{\text{max}}/2}} = 50\%$$

$$\text{Efficiency}(\%) = 100\frac{P_{\text{out}}}{P_{\text{supply}}} = 100\frac{(V_{\text{supply}} - V_{\text{PSS}})^{l_{\text{PSS}}/2}}{2V_{\text{supply}}^{l_{\text{PSS}}/2}} = 43.5\%$$

The power-dependent FOMs (for different values of $RF_{\text{amplitude}}$) for this amplifier class are also shown in Figure 1.9b. The previously established onset of current compression is clearly seen in the gain, but no distortions are present at any other output powers. The efficiency, on the other hand, caps at a dissapointing 43.5% due to the aforementioned necessary supply offset as derived from the chosen PSS boundary region. Even worse, the efficiency swiftly diminishes for lower output powers, dragging the average efficiency to quite a low level when a signal with variable envelope power is employed as the input excitation signal.

In conclusion, the gain of a Class-A amplifier is truly the highest of all single-transistor classes. Furthermore, and arguably even better, the gain is flat in function of the output power thus no additional pre-distortion techniques for gain flattening are required. Thus in fields and applications where gain and linearity are the defining requirements, Class-A amplifiers rule supreme. However, when some higher degree of power efficiency is required, other amplifier classes can give some much needed solace from the meager maximal 50% presented by the Class-A architecture.
1. Introduction to Dynamic Power Supply Transmitters

(a) Time-domain voltage $v_{ds}$ (■) and current $i_{ds}$ (■) waveforms of a Class-A Power Amplifier for different input powers. Higher amplitudes showcase compression of the current waveform due to a small excursion into the triode boundary region.

(b) Input-output Gain (■) and Efficiency (■) of a Class-A Power Amplifier in function of the device's output power $P_{out}$. Maximum efficiency is about 43.5%, which deviates from the optimum (= 50%) due to the presence of non-zero knee voltage.

Figure 1.9.: Time-domain waveforms, input-output gain and efficiency belonging to a Class-A PA with non-zero knee voltage as given by the simulation framework shown in Figure 1.7.
1.4. Classic Power Amplification

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{ds} (V_{rms})$</th>
<th>$I_{ds} (A_{rms})$</th>
<th>$Z_{load} (\Omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$V_{supply}$</td>
<td>$\frac{I_{max}}{\pi}$</td>
<td>/</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$\frac{V_{supply}}{\sqrt{2}}$</td>
<td>$\frac{I_{max}}{2\sqrt{2}}$</td>
<td>$\frac{2V_{supply}}{I_{max}}$</td>
</tr>
<tr>
<td>$kf_0$ (\forall k \geq 2)</td>
<td>0</td>
<td>$\frac{-2I_{max} \cos(k\pi/2)}{\sqrt{2}(k-1)(k+1)\pi}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 1.2.: Theoretically ideal values for voltage and current at the device's drain for a Class B amplifier. The voltage is 0 for $k \geq 2$ (with $k$ belonging to $\mathbb{Z}$, the set of all integer numbers), while the current is only zero for odd values of $k$. As a result, the impedance is either constrained to zero, when $k$ is odd, or is a member of the universal set $\mathbb{U}$, when $k$ is even.

**Class-B**

The primary reason for the low efficiency of Class-A amplifiers is that both voltage and current are non-zero at all time instants. A possible way to increase the device’s efficiency is by reducing the bias voltage from its Class-A value. In that case the drain-source channel only conducts current for a fraction of the total period, decreasing the amount of energy that is wasted as a result. A special case is found when the bias voltage is chosen equal to the threshold voltage $V_t$. In that case, the device’s channel only conducts half of the time and the resulting amplification mode is generally known as a Class-B amplifier.

The resulting current waveform, a halfwave rectified sinusoid, has some very attractive spectral properties as is summarized in Table 1.2. Indeed, the exact same fundamental load impedance is found as for Class-A, while all higher harmonics can still be terminated in zero impedances. However, the current at these higher harmonics is only zero at odd harmonics, meaning that the resulting amplifier cannot be considered a linear device. Equivalent to the issues faced previously for the Class-A amplifier, the triode boundary region needs to be respected for non-zero knee voltage devices. The exact same decrease in $Z_{load}(\omega_0)$, as for a Class-A device, becomes a necessity and the ensuing Class-B loadline can be seen in Figure 1.10 alongside the chosen –30dB PSS boundary region.
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.10: Loadline of a Class-B PA generated using the simulation setup shown in Figure 1.7. Also shown is the boundary region, to be respected for linearity purposes, belonging to a PSS of $-30\text{dB}$.

Time-domain waveforms for Class-B, for different values of the input amplitude $RF_{\text{amplitude}}$, are shown in Figure 1.11a. An important consequence of the decreased bias voltage is that the input amplitude should be doubled, in comparison to Class-A, to 4V to properly reach the upper input boundary ($V_{g,s,\text{max}}$). The resulting FOMs are defined at this maximum amplitude as such:

**Power Gain (@ $RF_{\text{amplitude}}^{\text{MAX}}$)**

$$G_{\text{in} \rightarrow \text{out}}^{\text{ideal}}(\text{dB}) = 10\log_{10}(0.5 \left(\frac{V_{\text{supply}}}{37.3\ \Omega}\right)^2) - 10\log_{10}(0.5 \left(\frac{RF_{\text{amplitude}}^{\text{MAX}}}{50\ \Omega}\right)^2) = 18.2\ \text{dB}$$

$$G_{\text{in} \rightarrow \text{out}}(\text{dB}) = 10\log_{10}(0.5 \left(\frac{V_{\text{supply}} - V_{\text{PSS}}}{32.9\ \Omega}\right)^2) - 10\log_{10}(0.5 \left(\frac{RF_{\text{amplitude}}^{\text{MAX}}}{50\ \Omega}\right)^2) = 17.5\ \text{dB}$$

**Efficiency (@ $RF_{\text{amplitude}}^{\text{MAX}}$)**

$$\text{Efficiency}^{\text{ideal}}(\%) = 100 \frac{P_{\text{out}}^{\text{ideal}}}{P_{\text{ideal, supply}}} = 100 \frac{V_{\text{supply}} I_{\text{max}}/2}{2V_{\text{supply}} I_{\text{max}}/\pi} = 78.5\ %$$

$$\text{Efficiency}(\%) = 100 \frac{P_{\text{out}}}{P_{\text{supply}}} = 100 \frac{(V_{\text{supply}} - V_{\text{PSS}})/\pi}{2V_{\text{supply}} I_{\text{PSS}}/\pi} = 68.5\ %$$
The power-dependency of these FOMs, as achieved for the Class-B amplifier, are depicted in Figure 1.11b. As can be clearly seen the gain is still perfectly flat, but again exhibits some compression at higher output powers. However, due to the doubled input requirement the gain loses about 6dB in comparison to the equivalent Class-A gain metric. Luckily, the resulting peak efficiency has in effect been greatly increased to a delightful 68.5%. The swift decrease in efficiency, in function of the output power, is unfortunately inherited from the Class-A topology. This results in average efficiencies for the device that are quite unacceptable when excited by envelope modulated communication signals.

In conclusion, when efficiency is a more important criteria than linearity while gain is still a commodity in abundance, Class-B can be considered the superior choice to Class-A. The gain is still perfectly flat, as seen in Figure 1.11b, and thus no additional pre-distortion techniques are, in theory, required to flatten out any gain distorsion.

Class-AB & Class-C

Between the conduction angles of Class-A and -B and below the angle of Class-B, lies a continuum of classes each resulting in viable amplifier topologies that could also be used in practice. The necessary fundamental load impedances $Z_{\text{load}}(\omega_0)$ and bias voltages for operation along this continuum are shown in Figure 1.12 together with specifically named regions of operation. The resulting FOMs for each of these classes is similarly shown in Figure 1.13, in function of the conduction angle, at the maximum input amplitude $RF_{\text{MAX}}\text{amplitude}$, which is also dependent on the conduction angle due to the increase in amplitude required to still reach the upper input boundary $V_{gs,max}$.

While these classes have their roles in practice, there are several reasons why they are considered of lesser interest in this work:

- Flatness of the input-output Gain, in function of the input power, is only a given when operating at Class-A or -B. When working at other classes on the continuum, additional work and complexity would be required for gain linearization when using them in single transistor topologies.
- While efficiency greatly increases when going below a conduction angle of 0.5, the required maximum input amplitude also increases significantly. At RF, at which transistor gain is already a limiting factor [Zhao 13], this decrease of the PA’s gain might be unacceptable.
1. Introduction to Dynamic Power Supply Transmitters

(a) Time-domain voltage (■) and current (■) waveforms of the Class-B Power Amplifier.

(b) Input-output Gain (■) and Efficiency (■) of the Class-B Power Amplifier in function of the device’s output power $P_{\text{out}}$. Maximum efficiency was about 68.5%, which deviates from the optimum ($= 78.5\%$) due to the presence of non-zero knee voltage.

Figure 1.11: Time-domain waveforms, input-output gain and efficiency belonging to a Class-B PA with non-zero knee voltage as given by the simulation framework shown in Figure 1.7.
Figure 1.12.: Necessary load impedances (■) and bias voltages (■) required for operation on the Class-A, -B and -C continuum. Specific named regions for the conduction angle are also depicted for clarity’s sake.

Figure 1.13.: Efficiency (■) and input-output Gain (■) of the Class-A, -B and -C continuum at the maximum input amplitude at $RF_{\text{MAX}}^{\text{amplitude}}$ in function of the conduction angle.
1.5. Modern Communication Signals

As seen previously for Class-A and -B transistor topologies, the PA efficiency is only maximum at a single (maximum) input amplitude and swiftly drops when going to lower input amplitudes. This is quite acceptable when the input signal is just a simple sinusoid, but has questionable merit in practice, when the input signal becomes a modulated signal with an input amplitude that changes in function of the time. As a result, it is actually not the maximum efficiency that is of interest, but the weighted averaged efficiency instead. This begs the question: How does a classic PA perform when excited with a modern communication signal? In this work, such an input signal is assumed to be an IQ-modulated communication signal of the following form:

\[
    v_{\text{in}}(t) = \Re \{ (i(t) + jq(t))e^{-j\omega_0 t} \} = i(t)\cos(\omega_0 t) + q(t)\sin(\omega_0 t) \tag{1.6}
\]

in which:

- Both the so-called in-phase \(i(t)\) as the quadrature signal \(q(t)\) are assumed to be band-limited normally distributed stochastic variables with zero mean and a standard deviation equal to \(\sigma\).
- \(\omega_0\), as was defined before, is the fundamental pulsatance around which the spectral content of the modulation is upconverted (by the multiplication).

As defined by the normal distribution’s properties, 99.7% of the data samples of both \(i(t)\) and \(q(t)\) lies inside of the closed interval \([-3\sigma; +3\sigma]\) and as a result, this range indirectly defines the extent to which the PA device should be able to properly amplify the incoming signal without going into compression. In other words, all of the input amplitudes should be below the device’s maximum input amplitude \(RF_{\text{MAX}}\) to avoid violating the triode boundary region. This is commonly known as backing-off and assures that there will be a discrepancy between the efficiency peak of the PA and the actual peak at which the signal’s amplitude has the highest probability.

The distribution of this time-varying amplitude can be found when analyzing the input signal envelope \(e_{\text{in}}(t)\), which is defined as such:

\[
e_{\text{in}}(t) = |i(t) + jq(t)| = \sqrt{i(t)^2 + q(t)^2} \tag{1.7}
\]

The resulting random variable is Rayleigh distributed by definition since both in-phase and quadrature signals are normally distributed random variables with zero mean and identical standard deviations [Cast 88]. This signal envelope, alongside the actual RF input voltage \(v_{\text{in}}(t)\), is shown in Figure 1.14 for demonstration purposes.
1.5. Modern Communication Signals

Figure 1.14.: IQ-modulated communication signal $v_{in}(t)$ (■) alongside its envelope signal $e_{in}(t)$ (■) as an example of a modern communication signal with time-varying envelope amplitude that is to be amplified by a PA.

Figure 1.15.: Discrepancy between power efficiency of a classic Class-B PA (■) and the Probability Density Function (PDF) of the power distribution of an IQ-modulated input signal (■) as specified in Equation 1.6.
1. Introduction to Dynamic Power Supply Transmitters

The resulting Rayleigh distributed PDF is shown superimposed with the Class-B efficiency curve in Figure 1.15. As can be easily seen the efficiency peak does not coincide with the probability peak of the signal amplitude’s distribution. The maximum efficiency of the Class-B PA is 68.5% while the weighted average efficiency, using the modulated signal’s PDF as the weight, is only a miserable 25.3%, all because considerable backing-off had to be assured to accommodate for the communication signal’s envelope distribution.

An important signal metric that aims to give a value for this degree of backing-off, required for a particular modulation, is the Crest factor:

\[ C = \frac{\max(|e_{in}(t)|)}{\text{rms}(e_{in}(t))} \]

or more commonly it’s logarithmic equivalent, namely the Peak-to-Average Power Ratio (PAPR):

\[ \text{PAPR} = 20\log_{10}(C) \]

The PAPR gives the difference between the peak and the average power of the modulation’s distribution and is according to extreme value theory [Cast 88] found to be, on average, about 11.5 dB when working with the communication signals as proposed in Equation 1.6. This value is similarly evident in Figure 1.15 when comparing the power values at the upper tail of the distribution versus the point at which the highest power probability is achieved. Furthermore, a well-known result of signal theory is that the channel capacity (= the amount of information that can be reliably transmitted) is maximized when communications signals are employed that belong to a gaussian distribution [Cove 06]. Random variables of this type are in possession of an infinite PAPR which necessitates the choice of a realistic upper boundary condition. As such, the 99.7% boundary condition is employed and to be respected in this work.

In conclusion, classic amplifier classes have limited use for today’s modulated signals as their efficiency is only sufficiently high at a single output power. As previously shown in Figure 1.15, one would prefer an efficiency metric that is more forgiving towards the points at which the signal’s probability is highest instead. Multiple advanced architectures, commonly known as ‘efficiency enhancement techniques’, exist that attempt to alleviate this problem and this work will focus on the particular subset that aims to use clever manipulation of the supply and/or the bias voltages as a way to recover the device’s power efficiency as is the subject of subsequent sections.
1.6. Dynamic Power Supply Transmitters

Classic power amplification only allows for decent efficiency at a single peak output power. The main reason for this shortcoming is that the exact same supply voltage is provided to the device, irregardless of the actual amount of input signal that is to be amplified. As is visible in both Figures 1.9a and 1.11a, the supply current already closely follows and scales with the device’s instantaneous power requirements. Thus, a possible way to rectify this issue might lie in likewise decreasing the supply voltage to lower levels whenever possible. Such an endeavour is a potentially dangerous undertaking as the supply voltage has strictly defined boundaries, in the form of the PSS boundary curves, that have to be respected lest the supply voltage would directly start influencing the output waveforms. Or, alternatively, this direct influence might be welcomed and exploited as well. Nevertheless, this leads to an extension of the previously defined Gain metric of Equation 1.2, which now also receives a term corresponding to the supply voltage. As a result, the transmitter is now assumed as a 3-port device instead and the resulting output power $P_{\text{out}}$ becomes of the following form:

$$P_{\text{out}} = \frac{\partial P_{\text{out}}}{\partial P_{\text{in}}} P_{\text{in}} + \frac{\partial P_{\text{out}}}{\partial V_{\text{supply}}} V_{\text{supply}}$$

in which the second term was previously assumed to be non-existent and attempts to describe the direct influence of the supply variation on the output power. Such a 3-port device, in which the supply port is considered as an additional independent and controllable input port, is commonly known as a Dynamic Power Supply (DPS) transmitter.

Depending on the actual values of these partial derivative terms, DPS transmitters can be subdivided in three distinct sub-categories [McCu 15]:

1. **Envelope Tracking**

$$\frac{\partial P_{\text{out}}}{\partial V_{\text{supply}}} = 0$$

$$\frac{\partial P_{\text{out}}}{\partial P_{\text{in}}} = G_{\text{in} \rightarrow \text{out}}$$

(1.8)

Thus while the supply voltage is modulated in function of the input signal’s waveform, the output power is only dependent on the input power. The 3-port gain, in this category, only consists of the input-output gain $G_{\text{in} \rightarrow \text{out}}$ as was the case for classic power amplification. In effect this means that, while the supply voltage is being decreased, operation in Envelope Tracking (ET) still requires that the PSS boundary curve is respected at all costs.
1. Introduction to Dynamic Power Supply Transmitters

2. Polar Modulation

\[
\frac{\partial P_{\text{out}}}{\partial P_{\text{in}}} = 0
\]

\[
\frac{\partial P_{\text{out}}}{\partial V_{\text{supply}}} = G_{\text{supply} \rightarrow \text{out}}
\] (1.9)

Having no influence of the input power on the device’s output means that the device should be operated fully in the triode region. Such an arrangement is known as Polar modulation (PM) as the communication signal, that is to be amplified, is separated into its polar coordinates \(e_{\text{in}}(t)\) and \(\phi_{\text{in}}(t)\) when adopting this technique (as discussed in depth later). In general, one would prefer to have a supply-output gain \(G_{\text{supply} \rightarrow \text{out}}\) that is flat across all supply voltages and gives unitary gain from supply voltage to the output voltage’s envelope.

3. Hybrid Combinations

\[
P_{\text{out}} = \frac{\partial P_{\text{out}}}{\partial P_{\text{in}}} P_{\text{in}} + \frac{\partial P_{\text{out}}}{\partial V_{\text{supply}}} V_{\text{supply}}
\] (1.10)

Some DPS transmitters don’t respect neither the basic principles for ET nor for PM and have their region of operation somewhere in-between. This category is quite broad and depends on other criteria that don’t tend to put any direct importance/emphasis on respecting or disrepecting the PSS boundary curves. Some possible, and relevant, criteria include:

- Enforcing constant gain across all relevant input powers.
- Choosing supply voltage/input power combinations that give maximum efficiency.

As will be discussed later, this categorical distinction between pure ET and these hybrid combinations is hardly ever made in scientific literature. The consequence is that the naming convention ET is also frequently used to indicate a transmitter of the hybrid variety [Kim 11].

All three sub-categories have their own merits and faults which will be discussed in due detail. Subsequent sections aim to take the previously established Class-A and-B PAs and start modulating their supply voltages in such ways that transmitters with superior weighted average efficiencies, than their classic counterparts, are conceived.
1.7. Envelope Tracking

According to the basic principle of ET the device’s supply voltage can be diminished only to that extent at which the time-domain waveforms enters the triode region. Such a constraint is of course dependent on the choice of the PSS boundary region which is, in this case, chosen as being $-30\text{dB}$ as before. As there is still a slight dependence on the supply voltage, even when working strictly in the saturation region, the ET basic principle will in fact never be perfectly satisfied. The unfortunate consequence is that some slight gain variation ($\Delta G_{\text{in}\rightarrow\text{out}}$) will be observed even though the boundary region is never breached.

Pertinent to the entire concept of ET (and for hybrid combinations as seen later) is the so-called shaping function, defined as such:

**Shaping function** the Static Non-linearity (SNL) relationship between $P_{\text{in}}$ and the supply voltage $V_{\text{supply}}$ that obeys the chosen $V_{\text{PSS}}$ constraint.

For ET this simply means that the following procedure has to be respected, starting with $P_{\text{in}}$ being equal to the peak power at which ET operation is desired:

1. Modify the supply voltage $V_{\text{supply}}$ until the minimum value of the time-domain voltage waveform $v_{ds}(t)$ is equal to $V_{\text{PSS}}$, being a value that lies on the chosen PSS boundary.

2. Decrease the input power $P_{\text{in}}$ and go back to step 1.

This simple procedure is now to be used for extracting the ET shaping function for both the Class-A and -B PAs discussed earlier.

**Class-A**

**Naive implementation**

As can be seen in Figure 1.16a, directly implementing the above procedure reveals some immediate problems. The transmitter’s operational class shifts from Class-A, at maximum input amplitude, towards a mode of operation in which excessive quiescent current $I_q$ is required for the amount of high-frequency output power that is actually being generated by the device. Indeed, there is still a lot of headroom available at the lower current values that is not used by the time-domain current waveform. Ideally the device would stay in Class-A operation, resulting in a time-domain current waveform that would be able to fully swing between and $0$ and $I_{\text{PSS}}$, being the current value of a point that lies on the PSS boundary curve, for all swept values of the supply voltage $V_{\text{supply}}$. 

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(a) Loadline curves (■) of a first naive attempt at ET for Class-A, achieved by solely modifying the supply voltage. The resulting supply-modulated transmitter leaves Class-A operation and ends up in an operational mode with an excessive appetite for quiescent current $I_q$.

(b) Modifying the bias voltage $V_{bias}$ alongside the supply voltage $V_{supply}$ assures that Class-A operation is achieved across the entire input power range. The resulting loadline curves (■) achieve full-swing inside their given boundaries.

Figure 1.16.: Resulting loadline curves of both the naive and correct implementation of the ET technique in the case of a Class-A PA. Also shown is the PSS boundary region, to be respected for linearity purposes, belonging to a PSS of $-30$dB.
1.7. Envelope Tracking

Enforcing the transmitter to stay in its designated Class can be accomplished by modifying the bias voltage $V_{\text{bias}}$ as well. The resulting loadline curves, as shown in Figure 1.16b, now make full use of the available swing and stay in the Class-A mode of operation across the entire range. However, this is also a clear disadvantage of using Class-A as an ET candidate, as additional hardware has to be designed to be able to modulate the bias voltage alongside the supply voltage, in a synchronized fashion. Regardless, a shaping function can be obtained, using the previously proposed procedure, that results in a SNL that can be directly implemented in the simulation example and is depicted in Figure 1.17. Performance metrics of both the classic Class-A PA and the correctly implemented ET device are shown in Figure 1.18. As is evident, efficiency is greatly improved across the entire output range in comparison to its classic predecessor.

Unexpectedly, and less intuitive, is that the ET gain actually varies in function of the output power $P_{\text{out,dBm}}$ and is not a constant value as was previously postulated in Equation 1.8. This is an unfortunate side-effect of choosing the PSS metric at $-30\,\text{dB}$, instead of $-\infty\,\text{dB}$ (which is infeasible for efficiency and realistic purposes) as is actually demanded by the basic principle of ET. When decreasing the supply voltage, the device slowly, but surely drops out of ET operation and therefore some kind of lower bound is to be set into place [McCu 15].

Figure 1.17.: Extracted shaping functions for the unbounded (■) and the lower-bounded (■) implementation of the ET technique for a Class-A PA (achieved by modulating both $V_{\text{bias}}$ and $V_{\text{supply}}$).
1. Introduction to Dynamic Power Supply Transmitters

(a) Input-output gain $G_{\text{in} \rightarrow \text{out}}$ for a Class-A PA (■), compared alongside both the unbounded (■) and the lower-bounded (■) implementation of the ET technique for the exact same class.

(b) Comparison between the efficiency for a classic Class-A PA (■) with a fixed supply voltage ($V_{\text{supply}}=28\,\text{V}$), the unbounded ET implementation (■) and a lower-bounded approach (■) with hard supply clipping at 4.25 V. Also shown is the PDF belonging to an IQ-modulated communication signal (■).

Figure 1.18: Comparison of FOMs for a Class-A PA with fixed $V_{\text{supply}}$, implementation of ET without any bounds on the supply voltage and an implementation using the lower-bounded approach which corresponds to hard clipping of the supply voltage at 4.25 V.
From an intuitive standpoint this variation can be explained as such: A fixed degree of current compression, as set by the PSS, is allowed at each and every supply voltage value, while the fundamental time-domain current waveform decreases in a proportional fashion to this same value. Relatively this means that smaller output powers have a higher percentage of compression and thus result in, seemingly, a lower amount of input-output gain. In this case, a ET gain variation of around 1dB was deemed an acceptable sacrifice for the increase in efficiency offered by implementation of the ET technique. As a result, the supply voltage $V_{\text{supply}}$ now needs to lower-bounded at 4.25V to obey this constraint and the modified shaping function is shown in Figure 1.17 alongside the old one. As is seen in Figure 1.18, the input-output gain is now properly constrained to a maximum big of a problem as long as the drop-off of efficiency lies at a lower power than the peak probability of the communication signal to be amplified.

**Class-B**

The bias operating point of a Class-B PA does not shift depending on the supply voltage and, as a result does not require any additional modulation of its bias voltage of as is the case for Class-A. Thus, implementation of the ET technique is pretty straight-forward in the case of a Class-B PA and the resulting loadline curves are shown in Figure 1.19.
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Figure 1.20.: Resulting shaping functions for the implementation of ET in the case of a Class-B PA for unbounded supply (■) as well as both lower-bounded approaches at 13.25 V (■) and 3.75 V (■) respectively.

As before, as was the case for the Class-A PA, the input-output gain takes a thumble for low supply voltages and, preferably, needs to be constrained to some lower bound. In this case, constraints on the input-output gain variation/ripple were chosen at 0.1 dB at 1 dB, to check the influence of this constraint, and the resulting lower bounds on the supply voltage $V_{\text{supply}}$ were found to be 13.25 V and 3.75 V respectively. The modified shaping functions are shown in Figure 1.20, together with the unbounded one.

The resulting FOMs are shown in Figure 1.21 for all relevant implementations. While the gain ripple was contrainted to a variation of only 0.1 dB, the resulting efficiency of this particular implementation drops way too fast, before the actual peak of the communication signal’s PDF (as shown in Figure 1.15) is reached. Hence, accepting some ripple on the gain is a necessity to enjoy from the benefits in efficiency improvement of the ET technique. When the exact distribution of the communication signal to be amplified is known, a trade-off can be made between allowable gain ripple and efficiency increase that makes the choice of lower bound a balanced and premediated choice.
Figure 1.21.: Comparison of FOMs for a Class-B PA with fixed $V_{\text{supply}}$, alongside different possible implementations of the ET technique belonging to this particular class.
1.8. Polar Modulation

The previously discussed ET technique avoids any intrusion in the triode region and, thus only works in the saturation region, in which influence of perturbations of the supply voltage into the triode region is demarcated by a certain PSS threshold. In contrast, Polar modulation (PM) aims to remove all influence of the actual input and aspires to achieve an output that is solely dependent on the supply voltage $V_{\text{supply}}$. This means that the loadline curves should preferably stay inside of the triode region for at least a significant percentage of their period.

For PM, as the name implies, the input communication signal is split into its polar coordinates, as such:

$$i(t) + jq(t) = e_{\text{in}}(t)e^{j\phi(t)}$$

where $e_{\text{in}}(t)$ was defined previously (in Equation 1.7) as the input signal envelope and $\phi(t)$ is the time-varying phase component between inphase and quadrature component. Since no input-output gain is possible whatsoever, the time-variation on the device’s output amplitude is exclusively dependent on the baseband supply voltage. The phase, as was the case for ET, cannot be enforced at the supply side, since baseband signals are real signals without any quadrature component as such the RF signal at the device’s gate ($V_{gs}$) is responsible for passing along the phase component to the device’s output.

Different possible variations on PM transmitters are present in the literature [McCu 15], but going through all of them would be beyond the scope of this work. Consequently only amplifiers working in the so-called Class-E operational mode are introduced, mainly because these devices are, theoretically at least, capable of amplifying at efficiencies close or even equal to 100%. Secondly, these devices do not require that the amplitude and DC-operating point at the device’s gate is modulated during operation, something which is, unfortunately, required for several other possible implementations of the PM technique [McCu 15].

Class-E

The main operating principle behind Class-E requires that the RF transistor can be considered as an ideal switch. At RF frequencies and above such an assumption is quite dangerous as several device characteristics, such as slew-rate, on-resistance, etc., actively hinder the validity of such a claim. Crucial to the entire operation is the device’s drain-source capacitance $C_{ds}$ that, as seen later, sets a hard frequency limit above which Class-E operation can simply not be achieved by the RF transistor (or only in very approximate manner [Made 95]).
1.8. Polar Modulation

Figure 1.22: Heavily simplified schematic, in which the transistor is considered as a switch, of the modified simulation testing environment (as fully shown later in Figure 1.24) used as a demonstrator for Class-E operation.

In the assumption that the transistor behaves as a perfect switch, the simplified schematic shown in Figure 1.22 is used for demonstration purposes. The operational principle of an ideal Class-E can be summarized as alternating between two distinct phases:

1. **The switch is closed** \( (R_{\text{closed}} = 0 \Omega) \)

   Drain-source voltage \( v_{\text{ds}} \) at the device’s internal node immediately drops to zero due to the switch’s low on-resistance (0Ω in this case). An increasing amount of current \( i_{\text{ds}} \) flows through the closed switch, which is frequency limited by the LC-resonator at the device’s load. This ideal LC-resonator blocks all harmonic frequency transfer between load and switch and only allows the appearance of spectral content at the PA’s center pulsatance \( \omega_0 \).

2. **The switch is open** \( (R_{\text{open}} = +\infty \Omega) \)

   No current \( (i_{\text{ds}}) \) can flow through the switch, but the components of the LC-resonator still contain energy. Thus, to obey transient constraints, the LC-resonator will start pulling current out of the device’s drain-source capacitance \( C_{\text{ds}} \). The voltage at the device’s internal node will start rising as a result since the LC-resonator only pulls the amount of current required to obey its transient.
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.23.: Time-domain waveforms corresponding to the Class-E PA corresponding to the simulation setup shown in Figure 1.22, for different values of the supply voltage $V_{\text{supply}}$. Depicted are the drain-source voltage $v_{\text{ds}}$ (■), the drain-source current $i_{\text{ds}}$ (■) and the current flowing through the device’s parasitic output capacitance $i_{\text{cap}}$ (■).

Power dissipation only happens when both current and voltage are different from zero at the same exact moment in time. Thus to allow for maximum possible efficiency, the switch can only be flipped at points in time at which the drain-source voltage $v_{\text{ds}}$ is strictly zero, while the current only needs to be zero when the switch is opened. This due to the fact that the current is unable to dissipate anywhere else than the device’s load when the switch is open. The resulting constraints on the design parameters are the following:

- The gate-source voltage $v_{\text{gs}}$ should, ideally, be a perfect square wave, either fully turning the device on or off [Crip 06] with a minimum of rise and fall time.
- Additionally, a significant imaginary part is required at the device’s load impedance $Z_{\text{load}}$ to phase-shift the drain-source voltage and drain current relatively from each other to a position in which there is minimal overlap.

Unfortunately, real RF transistors do not behave as perfect switches, mainly due to the existence of a finite on-resistance and the device’s output capacitance. Similarly problematic is that generation of perfect square waves can be quite a challenge that, although feasible, is preferably avoided at RF frequencies. In this work, the approach proposed in [Made 95] is adhered to as equations are given that take these non-idealities into account and greatly simplify the steps required to end up with a working design.
Figure 1.24.: Modified simulation setup for the waveform engineering testing environment using the HB simulator of ADS, shown with requisite impedances and settings required for Class-E operation.
According to this approach, the maximum frequency below which Class-E operation is still available is given by [Made 95]:

\[ f_{\text{max}} \simeq \frac{I_{\text{max}}}{56.5C_{ds}V_{\text{supply}}} = 862\text{MHz} \]

in which:

- the maximum current \( I_{\text{max}} \) and supply voltage \( V_{\text{supply}} \) are chosen to have identical values as in previous sections, respectively 1.5 A and 28 V, again to respect the values given for Cree’s CGH40006P.
- \( C_{ds} \) is the device’s output capacitance and is equal to 1.1 pF, chosen identical to the value reported for Cree’s CGH40006P [Cree 15].

Choosing the operational frequency \( f_0 \) above this value only allows for operational modes that approximate Class-E behaviour, and thus the frequency employed for previous examples, which was chosen as 1GHz for simplicity’s sake, should be lowered appropriately. In this case, the fundamental frequency was lowered to 500MHz mainly because this value gives a rational period. Due to Class-E design requisites, the device’s load impedance \( Z_{\text{load}} \) is to be chosen infinite at harmonic pulsatances \((2\omega_0, 3\omega_0, ...)\) while having following value at the fundamental pulsatance:

\[ Z_{\text{load}}(\omega_0) = \frac{\kappa_0}{\omega_0 C_{ds}} e^{j\theta_0} = (53.13 + j61.23)\Omega \]

with:

- \( \kappa_0 \) and \( \theta_0 \), two constants respectively equal to 0.28015 and 0.85613 as presented in [Made 95].
- \( \omega_0 \) is the input signal’s fundamental pulsatance and is chosen at \( 2\pi \times 500\text{MHz} \).

The complete ADS simulation setup for this particular Class-E design can be found in Figure 1.24. Of particular interest is the choice of the input’s DC-operating point and amplitude which were chosen exactly the same as those of a classic Class-B PA. These values permit the transistor to behave as an open circuit and an approximate closed circuit for half of the time as is required for Class-E operation.

As before the HB simulator was exploited and the resulting time-domain waveforms can be seen in Figure 1.23. Alarmingly, the drain-source voltage has an extremely high peak (about 95 V), higher than the reported breakdown voltage of Cree’s CGH40006P which is equal to 84 V. Thus, the device would destroy itself when operating in this particular mode and some special care should be taken to reduce this peak in practice by, for example, reducing the maximum supply voltage or introducing resistive elements to bring down the resonance peak.
1.8. Polar Modulation

Interestingly to note, is that Cree’s CGH40006P is a Gallium Nitride (GaN) transistor, a technology that is already flaunted as having an exceptionally high breakdown voltage in comparison to other technologies. Thus while this particular design fails, other technologies would have even more trouble staying beneath their respective breakdown voltages.

The resulting loadline curves are shown in Figure 1.25 and showcase the loadline constraints of all PM transmitters. As can be clearly seen, the loadline curves are allowed to transition through the saturation region, but are only allowed to spend a minimum amount of time here lest the efficiency would take a hit. For most of the period, the loadline resides inside of the triode region (low voltage, but high current) or the horizontal zero-axis (high voltage, but low current) which allows the device to be as efficient as possible.

As this device is one belonging the PM transmitter variety, it is necessary to verify the design’s FOMs in function of different $V_{\text{supply}}$ instead of the input amplitude as was done in the case of ET. As a result, the gate-source voltage is left at its maximum value such that the transistor consistently behaves as an approximate switch. The resulting FOMs are shown in Figure 1.26 for both the efficiency and the supply-output gain $G_{\text{supply} \rightarrow \text{out}}$. Peak efficiency of this design is about $93.7\%$ at maximum supply voltage and seemingly increases even further in function of decreasing supply voltage.
1. Introduction to Dynamic Power Supply Transmitters

(a) Comparison of different efficiency metrics for a Class-E in function of the device’s output power $P_{\text{out}, \text{dBm}}$. Due to the large input power, the normal efficiency metric (■) results in significant exaggeration of the efficiency metric, especially at lower output powers. Both PAE (■) and TAE (■) give more truthful metrics, but the PAE drops to values below 0%, when the device’s output power becomes lower than the input power.

(b) Device’s supply to output gain $G_{\text{supply}\rightarrow\text{out}}$ in function of the output power which should, for an ideal PM transmitter, be zero across all output powers. The supply voltage $V_{\text{supply}}$ is thus transferred in almost perfect 1 : 1 ratio to the device’s output power.

Figure 1.26.: Different FOMs for the example Class-E PA showcasing both different possible efficiency metrics and the device’s flat supply to output gain.
Unfortunately the classic efficiency metric, as given in Equation 1.3, is quite optimistic and gives a exceedingly large exaggeration of the device’s actual efficiency as it does not take into account the large input power required for Class-E operation. The PAE (Equation 1.4) gives a more truthful metric, but drops below 0% when the device’s output power becomes lower than the input power. Solace can be found by using the TAE metric (Equation 1.5) as this metric correctly gives the variable efficiency in function of the supply voltage without over- or underestimation [McCu 15].

The device’s gain \( G_{\text{supply→out}} \) is almost perfectly flat as is required by the PM’s basic principle.

Some important, additional, issues make these kind of transmitters harder to realize than is evident on first sight:

- **Signal alignment is critical**

  While alignment of the dynamic supply voltage is extremely important in ET for achieving the desired linearity and efficiency target, a certain tolerance is allowed on the alignment of supply and input path as they only have a secondary influence on the actual signal integrity. In contrast, the slightest amount of misalignment in a polar arrangement has a direct impact on the signal integrity. Each signal path only contains either the amplitude or phase information of the modulated signal, thus any differences in path length or dynamics between these paths totally corrupts the inphase and quadrature signals contained within.

- **Realizing zero crossings is hard**

  At some points the signal envelope will inevitably drop to zero. Ideally, one would want the transmitter to correctly realize such a zero crossing at the device’s output. Decreasing the supply voltage to zero should, on a theoretical level, lead to a zero signal at the output, but unfortunately the overdriven input signal has the tendency to leak through the gate. Such a thing is not observed in the simplified model architecture as depicted in Figure 1.24 as this model does not contain any parasitic energy transfer from input to output. Some special care, either by avoiding zero crossings altogether in the modulated signal or by augmenting the device’s architecture, is necessary to make sure that distortion of this kind is circumvented.

In conclusion, there are several potential problems with this particular mode, ranging from the extremely high drain-source voltage peaks to input leakage. The weighted average efficiency (derived using the TAE formula) is about 75.45% and is impossible to ignore. Reaching these kind of metrics is very much impossible for the ET transmitters discussed in previous section.
Most of the scientific literature on ET makes no distinctions nor stands still on notions such as defined in Equations 1.8 and 1.9. As a result, the transmitters discussed in most works have the tendency to directly and significantly violate the main ET principle, given in Equation 1.8, and can therefore not be considered as thus. These type of amplifiers lie somewhere inbetween PM and pure ET, and are in effect hybrid combinations of some sort. These type of transmitters aren’t designed with respecting the PSS boundary in mind, but with a totally different goal. For example, a DPS transmitter could be designed by taking only those combinations of supply voltage and input power that result in maximum efficiency of the device. Such a device is maximally efficient, but will most likely be unusable for practical applications due to the significant amount of compression required to reach these high efficiency peaks. Therefore, this kind of hybrid combination, maxing out of efficiency by sacrificing everything else, will not be considered for purposes of this work.

However, a hybrid combination that is of further interest is the so-called constant joint transfer function. Such a device aims to achieve a constant gain, independent of the output power $P_{\text{out, dBm}}$, using the lowest $V_{\text{supply}}$ possible and, hence, gives a device that is efficient as can be, given the gain constraint. Important to note is that this particular gain is not an input-output gain $G_{\text{in} \rightarrow \text{out}}$ nor a supply-output gain $G_{\text{supply} \rightarrow \text{out}}$, but instead a weighted sum of both as defined in Equation 1.10.

**Constant Joint Transfer Function**

For achieving a constant joint transfer function, the procedure is quite similar to the one employed for ET, again starting with the maximum $P_{\text{in}}$, but with some minor alterations:

1. Lower the supply voltage $V_{\text{supply}}$ until the transmitter’s gain (being the sum of both supply and input contributions, as in Equation 1.10) just barely achieves the prerequisite gain constraint $G_{\text{goal}}$.

2. Decrease the input power $P_{\text{in}}$ and go back to step 1.

To avoid the additional hardware and time required for modulating the bias voltage of Class-A device, this hybrid technique was exclusively demonstrated starting from a classic Class-B PA only. Showcasing the necessary procedure was done using a constraint of $G_{\text{goal}} = 16.62 \, \text{dB}$, which corresponds to 1 dB less than what would be achievable by a classic Class-B PA. Other goals are equally feasible and represent a trade-off between efficiency on one hand and gain/linearity on the other hand.
1.9. Hybrid Combinations

Figure 1.27.: Two-dimensional sweep depicting the device’s combined gain in function of both the supply voltage \( V_{\text{supply}} \) and the input power \( P_{\text{in,\,dBm}} \). The chosen gain constraint \( G_{\text{goal}} \), having a value of 16.62 dB, is shown as well.

Traditionally, this procedure goes hand-in-hand with a simulation involving a two-dimensional sweep (sweeping both the device’s input power and supply voltage) as is shown in Figure 1.27 [McCu 15]. The chosen gain constraint is also shown and such a figure allows the designer to easily see if the chosen gain is achievable/feasible for the relevant device. The extracted shaping function is shown in Figure 1.28 together with the shaping function extracted previously for the lower-bounded ET transmitter, also based on a classic Class-B PA. As is immediately evident the hybrid approach employs much lower supply voltages for the same input power and therefore most definitely violates the PSS boundary region. The loadline curves shown in Figure 1.29 confirm this hypothesis as the drain-source current starts clipping at low drain-source voltages. It is the designer’s choice if the resulting non-linear compression is worth the increase in efficiency.

Due to the higher degree of compression, the efficiency of the constant joint transfer function is higher across the board as is to be expected. Table 1.3 gives the weighted average efficiency for different possible values of the gain constraint \( G_{\text{goal}} \) alongside the average efficiencies of other DPS transmitters that were previously discussed in this chapter. As expected, hybrid combinations only start to achieve higher average efficiency in comparison to the envelope tracking solution in the case of lower (= more compressed) values of the gain constraint.
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.28: Resulting shaping function for the implementation of the constant joint transfer function with $G_{\text{goal}} = 16.62\, \text{dB}$ (■), compared to the shaping function of the lower-bounded ET amplifier, lower-bounded at 3.75 V, introduced previously (■).

Figure 1.29: Loadlines curves of the constant joint transfer function, corresponding to a constant value of 16.62 dB, do not respect the $-30\, \text{dB}$ PSS boundary curve and cross into triode territory, clipping the time-domain current waveforms as a result.
1.9. Hybrid Combinations

<table>
<thead>
<tr>
<th>Operational mode</th>
<th>Weighted Average Efficiency (%)</th>
</tr>
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<tr>
<td>Class-B</td>
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<tr>
<td>Hybrid Combination ($G_{\text{goal}} = 17.62,\text{dB}$)</td>
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<tr>
<td>Lower-bounded ET ($@, 3.75,\text{V}$)</td>
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<tr>
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</tr>
<tr>
<td>Class-E</td>
<td>75.5</td>
</tr>
</tbody>
</table>

Table 1.3.: Comparison of weighted average efficiencies of different DPS transmitters based on the classic Class-B PA ordered from lowest to highest.

FOMs of the new hybrid transmitter are compared with the lower-bounded ET transmitter in Figure 1.30, as well as the metrics of the classic Class-B PA. While the gain for this new hybrid operational mode is indeed flat, it is essential not to forget that this gain is sensitive to the power supply (since its loadline curves breach the boundary region). The gain reported for the ET transmitter, whilst not flat in function of the output power, is to a certain extent insensitive to variations on the power supply by definition which might be an advantage in cases where large ripples on the supply are expected.
1. Introduction to Dynamic Power Supply Transmitters

Figure 1.30.: Comparison of FOMs belonging to a classic Class-B, a lower-bounded ET implementation of the same Class as introduced previously, see Section 1.7, and a hybrid design employing a constant joint transfer function with $G_{\text{goal}} = 16.62$ dB.

(a) Comparison of gain of the hybrid design (■), the input-output gain of a classic Class-B (■) and the input-output gain of the lower-bounded ET design (■).

(b) Comparison of the efficiency of the hybrid design (■) alongside the efficiency of a classic Class-B (■) and the efficiency of the lower-bounded ET design (■). Also shown is the PDF belonging to an IQ-modulated communication signal (■).
1.10. Conclusions

Several possible ways to ‘enhance’ the efficiency, based on modulating one or more DC-operating points, were introduced and verified using an extremely simple simulation example. Depending on the design requirements and constraints, a particular DPS technique can be picked and, quite easily, implemented using the design methodology employed in this chapter. For example, if a design at very high frequencies is desired, the Class-E technique is most likely infeasible due to the fixed and unchangeable size of the transistor’s output capacitance.

While these devices report very high efficiencies, the modulated supply and bias voltages required were seemingly generated by a fictional 100% device and this is a flaw which still is to be rectified in the next chapter.
2. Simulation & Design of Dynamic Power Supply Transmitters

Using a simplified theoretical model, as in previous chapter, is useful to introduce and consolidate the basic concepts of power amplification design. However, an actual commercial model of a transistor deviates from these simplified properties and thus makes the design of a Dynamic Power Supply (DPS) transmitter more difficult. In this chapter a Cree’s CGH40006P transistor device is exploited as the device on which to verify the theoretical basis discussed in previous chapter. One of the more suspicious absentees from previous chapter is the chapter’s namesake, the actual DPS. This device, necessary to modulate the bias and/or supply voltage, still requires an adequate introduction and is, in this work, implemented in the form of a Buck converter. Lastly, both components have to be connected in an appropriate manner such that the overall (= total) efficiency of the complete device increases.

This chapter’s main goal is to introduce the reader to some of the more dire problems encountered when both Power Amplifier (PA) and DPS have to actually work together. More specifically, the exact requirements on the interface between these components is not well-defined at all and needs to be handled with appropriate care to obtain the increased efficiency expected on the basis of the theoretical framework. Ultimately, the designs discussed here serve as test cases for the modelling techniques proposed in chapter 4 that aim to answer these questions in even more depth.
2. Simulation & Design of Dynamic Power Supply Transmitters

Figure 2.1.: Full-blown schematic of a DPS transmitter, this time with an actual DPS connected to the device’s extrinsic drain. Also shown are the necessary input and output matching networks (IMN & OMN) for forcing the device into the wanted operational class as well as the required input source with the double input voltage $v_{\text{in}}$ to remain consistent with definitions introduced in previous chapter.

2.1. Problem Statement

Previous chapter discussed important methods to enhance the efficiency of a PA by modulating and changing the DC-operating point(s) in a structured manner. This discussion accumulated into the inevitable design of several DPS transmitters of different categories using a simplified mathematical model.

An important, but deliberate, absentee of the whole story was the DPS itself, whose job is to actually modulate the DC-operating point in a, hopefully, synchronized and efficient fashion. This additional, but very much crucial, component is now to be discussed and a possible implementation is given in the form of a Buck converter. However, this is only half the story as naively connecting this Buck converter to the device’s drain, as is shown in Figure 2.1, without taking into account the impedances at this interface results in an inferior design. As will be seen, the device’s intrinsic drain $v_{\text{int}}^{\text{ds}}$ (as depicted in Figure 1.5) actively resists the supply voltage that is enforced by the DPS due to second order harmonic generation of the internal non-linear current source of PA. Forcing this intrinsic node to listen and follow the exact baseband waveforms, as specified by the non-linear shaping function’s output, is a necessary pre-condition to reap the actual benefits of the theoretical foundations introduced in Chapter 1.
2.1. Problem Statement

An important defect of the mathematical model, introduced in previous chapter in Equation 1.1, is that this device's baseband drain impedance does not possess any dynamics whatsoever. This important impedance, named $Z_{PA}$ from this point onwards, is the impedance seen by the DPS when looking into the PA’s drain connection ($Z_{PA} = \frac{V_{PA}}{I_{PA}}$ as per Ohm’s law and only valid if the DPS is considered as an ideal voltage source) and is dependent on a multitude of parameters, such as the device’s average input power, the operational mode and others. Having a more realistic model of these baseband dynamics is essential to make sure that the device’s components not only work independently, but also behave as expected when the DPS is connected and actually has to cooperate with the PA as a whole. To accommodate such an endeavour, the large-signal model for Cree’s CGH40006P is employed to design a set of DPS transmitters, each belonging to its own category, using the theoretical groundwork that was already laid out in the previous chapter. Ultimately, this chapter aims to answer some very important and practical issues:

- How can this DPS be designed/modified in such a way that the total efficiency of the complete transmitter (PA + DPS) is close to the theoretical expectations?
- How to connect and design the DPS in such a way that the desired modulated supply voltage actually arrives in an uncorrupted manner at the device’s intrinsic node?
- Can the non-linear shaping function be modified and/or expanded in such a way that it includes, in some way or form, a dependency on these baseband dynamics?

As is common in design methodologies, these questions don’t really have a unique answer and depend on several trade-offs that have to be carefully weighted by the designer. This particular chapter aims to discuss these issues without resorting to any advanced modelling or identification techniques and to get the job done by solely relying on the simulation techniques available in Keysight’s Advanced Design System (ADS). Such an approach has its limitations, but serves as a cautionary introduction to the problems at large.
2. Simulation & Design of Dynamic Power Supply Transmitters

2.2. The Buck Converter

In this work, the DPS will be implemented in the form of a Buck converter [Seba 14b], a highly efficient Switch-Mode Power Amplifier (SMPA) that consists of two transistors that down-convert a fixed DC voltage supply to a lower, but time-varying voltage as is depicted in Figure 2.2. These transistors are operated as switches and are thus only ever allowed to be fully turned on or off, which is the primary cause for this circuit’s extremely high efficiency metric. Due to this on/off constraint, the wanted output voltage has to be transformed to its Pulse Width Modulated representation as is demonstrated in Figure 2.3 for the gate voltage at the upper transistor ($V_{up}$). The lower transistor’s gate voltage ($V_{down}$) is an inverted version of the voltage at the upper transistor as to never allow both transistors to conduct at the same time instances.

The Buck’s internal node, positioned inbetween the upper and lower transistor contains the wanted spectral content, but also odd multiples of the clock frequency and up-converted versions of the baseband signal harmonics around these odd multiples. Suppression of these unwanted signal harmonics is achieved with the help of a low-pass output filter that, inevitably, influences the modulated supply voltage in some way or form and thus has to be adequately chosen to have minimal impact on the signal integrity.

Figure 2.2.: Simplified schematic of a one-phase Buck converter including the requisite output filter to suppress the clock frequency.
2.2. The Buck Converter

One of the disadvantages of this particular DPS type are the limitations on output signal bandwidth imposed by the clock frequency $f_{\text{clock}}$:

- Higher $f_{\text{clock}}$ results in a less strict filter characteristic and less error in correctly reconstructing the wanted supply voltage at the device’s output, but decreases efficiency due to increasing switching losses of the transistor devices.

- Lower $f_{\text{clock}}$ equates to more strict filter roll-off and steepness constraints for the filter characteristics and a higher reconstruction error, but increases the device’s efficiency.

It is for this reason that a SMPA, such as a Buck converter, is mostly augmented with a linear regulator that takes care of the higher frequency content such that the broadband supply voltage might be reconstructed with minimal error [Wang 14]. However, there’s ample motivation to get rid of this linear regulator as the loss in efficiency and increase in circuit complexity are unwelcome additions. Recent advances in transistor technology have greatly decreased the switching losses of the involved transistors and make the addition of a linear regulator obsolete for lower envelope bandwidths [Shin 13, Saka 17]. Unfortunately, these advances still greatly lag behind the bandwidth requirements for modern communication frequency ranges, some of which demand modulation bandwidths in excess of 100MHz, such as signals working at the $K_a$-band (26.5GHz $< f_0 < 40$GHz) of 5G NR standards [Dahl 18].

Figure 2.3.: The wanted modulated supply voltage $V_{\text{supply}}$ (■) is converted to its Pulse Width Modulated representation $V_{\text{up}}$ (■) and applied at the upper transistor’s gate for efficiency purposes.
2. Simulation & Design of Dynamic Power Supply Transmitters

Important to note is that other highly efficient DPS techniques exist in which the spectrally wide signal envelope is not applied directly, but a lower bandwidth equivalent is tracked instead. An example of this technique is the usage of an Analog to Digital Convertor (ADC)-like structure as a DPS. Such a device transforms the envelope signal into a series of discrete voltage levels that encapsulate the underlying signal envelope and are able to track communications signals with much higher modulation bandwidths [Flor 15]. This particular technique will not be pursued as, according to the definitions in [McCu 15], it cannot be considered a DPS transmitter in the truest sense, since the envelope is not tracked continuously. In effect, this PA device is more akin to a multi-level Class-G PA instead [Wolf 17].

**Output Filter Design**

In [Seba 14b] different possible filter types are compared for use as DPS output filters (Butterworth, Bessel-Thomson and Legendre-Papoulis) and due to its superior quadratic tracking error, the Legendre-Papoulis design comes out on top. As a result, a fourth-order Legendre-Papoulis is employed to suppress the Buck converter’s clock frequency down to acceptably small levels. The switching mode transistor stack behaves as a nearly ideal square-waveform source generator and can thus be approximated by an ideal voltage source as is the case in Figure 2.4, where the Buck converter is connected directly to the PA’s transistor drain. Connecting the DPS in such a manner to the PA only works correctly in the assumption that the PA’s baseband impedance \(Z_{PA} (= \frac{v_{ds}}{i_{PA}})\) is perfectly matched, across the entire baseband bandwidth, to the design impedance \(R_{design}\) of the output filter, which is a questionable claim at best.

![Diagram of DPS and PA connection](image-url)
Figure 2.5.: Simulation setup for the two-phase Buck converter with a fourth order Legendre-Papoulis filter implemented in ADS and using the Transient simulator.
2. Simulation & Design of Dynamic Power Supply Transmitters

Simulation setup

One of the bigger disadvantages of a Buck converter is the strict limitation on bandwidth. For a one-phase Buck converter, as considered previously, this limit is set in stone by the clock frequency $f_{\text{clock}}$ which has to be adequately suppressed by the output filter’s stopband alongside the up-converted input spectra. A possible way to increase this limit is to introduce the concept of multi-phase Buck converters. These more complex architectures contain multiple transistor stacks connected to the same output filter, instead of the single transistor stack employed by a one-phase Buck converter. By modifying the input signal of the $i^{th}$ stack with a phase shift of $2\pi(i-1)/N$ (with $N$ being the number of phases), harmonic multiples of the clock frequency can be successfully suppressed as discussed in [Seba 14a].

In this work, a two-phase Buck converter with a fourth-order Legendre-Papoulis filter is employed as the DPS of choice. Using two phases in the Buck converter removes odd multiples of the clock frequencies from the internal voltage spectrum and thus both loosens the design constraints on the filter characteristic and increases the available DPS bandwidth. The resulting ADS schematic of two-phase Buck converter, simulated using the Transient simulator with trapezoidal integration method [Kund 95], can be found in Figure 2.5.

For both the upper and lower transistors, EPC’s EPC2014c are used as this device enables the design of highly efficient Buck Converters with DPS purposes in mind [Effi 19b]. The clock frequency $f_{\text{clock}}$ is chosen as 50MHz and the filter’s cutoff frequency $f_{\text{cutoff}}$ is, as a direct consequence, set to 25MHz. Generation of the necessary switched transistor gate voltages was done using a Dataflow simulation which took care of assigning the correct delay between input signals of both phases as well as the introduction of optimized dead times. Due to dynamic influences of the parasitic elements in the Buck’s transistors, switching the input signal to zero doesn’t immediately result in the same result at the output. This could lead to time instances where, albeit for a short time, both the high and low transistors of the Buck stack are conducting current which leads to much lower efficiency. Increasing the dead time, the time during which both input signals are zero, is the preferred way to assure that both devices are never conducting at the same time. On the other hand, increasing the dead time too much leads to other problems as well [Effi 19a]. In this case, the ideal dead times were found empirically by tuning the relevant parameters in the simulation.
2.2. The Buck Converter

![Figure 2.6](image)

Figure 2.6.: Efficiency of the simulated two-phase Buck converter, shown in Figure 2.5, in function of the swept DC-value of the wanted supply voltage $V_{\text{supply}}$. Discrete efficiency stepping behavior is due to a combination of finite sampling frequency $f_{\text{sampling}}$ and dead times.

Lastly, the sampling frequency $f_{\text{sampling}}$, setting the fixed timestep $t_{\text{sampling}} (= 1/f_{\text{sampling}})$ of the Transient simulation as result, is chosen as 4GHz to be able to correctly simulate the device’s switching behavior without crashing the simulator. Higher sampling frequencies are possible but lead to simulations that take considerable amount of time without any significant change in simulation result. Lower sampling frequencies fail to converge due to the steepness of the device’s switching slopes and the fixed nature of the timestep (as required for the identification framework, as discussed in Chapter 3). To remove any transient behaviour, 3.2 periods are simulated and the initial fractional part of 0.2 is discarded.

To verify the resulting device’s performance, both the efficiency as well as the static and dynamic tracking errors are checked by sweeping the DC-value of the wanted supply voltage $V_{\text{supply}}$. In both cases the Buck converter is terminated into a fixed load impedance that coincides with the filter’s design impedance $R_{\text{design}}$. The device’s efficiency is shown in Figure 2.6 and clearly shows the superiority of SMPA efficiency (in contrast with linear regulators), especially at higher supply voltages. The efficiency takes a hit at lower voltages due to the proportionately increased contribution of switching losses at these output voltages. Of further interest is the discrete stepping behavior which is an unavoidable consequence of working with finite sampling frequency and non-zero dead times.
2. Simulation & Design of Dynamic Power Supply Transmitters

Figure 2.7.: Output voltage $V_{PA}$ (■) of the simulated two-phase Buck converter, shown in Figure 2.5, in function of the swept DC-value of the wanted supply voltage $V_{supply}$ (■). Discrete output voltage stepping behavior, as well as the inability to produce voltages below $\sim 3V$, is due to a combination of finite sampling frequency $f_{sampling}$ and dead times.

The resulting static tracking performance for the exact same sweeping experiment is depicted in Figure 2.7 and reveals an additional issue that is to be rectified. As can be seen the output voltage $V_{PA}$ increasingly deviates from the requested voltage $V_{supply}$ at increasingly higher supply voltages. Such a deviation is expected and is, in general, removed by static pre-compensation i.e. by placing an inverted version of this Static Non-linearity (SNL) in front of the DPS.

Dynamic tracking performance was also inspected by exciting the device with a 50MHz multisine, see Chapter 3, with a DC-offset of 14V. The resulting Frequency Response Function (FRF) is shown in Figure 2.8 and possesses a minimal inband amplitude ripple and almost no non-linear phase deviation. This deviation from the expected perfectly flat filter characteristic can be blamed in part due to frequency warping caused by the integration method as well as the fact that the transistor stacks only behave in an approximative fashion as ideal voltage sources. Similarly as for the static tracking error, this dynamic deviation can be removed by a pre-compensation, this time with an inverted linear time-invariant block instead. Nevertheless, the simulated Buck converter is shown to be able to adequately track a dynamic supply voltage.
Figure 2.8.: Amplitude and phase of the FRF of the Buck converter with a fourth-order Legendre-Papoulis filter with $f_{\text{cutoff}} = 25\,\text{MHz}$ perfectly matched into its design impedance $R_{\text{design}}$. A large part of the linear phase delay, about 85 samples ($= 21.25\,\text{ns}$), was removed from the phase metric to reveal the underlying non-linear phase. The resulting standard deviation, obtained by exciting the system with 7 different phase instances of the same input multisine, is also shown using $\times$ markers.
Conclusions

Due to the presence of both static and dynamic tracking errors, a pre-compensation block has been added to the baseband/envelope path of the DPS transmitter depicted in Figure 2.1. Such a pre-compensation unit is implemented in the form of a Wiener block structure. Together with the already present non-linear shaping function, this forms a Hammerstein-Wiener block structure as seen in Figure 2.9. Previous discussions were based on the assumption that the Buck converter’s output filter was perfectly matched to the baseband impedance \( Z_{PA} \) presented by the PA’s extrinsic drain node. Unfortunately, this drain impedance is not a fixed impedance at all and exhibits both frequency- and power-dependent behaviour. This begs the question: how should the output filter’s design impedance actually be chosen? Close to the averaged baseband impedance? Or what kind of trade-off should be respected?

As explained in [McCu 15], there’s a large indication that the output filter’s design impedance \( R_{design} \) is actually best chosen as small as possible and this for following practical reasons:

- to ensure baseband stability at the DPS transmitter’s output, and
- to have the necessary source strength to impose the wanted supply voltage \( V_{supply} \) on the PA’s drain node.

Evidently, not perfectly matching the \( R_{design} \) with the impedance \( Z_{PA} \) presented by the PA results in the appearance of additional ripple and phase non-linearity that has to be reduced to acceptable levels by the pre-compensation unit. There are limitations on the effectiveness of these pre-compensation components though as, for example, sharp resonance peaks are difficult to remove accurately. As always, the actual solution to this matching problem lies somewhere in between and results in some sort of trade-off as discussed in the subsequent sections.
2.3. A Slightly More Realistic Transistor Model

While the simplified transistor model used in Chapter 1 is ideal for demonstration purposes, it does not exhibit some of the more practical impairments expected of actual transistors, such as baseband dynamics, thermal effects and so on. As such, a more realistic model is to be used in the remainder of this work to be able to model these phenomena in some way or form. For this purpose Cree’s CGH40006P is used to design a more practical simulation example [Cree 15]. This particular transistor employs the Gallium Nitride (GaN) transistor technology and the advantages, inherent to this particular technology, can thus be exploited. Such advantages include the especially high breakdown voltage $V_{br}$ of these devices (84 V in this case) which moves the optimum load impedance for power amplification purposes closer to the middle of the Smith chart [Peng 12, Reza 13]. Trapping phenomena, one of the more exotic characteristics exhibited by the GaN transistor technology, are unfortunately not included in this model.

Access to the device’s intrinsic drain voltage and current is provided by the model and, as a direct result, the waveform engineering technique can be employed to find the requisite load impedances at the device’s extrinsic drain. However, without access to any other internal parts of the model, the matching procedure amounts to trial-and-error iterative tuning of the extrinsic impedance values.

IV-curves of this particular transistor are extracted using a similar simulation setup as in Figure 1.2 and are shown in Figure 2.10. The drain current of the device drops as a function of increasing drain-source voltage which is solely caused by thermal effects (in this model). In an effort to split the triode and saturated region of this transistor, the Power Supply Sensitivity (PSS) boundary regions of the exact same magnitudes as in Chapter 1 are derived and plotted on top of the simulated IV-curves. As can be seen the PSS boundary curve of $-70\,\text{dB}$ has significant issues at lower current values. At those current levels, the boundary curve jumps to unattainable voltage levels, close to the breakdown voltage, that make the design unfeasible for the $-70\,\text{dB}$ constraint.

**Simulation Setup**

As the necessary theoretical foundations were already discussed in Chapter 1, subsequent sections mainly focus on anything that strictly deviates from the idealistic considerations taken in that chapter. In an effort to streamline the PA design methodology, a generic matching network was used at both the device’s input and output that is able to correctly match both extrinsic nodes at the fundamental frequency and the second harmonic.
(a) Drain-source current $i_{ds}$ in function of the drain-source voltage $V_{ds}$ for increasing levels of the gate-source voltage. Possible boundaries between the perceived triode and saturation regions, corresponding to Power Supply Sensitivity (PSS) with values of respectively $-30\text{dB}$ (■), $-50\text{dB}$ (■) and $-70\text{dB}$ (■), are also shown.

(b) Drain-source current $i_{ds}$ in function of the gate-source voltage $V_{gs}$ for increasing levels of the drain-source voltage.

Figure 2.10.: IV-curves of Cree’s $CGH40006P$ transistor model acquired using a similar simulation setup as shown in Figure 1.2.
2.3. A Slightly More Realistic Transistor Model

Harmonic Balance

\[ \text{V}_{\text{DRAIN}} \]
\[ \text{V}_{\text{GATE}} \]
\[ \text{V}_{\text{IN}} \]
\[ \text{V}_{\text{OUT}} \]
\[ R \]
\[ \text{HARMONIC BALANCE} \]

Figure 2.11.: Simulation setup for the iterative matching procedure, with extrinsic impedance values shown for the Class-B design, simulated using the Harmonic Balance (HB) simulator.
The resulting simulation testbed is shown in Figure 2.11 as well as the necessary mathematical equations, used in an effort to greatly simplify the matching procedure as found in [Cola 06, Gian 09].

The proposed matching network structure, as seen in Figure 2.11, provides a convenient connection point for the supply and bias voltage at a quarter wavelength stub (shown as $TL3$ and $TL2$ respectively) that dually serves as a short circuit for the second harmonic. It is at this exact interface that the previously discussed DPS is to be connected. As the schematic presented in Figure 2.11 aims to serve a Class-B design, the DC-bias remains fixed at all time instances and a sufficiently large capacitor (serving as a short circuit at the fundamental frequency $\omega_0$) can be added to the bias feeding point without any problem. This practice is also actively encouraged for baseband stability purposes as the bias voltage ripple, at baseband, is sufficiently shorted before any resonant behaviour can show up. Unfortunately adding a decoupling capacitor of similar magnitude at the supply interface is not recommended as this would lead to a significant loss of the modulated (baseband) supply current. Instead, decoupling Radio Frequency (RF) and baseband paths is achieved here by aid of the dynamic components present in the Buck converter's output filter. As a result, the baseband stability at the supply side is not assured and should be adequately verified in practice.

Additionally, a second delay line, to allow for independent control of the imaginary part of the second harmonic impedance, is also included at both the input and output (denoted as $TL9$ and $TL6$ respectively). Explicit tuning of the second harmonic impedance at the device's gate is a less common practice, but, as discussed in [Shar 18], has been proven to have, depending on the exact class of operation, an important influence on the transmitter's efficiency and thus should be treated with due care.

For matching purposes, it is much more convenient to transform the device's voltages and currents to power waves [Kuro 65], defined as such:

$$a_{in}(t) = \frac{v_{in}(t) + Z_0 i_{in}(t)}{2\sqrt{Z_0}}$$
$$b_{in}(t) = \frac{v_{in}(t) - Z_0 i_{in}(t)}{2\sqrt{Z_0}}$$

with equivalent expressions for $a_{out}$ and $b_{out}$. In the ideal case of perfect matching conditions both the input reflected wave $b_{in}$ and the output incident wave $a_{out}$ would be perfectly zero across all powers and frequencies. Such a device adheres to the so-called Single-Input Single-Output (SISO) condition as it can be completely described using only the input incident wave $a_{in}$ and output reflected wave $b_{out}$. Subsequent chapters will, for ease of use, assume this SISO condition as valid and it is thus imperative that the matching networks designed in this chapter are properly tuned to satisfy this assumption.
2.3. A Slightly More Realistic Transistor Model

The presence of an input matching network reveals a further discrepancy with the previous chapter as the actual input power arriving at the device’s gate is only equal to the supplied reference power if the input matching condition is perfect across the band. In this work, this potential input mismatch is considered as part of the system and should, as a result, be accounted for when extracting the DPS transmitter’s shaping function. Henceforth the reference input power, being the power that would arrive at the device's gate in the case of perfect input match, is denoted as $P_{\text{ref,dBm}}$ while the actual input power that arrives at the gate is still indicated as $P_{\text{in,dBm}}$. Similarly as for the shaping function the device’s power gain is now written as $G_{\text{ref→out}}$ to reflect the fact that the gain from reference input power to output power is the one that actually matters for this chapter’s purposes.

In the remainder of this section, a design of a classic class-B PA, working at a center frequency of $2140\text{MHz}$, is briefly discussed. Similarly, a Class-E is also designed albeit at a lower center frequency of about $500\text{MHz}$ due to frequency constraints imposed by the transistor’s output capacitance. Due to the increased complexity of having modulated signals at both the supply and the bias side, a Class-A design will not be considered in this chapter.

**Class-B**

As elaborately discussed in the previous chapter, the class-B fundamental load impedance can be readily derived after an adequate choice of PSS boundary curve. Identical as in previous chapter a value of $-30\text{dB}$, depicted in Figure 2.10, is chosen as a good compromise between efficiency and PSS. The required fundamental load impedance for Class-B operation then becomes:

$$Z_{\text{load}}(\omega_0) \approx 31.8\Omega$$

This is the load impedance that should be presented at the device’s intrinsic node while (intrinsic) impedances at all other harmonic frequencies need to be as close to zero as possible to obey Class-B operational requirements. As discussed before, only the extrinsic impedances of the device are controllable by the designer, while the intrinsic impedances are observable, but not directly tuneable. This gives rise to a manual tuning (and/or automatic optimization) procedure that aims to generate the correct time-domain waveforms at the intrinsic interface.

Meanwhile, the device's input interface is matched using a large-signal conjugate matching procedure which aims to suppress the input mismatch across the entire reference power range using a single fundamental impedance.
2. Simulation & Design of Dynamic Power Supply Transmitters

(a) Loadline curves of the designed Class-B PA obtained using the simulation setup shown in Figure 2.11, for different values of the reference input power $P_{\text{ref, dBm}}$. Also shown is the boundary region, to be respected for linearity purposes, belonging to a PSS of $-30$ dB.

(b) Time-domain voltage (■) and current (■) waveforms of the Class-B PA for different values of the reference input power $P_{\text{ref, dBm}}$ expressed in integer multiples of the fundamental period $T_0$ ($=1/f_0$).

Figure 2.12.: Loadlines and time-domain waveforms of the designed Class-B PA obtained by using the simulation framework as shown in Figure 2.11
2.3. A Slightly More Realistic Transistor Model

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<th>$Z_{\text{int_load}}$ (Ω)</th>
<th>$Z_{\text{source}}$ (Ω)</th>
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<td>$-46j$</td>
<td>$0.353 - 7.797j$</td>
<td>$-24j$</td>
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</tbody>
</table>

Table 2.1.: Required intrinsic and extrinsic impedance environment for achieving Class-B operational requirements.

Input matching of the reactance at the second harmonic is used to compensate, or “resonate out”, the second order non-linear term of the device’s input varactor and results in an almost purely sinusoidal input current [Whit 98].

Supply voltage was chosen fixed at 28V as recommended by Cree, whilst the bias voltage was tuned slightly above the device’s threshold voltage $V_t$ (=−3.1V) to a value of −3.05V. Choosing this value results in much less non-linear compression at higher power levels, but puts the device in so-called deep Class-AB as is evident by the non-flat gain curve as seen later in Figure 2.13a. A curve which rises at lower amplitude levels is one of the indications that Class-AB operation is observed. A Harmonic Balance simulation is executed for the entire range of input amplitudes ($RF_{\text{amplitude}}$) using the settings as given in Figure 2.11.

The resulting dynamic loadlines and time-domain waveforms for all reference input powers are shown in Figure 2.12a and 2.12b respectively, while the actual intrinsic and extrinsic impedances (as found when dividing the relevant voltages and current as per Ohm’s law) are found in Table 2.1.

Alongside the designed Class-B PA, which uses a fixed supply voltage, two designs with variable supply voltage are considered that use the fixed supply voltage design as their starting point.

1. **Envelope Tracking**

As in previous chapter, a Class-B PA can be transformed into an Envelope Tracking (ET) transmitter by solely modifying the supply voltage such that the rail-to-rail voltage swing fits into the zone demarcated by the PSS region boundaries. In this case and similar as before, a PSS boundary region of $-30$dB is chosen. Likewise, a gain variation of $\pm0.5$dB is chosen as to limit the gain ripple and sets a lower boundary on the possible decrease in supply voltage $V_{\text{supply}}$.

1. **Hybrid Combination** (constant joint gain transfer function)

Instead of attempting to obey the PSS boundary region, a composite “flat” gain of 17.5dB can also be chosen as a design constraint (see section 1.9). The resulting efficiency is potentially higher than for ET as a deeper excursion into the triode region is not punished, which, as a result, gives more non-linear compression.
2. Simulation & Design of Dynamic Power Supply Transmitters

(a) Reference-output gain $G_{\text{ref} \rightarrow \text{out}}$ for the designed Class-B PA with fixed supply voltage $V_{\text{supply}}$, compared alongside the gain corresponding to a lower-bounded implementation of the ET technique as well a hybrid combination, using a constant joint gain transfer function of 17.5 dB, based on the exact same class.

(b) Comparison of the efficiency of the designed classic Class-B PA with fixed supply alongside the efficiency of the lower-bounded ET design and the efficiency of the hybrid combination design.

Figure 2.13: Figures of Merit (FOMs) belonging to different possible implementations of the DPS technique compared to the FOM corresponding to a classic Class-B PA with fixed supply voltage (in blue) on which they are based.
2.3. A Slightly More Realistic Transistor Model

The necessary supply voltage in function of the reference input power, the so-called shaping table, can be found in Figure 2.14 for both designs. In the case ET, the shaping table is extracted by sweeping the reference input power and reducing the supply voltage until the loadline fits within the demarcations set by the PSS curve. On the other hand, the shaping table for the hybrid combination design is extracted using a two-dimensional sweep of both the reference input power and the supply voltage. Choosing combinations of supply and power that obey the wanted gain specification make up the points of the resulting shaping table.

Resulting gain and efficiency metrics for all three proposed designs can be found in Figure 2.13. As can be seen in these figures the gain ripple of the hybrid design is lowest, while the ET has considerable ripple (albeit constrained within the design specifications). Peak efficiency of the hybrid design is greatest, but gets overtaken by the ET efficiency at lower power levels. As discussed in the previous chapter, the peak efficiency is not that important for actual modulated signals as the peak probability of the signal, as defined by the Peak-to-Average Power Ratio (PAPR), lies about 11.5 dB lower (as discussed in Section 1.5). Efficiency of the ET design is higher at those power levels and the weighted averaged efficiency is comparable as a result.
Loadline curves of the designed Class-E PA for different values of the supply voltage $V_{\text{supply}}$. Also shown is the boundary region that divides triode and saturation regions belonging to a PSS of $-30\text{dB}$. The time spent in the saturation region, corresponding to switching losses, should be reduced as much as possible for Polar modulation (PM) purposes.

(b) Time-domain voltage (■) and current (■) waveforms of the Class-E Power Amplifier for different values of the supply voltage $V_{\text{supply}}$ expressed in integer multiples of the fundamental period $T_0$. In this case, the peak voltage values remain below the breakdown voltage limit ($V_{\text{br}} = 84\text{V}$), in contrast to the Class-E design that uses an idealized transistor device.

Figure 2.15.: Loadlines and time-domain waveforms of the designed Class-E PA obtained by using a modified version of the simulation framework shown in Figure 2.11.
2.3. A Slightly More Realistic Transistor Model

(a) Comparison of different efficiency metrics of the designed Class-E PA in function of the device’s output power $P_{\text{out,dBm}}$. The generic efficiency metric (■) results in significant exaggeration of the efficiency metric, especially at lower output powers where it breaches the 100% efficiency limit due to input leakage. Both Power-Added Efficiency (PAE) (■) and Total-Added Efficiency (TAE) (■) give more truthful metrics, but the PAE drops to values below 0% when the device’s output power becomes lower than the input power.

(b) Device’s supply to output gain $G_{\text{supply→out}}$ in function of the output power which should, for an ideal PM transmitter, be zero across all output powers. The supply voltage $V_{\text{supply}}$ is thus transferred in an almost perfect 1:1 ratio to the device’s output power except at lower output powers.

Figure 2.16.: Different FOMs for the designed Class-E PA showcasing all different possible efficiency metrics as well as the device’s almost flat supply-output gain.
2. Simulation & Design of Dynamic Power Supply Transmitters

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<th>Frequency (Hz)</th>
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<th>$Z_{\text{source}}$ (Ω)</th>
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Table 2.2.: Required intrinsic and extrinsic impedance environment for achieving Class-E operational requirements.

Class-E

Alongside the ET and hybrid designs, a PM design is also considered in the form of a Class-E PA. As discussed in previous chapter, the output capacitance sets firm constraints on the feasible center frequency $f_0$ and a value of 500MHz is chosen for simplicity’s sake. The exact same simulation testbed was used as for the previously considered Class-B design with the difference that the input amplitude $RF_{\text{amplitude}}$ now remains fixed at the maximum level. Bias voltage also remains fixed and is chosen at the exact same value as for the Class-B design ($= -3.05V$).

Resulting time-domain loadline curves as well as time-domain current and voltage waveforms are shown in Figure 2.15a and Figure 2.15b respectively. The peak voltage of this more realistic design stays below the breakdown voltage $V_{\text{br}}$ ($= 84V$). This is in contrast to the voltage waveforms extracted for the more “idealized” transistor device which breached the device’s breakdown voltage, destroying the device as a consequence. Having a less ideal switch thus actively helps the design in this case.

Efficiency and supply-output gain $G_{\text{supply} \rightarrow \text{out}}$ are shown in Figure 2.16. Due to having a more realistic device, the generic efficiency metric now starts to report an efficiency larger than 100% at lower output levels due to input-output leakage. As before, the PAE is also problematic as it drops to a negative percentage. As seen in equation 1.4, this is due to the fact that the input power becomes bigger than the output power at lower levels of the supply voltage. In the case of Class-E both the efficiency and the PAE metric should be avoided in favor of the more trustworthy TAE metric.

The supply-output gain $G_{\text{supply} \rightarrow \text{out}}$ has a problem with the same origin, namely that at lower output power levels the input power (which remains fixed) starts to become a significant contribution to the output spectral content due to leakage. Part of the input power leaks and this results in difficulties to realize zero-crossings of the signal envelope. However, in the range above 10dBm the gain behaves close to what is ideally expected from a Class-E design, namely being around 0dB and having no significant amplitude ripple.
2.4. Connecting DPS with PA

After separately designing a PA, in the form of a Class-B PA, and a DPS, in the form of a Buck converter, one might simply assume that connecting the two together would be sufficient to finalize the design. This is simply not the case as the output impedance of the DPS $Z_{\text{DPS}}$ is most likely severely mismatched to the transmitter’s supply impedance $Z_{\text{PA}}$. To easily verify this mismatch, the baseband FRF between the reference voltage input of the DPS and the actual output voltage as present at the device’s drain connection can be measured. Ideally this transfer function is both flat in amplitude and in possession of a linear phase characteristic. Such an FRF was already extracted for the Buck converter in Figure 2.8, but only in an ideal situation in which the Buck converter is terminated in its design impedance $R_{\text{design}}$. In practice, this FRF contains both resonance and anti-resonance peaks as well as non-linear phase characteristics [Minn 09, Le G 11, Hove 12] which compromises the DPS transmitter’s signal integrity and efficiency.

For this purpose the earlier designed Class-B PA together with a Legendre-Papoulis output filter (connected at the $V_{\text{PA}}$ node as defined in Figure 2.11) is simulated using HB. Alongside this simulation a so-called small signal simulation is executed. This simulation adds an additional baseband sinusoidal input to the supply voltage that can be swept in frequency. As before, the resulting small signal voltage at the device’s internal node ($V_{\text{drain}}$) can be used to extract the baseband dynamics of this setup. The design impedance is now swept from 1Ω to 10Ω, in steps of 10Ω and the resulting baseband FRF are shown in Figure 2.17. As can be seen in the topmost figure, lower values of the output impedance create severe resonances due to impedance mismatch. Hence, one might assume that choosing the output impedance in such a way that the non-linear phase and resonance peaks are minimized would be the best choice.

An additional constraint that needs to be taken into account is the fact that the output filter (and thus the DPS as a whole) needs to be able to enforce the supply voltage at the device’s internal node. In an attempt to verify this second constraint an Envelope simulation was done around the transmitter’s center frequency $f_0$ (= 2.14GHz). At the device’s input a complex time-domain signal is up-converted to the fundamental RF center frequency, resulting in a 3-tone signal that is bound to create intermodulation products and non-linear skirts at the transmitter’s output. In the same simulation, an actual multitone signal was added to the supply voltage (together with the already present DC-value of 28V) and the resulting internal spectra for both a lower and higher output filter impedance are shown in Figure 2.18, alongside the reference supply voltage.
Figure 2.17: Baseband dynamics for different values of the output filter’s $R_{\text{design}}$ extracted using a small-signal simulation of the previously design Class-B PA. Resonance peaks and non-linear phase characteristics appear at smaller values of $R_{\text{design}}$ due to the impedance mismatch.

Figure 2.18 shows that the resulting internal baseband spectra for the high impedance output filter contains significant 2nd harmonic contributions of the 3-tone as the power supply is not “strong” enough to enforce the voltage at this node due to the high output impedance. Meanwhile, the internal baseband spectra for the low impedance output filter ($R_{\text{design}} = 1\Omega$) has lesser issues with non-linear contributions, but contains large resonance peaks as clearly seen in the amplification peaks of the simulation noise.

Most papers discussing output filter design of DPS consider the impedance $Z_{\text{PA}}$ to be in the range of $\sim 1\Omega$ (or higher, but pretty much always below $10\Omega$) in which case both second harmonic contributions can be suppressed and resonance peaks are avoided. Such an assumption is mostly valid when working with transistor devices with a low supply voltage.
One of the main advantages of using GaN transistor is their significantly higher breakdown voltage. This allows designers to use a much higher supply voltage without putting the device in any danger of breaking down. The usage of a higher supply voltage directly results in a higher output impedance of the device that is much closer to the wanted 50Ω impedance environment to which the output of the amplifier is to be matched. The presence of a reduced matching ratio (30Ω → 50Ω vs. 1Ω → 50Ω) greatly simplifies the design of a matching network that matches the circuit over a broad range of frequencies [Fook 90].

In the case of Cree’s CGH40006P, the wanted output matching impedance is around 30Ω in both the Class-B and Class-E designs considered in previous sections. Unfortunately, the advantage of using such a high supply voltage now becomes a double-edged sword. As per the theoretical Class-B voltage & current spectral values (as listed in Table 1.2), the factor between both the RF and baseband impedance is $\frac{\pi}{2}$, resulting in a theoretical value of about 58.6Ω (and about 50Ω for the more realistic Class-B designed previously in this chapter). Matching the filter’s output impedance at such a high value is not an option due to the presence of excessive intermodulation products at the baseband side.

Some kind of trade-off has to be made; i.e. a measure of non-ideality on the matter of the DPS’s input-output FRF is to be allowed to make the design feasible.

To summarize, the design parameter $R_{\text{design}}$ of the DPS’s output filter needs to obey following (contradictionary) constraints:

- Needs to be **small** such that the power supply can be considered as a perfect voltage source. This is important for the transmitter’s stability as the DPS has to be able to attenuate any baseband ripple the PA might generate lest oscillations appear at the supply feeding interface [McCu 15].

- Has to be **matched** to the extrinsic drain impedance as to create a well-behaved (= flat + linear-phase) FRF such that signal integrity is maintained.

The resulting baseband dynamics thus won’t behave in an ideal fashion and some kind of compensation mechanism is required to stop these non-ideal baseband dynamics from corrupting or compromising the signal integrity of the DPS transmitter. Such compensation is possible in a number of different ways that is to be discussed in the next sections.
Figure 2.18.: Internal baseband and reference spectra for two different values of the output filter’s design impedance $R_{\text{design}}$, extracted using an Envelope simulation of the Class-B PA. Due to severe impedance mismatch, resonance peaks and non-linear phase characteristics appear at smaller values of $R_{\text{design}}$. In both cases, second order harmonic contributions of the transistor device appear alongside the multitone and DC-tone.
2.4. Connecting DPS with PA

Figure 2.19: RLC-snubber to be added in-between the DPS and the PA’s supply interface to suppress a single resonance peak in the supply path.

**Passive Compensation**

One possible way to modify the non-ideal transfer function of the DPS is to add additional components at the internal terminal, mainly with the purpose of suppressing any resonance peaks that might appear in the transfer function [Aitt 10]. Such a network is commonly known as a snubber. A common implementation, namely the RLC-snubber, is shown in Figure 2.19. Other more advanced implementations exist, but are not considered for purposes of this work.

**Advantages:**

- No additional computational resources need to be provided.

**Disadvantages:**

- Additional hardware needs to be added to the circuit, resulting in a higher bill of components and circuit space.
- No possibility to easily modify the component values or circuit implementation.

**Active Compensation**

As proposed in Figure 2.9, one could also attempt to pre-compensate the unwanted baseband dynamics by adding a well-chosen Linear Time-Invariant (LTI) filter alongside the already present SNL used for static error tracking. The shaped reference supply voltage is thus filtered by a Wiener pre-compensation block before actually being applied to the Buck converter’s input. Evidently, such an approach has again advantages and disadvantages as listed here.
Advantages:

• No additional hardware needs to be added to the circuit.
• Filter characteristics of the pre-compensation filter can be modified at run-time or can at least be re-estimated without having to modify any physical circuit components.

Disadvantages:

• Additional computational resources are needed to estimate and apply the pre-compensation filter at run-time. Albeit this estimation could potentially be merged with the already present pre-distortion algorithms.
• There are actual limits on the maximum amplitude ripple that is still pre-compensatable. On one hand, pre-amplification of the supply voltage is only feasible as long as the resulting time-domain waveform is still within the Buck converter’s available dynamic range ($0 – 28\text{V}$). On the other hand, suppression of frequency bins to a level that is too low just results in the amplification, by the DPS, of simulation or system noise.

Shared disadvantages

Both techniques also share some common disadvantages that need to be addressed:

• Very sharp resonances are extremely difficult to compensate: even a small deviation in frequency has disastrous consequences as this results in the creation of an additional anti-resonance alongside the already present resonance in the supply path’s total dynamics.
• The impedance $Z_{pA}$ dynamically (and statically) changes depending on the input and supply powers applied to the device. Hence, one simple LTI filter or snubber is most likely not sufficient to capture the entire dynamic range of the circuit.

In this work, an active compensation mechanism is proposed as the method of choice. Extracting the necessary pre-compensation filter characteristics requires both a simulation testbed and an identification procedure. Next chapters are dedicated to the introduction of the necessary identification framework, while the remainder of this chapter discusses how to combine both the Buck converter and the PA into a single simulation testbed.
2.5. Full Simulation Setup

Simulation of both the DPS and the PA in a single simulation testbed require some quite contradictory demands from the simulator that cannot be easily reconciled. On one hand, the chosen DPS architecture requires following capabilities of the chosen simulator:

- High sampling frequency \( f_{\text{sampling}} \) such that there’s a sufficient amount of samples in each clock period \( f_{\text{clock}} \) to ensure simulation convergence and accuracy.
- Only baseband \([0, f_{\text{sampling}}/2] \text{Hz}\) needs to be simulated.

As seen before, both demands can be easily satisfied by using the Transient simulator, a time-domain simulation method.

On the other hand, simulation of the PA requires that:

- Frequency bands around multiples of the center frequency \( f_0 \) need to be simulated.

Such demands can be met by either using the HB method, as used during earlier simulations with a single-tone, or using the Envelope simulator, a mixed-domain method.

A possible way to reconcile both components is to introduce a so-called Ptolemy dataflow co-simulation, in which both simulators are used in the same simulation, each of them simulating their own circuit components. The supply interface serves as the point at which both simulators need to be tied together which can be done by using a simple feedback network. The resulting simplified schematic for this co-simulation testbed is shown in Figure 2.20.

As can be seen in the proposed schematic, the PA’s drain interface impedance is represented by the controllable impedance \( Z_{\text{PA}} \) in the transient domain of the simulation. This impedance is tuned by a feedback network that measures the actual baseband current \( I_{\text{PA}} \) that flows into the Envelope simulation, upsamples it and delays it with a factor \( D \) to enforce causality. The integer number \( D \) accounts for the difference in sampling frequency between the Transient and Envelope simulations.

The integer number \( L \) represents the number of samples necessary to remove the integer part of the relative delay between the supply and the input branches such that both signals arrive in a synchronized fashion at the intrinsic device.

To avoid any issues with artificial frequency damping, the Trapezoidal integration method is chosen for the Envelope simulation.
Figure 2.20: Simplified schematic of the Ptolemy co-simulation combining the use of both Data Flow simulation and two different simulators. Connection between the Transient simulated DPS and the Envelope domain PA is achieved using a simple feedback network that modifies the output impedance of the DPS according to the supply current $I_{PA}$ requested by the PA. This allows for simulating all components of the DPS transmitter simultaneously and gives more accurate simulation results by taking into account the correct influence of each component on each other.
### Practical Application & Detected Issues

Initial simulation results were found to be promising, although some important issues are present in the proposed simulation framework:

- **Fixed time step makes convergence in Transient simulation unpredictable**
  
  One of the advantages of using Transient simulation is the availability of a variable time step, i.e. the time step can dynamically change depending on the slope of the signals that are being simulated, decreasing the chance that the simulation crashes due to convergence issues. The Buck converter contains fast switching behaviour, in the form of the Pulse Width Modulation (PWM) input at the transistor’s gates. Both the rising and falling slope of this square wave pose convergence issues when simulating the transmitter with a fixed time step as is required in this Ptolemy co-simulation testbed.

- **Excessive simulation time**
  
  Another point of contention is the excessive simulation time (longer than 1 hour) that is required for each simulation. This issue is closely related to the required fixed time-step of the Transient simulation controller and makes fast re-iteration of the device values and settings cumbersome.

- **The Buck converter is unable to track small variations of the supply voltage**
  
  Albeit less relevant for purposes of this chapter, a so-called tickler signal is a signal added for identification’s sake. Such a signal is made as small as possible as not to influence the device’s operation, but still large enough to be distinguishable from the noise floor. Due to the finite (discrete) resolution of the Buck converter’s PWM input signal, this tickler signal is unable to switch the supply voltage to another discrete output level.

  This issue is less dire when working with a modulated envelope since this increases the likelihood that the tickler signal accidently switches the supply voltage to another discrete output amplitude level. However, an excessive amount of consecutive simulations and averaging would be required to extract the tickler signal from the DPS’s output.

Due to the summarized issues the proposed dataflow framework is only to be used as a final check before going to real-world design. In the remainder of this work the Buck converter’s transistor stack is entirely removed from the schematic as a compromise to make the framework useable in practice. The DPS’s output filter is moved into the Envelope domain which allows one to get rid of the necessary Transient simulation controller. Evidently, this also removes the compatibility/convergence issues this simulator introduced to the framework.
2.6. Conclusions

This chapter discussed the more practical issues that appear when a DPS transmitter is to be designed in a slightly more practical environment. Such issues, include the design of the DPS and questions that arise when this power supply has to be connected to the PA. This includes the choice of the design impedance for the Buck converter’s output filter and the necessity to add an additional compensation mechanism, either in the form of a passive snubber component or, as is the preferred method in this work, the addition of a pre-compensation filter.

A simulation testbed was introduced that allows both the DPS and the PA to be simulated together, but due to time-constraints the simulation framework could not be exploited to its full extent.

Subsequent chapters aim to introduce the necessary theoretical foundations to create an adequate identification framework to extract the necessary pre-compensation filter.
3. Introduction to the Theoretical Framework

The purpose of this chapter is to set up and discuss the necessary system identification framework and elaborate on its application on the simulation setups introduced in the previous chapter. As such, several demonstrational examples are used alongside the theoretical formulas in an attempt to keep the discussion grounded into the realm of practical applicability. The ultimate goal of this chapter is to work towards a modelling technique capable of extracting the Dynamic Power Supply (DPS)’s baseband dynamics in some way or form.

In an initial part, the theoretical foundations of the Best Linear Approximation (BLA) are introduced and discussed in due detail. This includes concepts such as multisine excitations, the out-of-band BLA and parametrization of the non-parametric estimate, all of which are crucial components for understanding the eventual theoretical framework. Afterwards a small foray is made into the realm of parameter-varying identification methods, by introduction of Harmonic Transfer Functions (HTFs). While these are, in this case, a naive attempt at extracting the baseband dynamics, the introduction of parameter-variation into the model structure sets the stage for the next chapter in which these concepts are enhanced even further.

As in previous chapters, most of the underlying mathematical concepts and calculations are left to the extensive scientific literature on this subject, in particular to the work of Rik Pintelon and Johan Schoukens [Pint 12].
3. Introduction to the Theoretical Framework

Figure 3.1.: General model structure for an LTI system. Depicted are $a_{\text{in}}$ and $b_{\text{out}}$, both time-domain signals which respectively represent the input excitation signal and the system’s response to this particular excitation.

To avoid being overwhelmed by the inevitable Linear Parameter-Varying (LPV) model structures, this chapter’s structure slowly builds up the relevant theoretical concepts one at a time. The aim of this chapter is to gain an adequate understanding of the choices and assumptions made in the proposed model structure. While most of these assumptions might seem banal and redundant for the experienced reader they are discussed in an effort to cover all bases and leave no room for any ambiguity in the framework. Throughout these sections, relevant examples, applying the discussed theoretical framework on simulations, are presented wherever appropriate to address their application on DPS transmitters.

3.1. Linear Time-Invariant (LTI) Models

The LTI model structure is shown in Figure 3.1 together with the input excitation signal $a_{\text{in}}(t)$, applied by the user or, alternatively coming from a previous stage, and the output signal $b_{\text{out}}(t)$, consisting of the system’s response to the input. Direct breakdown of this model structure’s name reveals two important basic properties that should be obeyed by the system’s response to be considered part of the LTI model class:

1. **Linear**

   A linear system entails superposition, or simply put: the sum of outputs is equal to the output of sums, which can be written as such:

   $$G\{\alpha a_{\text{in},1}(t) + \beta a_{\text{in},2}(t)\} = \alpha G\{a_{\text{in},1}(t)\} + \beta G\{a_{\text{in},2}(t)\} \quad (3.1)$$

   with $a_{\text{in},1}(t)$ and $a_{\text{in},2}(t)$ being arbitrary input excitation signals, while $\alpha$ and $\beta$ are constant gain factors. $G\{\} \,$ is a linear operator that transforms the input excitation to the system’s output and is called the time-independent transfer function from this point onwards. The equation above should be valid for all possible combinations of excitation signals and gain factors for the system to be considered perfectly linear.
3.1. LTI Models

In real systems, superposition, and linearity as a consequence, can only be readily assumed at the smaller input powers. After ramping up the input power one begins encroaching on the device’s electrical limits and non-linear phenomena such as compression and intermodulation products will inevitably start appearing at the device’s output. Pushing power transmitters into compression is the modus operandi for reaching the desired power efficiency target and thus, as a result, disqualifies any chance for these devices to behave completely linear.

2. Time-Invariant

The concept of time-invariance guarantees that if the system’s output $b_{out}(t)$ is known for a certain input excitation $a_{in}(t)$ then shifting $a_{in}(t)$ in time with any constant time $T$, with $T \in ]-\infty, +\infty[\), has no effect on $b_{out}(t)$ besides also shifting it in time, or in the form of a formula:

$$b_{out}(t+T) = G\{a_{in}(t+T)\} \quad \forall T \in ]-\infty, +\infty[ \quad (3.2)$$

In reality, actual system behaviour drifts with temperature, age and other time-varying model-independent variables over time. A model-independent variable can be defined as any variable that is not included in the transfer function’s argument nor is implicitly accounted for by the transfer function itself. Most common sources of time-variance, such as age and temperature, are incredibly slow processes in comparison to the time-scales at which the actual excitation signal acts. For example, the system’s characteristics might change in the time-span of years due to wear and tear while excitation signals for measurement purposes last a few seconds at most, in which case assuming time-invariance of the system is easily justifiable. Unfortunately, DPS transmitters violate such a principle by design as the modulation of the transmitter’s DC-voltage(s) operates on the exact same time-scale as the signal envelope’s bandwidth.

Thus the LTI framework has questionable merit in the case of DPS transmitters which is unfortunate as the advantages inherent to LTI model structure’s simplicity cannot be underestimated. As such, the next section’s subject will be entirely focused on getting around this issue. However, before complicating matters even further, some further assumptions can be made on the exact nature of the excitation signals for simplification’s purposes. One of the more important advantages of the LTI assumption is that the system can be completely described by a uniquely-defined single function $g(t)$, as such:

$$b_{out}(t) = G\{a_{in}(t)\} = g(t) \ast a_{in}(t) \quad (3.3)$$
3. Introduction to the Theoretical Framework

in which \( g(t) \) is called the system’s impulse response and \( * \) is the convolution operator given by following formula:

\[
g(t) * a_{in}(t) = \int_{-\infty}^{+\infty} g(t - \tau) a_{in}(\tau) d\tau
\]

The origin of these infinite integration limits is due to the fact that both the system’s impulse response \( g(t) \) and the input excitation \( a_{in}(t) \) are supposedly different from zero for all time-instants \( t \) belonging to the open interval \( ]-\infty, +\infty[ \). In practical implementations this integral would have to be truncated to a finite value to be computable in simulation software and, unavoidably, result in the appearance of a truncation error. To alleviate this and further issues, the identification framework can be specified even further using additional assumptions on both the model and the specific signals employed.

**Additional assumptions**

When working with models that should be applicable to real-world systems, it is evident that the device’s impulse response doesn’t have the ability to react to any future events. As a result, for making the framework applicable to real-world systems, an additional assumption is introduced on the values of the convolution’s arguments for \( t < \tau \) as such:

3. **Causality**

   The system’s output signal \( b_{out}(t) \), measured at time \( t \), is only dependent on the input excitation signal \( a_{in}(t') \) for times \( t' \leq t \) and never on any future values of the input. Applying this constraint to the previously established convolution formula results in a direct constraint on the impulse response, namely that the impulse response has to be zero for all times \( t < 0 \). This results in following truncated form of the system’s LTI equation:

\[
b_{out}(t) = \int_{-\infty}^{t} g(t - \tau) a_{in}(\tau) d\tau = \int_{0}^{\infty} g(\tau) a_{in}(t - \tau) d\tau
\]

Alongside causality, it is also customary to assume that the system’s input only becomes non-zero at time-instances after a well-defined time \( t_0 \). The resulting integral becomes proper and can now be solved over the finite interval \([t_0, t]\). For simplicity’s sake, the convention \( t_0 = 0 \) is respected in most cases. As a result, the convolution integral becomes of the following form:

\[
b_{out}(t) = \int_{0}^{t} g(t - \tau) a_{in}(\tau) d\tau
\]
which is an integral that is easily solveable in modern simulation software except for the fact that the interval \([0,t]\) still contains an infinite amount of datapoints. To solve this issue, the input and output signals can be discretized with a certain sampling frequency \(f_{\text{sampling}}\), which is an exact and unique operation as long as these signals are band-limited and contain no spectral content above the Nyquist frequency \((= f_{\text{sampling}}/2)\). Hence, following assumption can be coined:

4. Discretization

All relevant signals in the model are assumed band-limited and thus can be discretized using a sampling procedure without loss of information. The previously defined continuous time convolution product now turns into a discrete sum:

\[
b_{\text{out}}(n) = \sum_{m=0}^{n} g(n-m)a_{\text{in}}(m)
\]

Any device’s impulse response is, unfortunately, not only dependent on the applied input excitation, but also on the system’s initial state. In this particular case, this initial state was assumed to be the system’s response to an infinitely long zero input excitation signal. Exciting the device with the non-zero input excitation \(a_{\text{in}}\) will result in a transient response that would, preferably, disappear after a finite amount of time. A guaranteed way to suppress the system’s transient, for stable systems, is to assume periodicity of the excitation signal.

5. Periodicity

Exciting the system with a periodic signal, possessing a period \(T\), assures that the system can slowly, but surely acclimatize to the chosen excitation signal. After a non-descript amount of time the start and end state of the response will be identical, as is innate to periodicity, such that only the steady-state response remains. Enforcing such a constraint has several advantages, including but not limited to the fact that the Discrete Fourier Transform (DFT) is exact under that condition [Pint 12]. The resulting output signal is thus only defined on an integer multiple of the signal’s period, as such:

\[
b_{\text{out}}(n+N) = \sum_{m=N}^{n+N} g_{N}(n+N-m)a_{\text{in}}(m)
\]

where \(N\) is the integer number of discrete time samples, each with length \(t_{\text{sampling}}\), that fit in a single signal period \(T\). Meanwhile, \(g_{N}\) is a periodic summation of the impulse response \(g\) and is, as a direct consequence, the steady-state part of the impulse response.
3. Introduction to the Theoretical Framework

Transformation to the frequency domain, by using the DFT, results in:

\[ B_{\text{out}}(k) = G_N(j\omega_k)A_{\text{in}}(k) \]

where the capitalized \( B_{\text{out}}, A_{\text{in}} \) and \( G_N \) represent the Fourier transformed versions of their respective discrete time-domain equivalents and the DFT is defined as:

\[ X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi kn}{N}} \]

Furthermore, \( k \) denotes a frequency bin that is part of the discrete frequency grid, while \( \omega_k \) is the specific pulsatance at this particular spectral bin. Pulsatance and discrete frequency bin are related to each other using the frequency resolution \( f_{\text{res}} \) in following manner:

\[ \omega_k = 2\pi k \frac{f_{\text{sampling}}}{N} = 2\pi k f_{\text{res}} \]

As a side-note, the reason for using the pulsatance \( \omega_k \) for the transfer function \( G_N(j\omega_k) \), instead of just the discrete frequency bin \( k \), is to emphasize that it is most definitely a continuous function that also exists at points inbetween these discrete pulsatance samples. The frequency-domain input and output signal, on the other hand, only exist at a discrete number of bins \( k \) due to their periodic nature.

As discussed previously, in the case of DPS transmitters, neither of the LTI’s basic properties has any validity whatsoever. Luckily, as will become evident in the next section, the LTI structure’s virtues can be recycled somewhat by introduction of the Best Linear Approximation (BLA).

3.2. The Best Linear Approximation (BLA)

DPS transmitters are most definitely non-linear devices. As became evident in previous chapter, ramping up the input power of these devices above a certain threshold results in significant compression and the creation of non-linear skirts around the amplified Radio Frequency (RF) output spectrum [Morg 06]. Instead of moving on to full-blown non-linear models, using increasingly complex structures, the introduction of the BLA allows one to preserve the simplicity of the LTI model structure while allowing one to estimate the “best” (= in least squares sense) linear approximation of the non-linear system around a well-defined Large-Signal Operating Point (LSOP).
3.2. The BLA

Figure 3.2.: Different possible, but equally valid, linearizations of the same Static Non-linearity (SNL) (■), but around totally different LSOP. Both input excitation signals have the exact same DC-operating point (= 0.5), but possess different PDF, one consisting of a uniform distribution (■) and the other belonging to a Gaussian distribution (■) with the same PSD.

This so-called LSOP is pertinent to the entire BLA framework and can be considered as an extension of the normal (small-signal) linear operating point, defined by the DC-operating point, by also taking into account several other input excitation attributes.

Large-Signal Operating Point (LSOP) The LSOP is the point around which the non-linear system is linearized and consists of, alongside the DC-operating point, following attributes of the input excitation signal:

- Probability Density Function (PDF)
- Power Spectral Density (PSD)

Excitation signals that possess the exact same attributes are said to belong to the same signal class and result in identical best linearizations [Pint 12]. An example of linearizations in the case of signals with different PDF is shown in Figure 3.2 for demonstration purposes.

Additionally, only non-linear systems that belong to the class of Wiener systems can actually be approximated by a BLA [Scho 05, Sche 80]. This particular class of systems is defined as:
3. Introduction to the Theoretical Framework

Figure 3.3.: Model equivalence between a PISPO system and the BLA.

**Wiener Class** A system belonging to the Wiener class of systems is in possession of the so-called Period-In Same Period-Out (PISPO) property, which can be defined as:

\[ a_{in}(t) = a_{in}(t + T) \iff b_{out}(t) = b_{out}(t + T) \]

which means that the output’s spectral content always falls on the exact same discrete grid as defined by the input excitation. This allows some pretty wild behaviour on the aspects of non-linearity, such as hard-clipping, but prohibits the existence of chaotic behaviour as well as the creation of subharmonics.

Since neither chaotic behaviour nor subharmonics are present in the case of DPS transmitters, the BLA can be used as a linearization mechanism as depicted in Figure 3.3. Hence, Equation 3.3 can be recycled, but with an additional term:

\[ b_{out}(t) = bla(t) \ast a_{in}(t) + d(t) \]

or when transformed to the frequency domain and using the exact same assumptions as in previous section (except for linearity):

\[ B_{out}(k) = BLA(j\omega_k)A_{in}(k) + D(k) \quad (3.4) \]

with \( D(k) \) (or \( d(t) \)) a non-linear distortion term that has noise-like qualities and has, by definition, the exact same period as the input excitation signal. This distortion term contains the part of the device’s output spectra that is not repeated when exciting the system with different excitation signal instances of the same LSOP, denoted as “realizations” from now on. Since this is a purely stochastic contribution to the BLA framework that can be proven to possess zero mean, it is solely characterized by its variance across these so-called realizations. Though this distortion term is of lesser interest here, it can, among other things, be used to find the relative non-linear contributions of different components in a complex circuit and, ultimately, pinpoint the component that generates most non-linear distortion in a so-called Distortion Contribution Analysis (DCA) [Coom 17a].
For estimation purposes the discrete frequency transformed BLA, as given in Equation 3.4, can be written as a matrix product:

\[ \vec{B}_{\text{out}} = \vec{BLA} \odot \vec{A}_{\text{in}} + \vec{D} \]

where:

\( \odot \) denotes the element-wise multiplication of two column vectors.

For correct estimation of both the BLA and the distortion term, several random realizations of excitation signals belonging to the same LSOP (\( R \) of them) have to be applied to the device’s input. These different realizations and their resulting output vectors then have to be stacked next to each other and the resulting matrices with dimensions \((T + 1) \times R\) are used to get an estimate of the BLA and its variance across realizations. The resulting variance can then be converted to the distortion term as they are closely related with each other [Pint 12].

**Multisine Excitations**

To apply the proposed BLA framework to the domain of DPS transmitters following demands have to be met:

- Using a periodic input excitation such that the identification procedure is greatly simplified.
- Using an input signal that belongs to the same LSOP group as the non-periodic communication signal that is used in the final design.

Both demands are contradictory and seem quite irreconcilable at first glance. Luckily, there exists a signal that has the exact properties required in the form of the so-called Random Phase Multisine (RPM). This periodic excitation signal consists of a sum of sinusoids each with a phase component chosen randomly from a uniform distribution. Furthermore it can be proven that for an RPM consisting of an infinite amount of tones this particular signal can be tailored to fit in any LSOP group imaginable, under the constraint that the PDF is Gaussian [Pint 12].
However, it is important to stress that the multisine is a fully deterministic signal and multiple “realizations” of the same multisine instance (taking a different random set of phases) are required to get similar stochastic properties as the aperiodic communication signal and thus converge towards the same BLA. It can be proven that this particular assumption becomes valid, in an asymptotic manner, when at least 7 (or more) different random phase realizations are employed [Pint 12].

In this work, two different kinds of RPM will be used, one having only spectral content at baseband frequencies around the DC-operating point, as is the case at the supply voltage’s side, and the other being a band-pass signal placed around the center frequency \( f_0 \), as is the case at the transmitter’s source port.

**Low-pass Multisine**

\[
    v_{\text{supply}}(t) = \sum_{k=1}^{T} \alpha_k \cos(2\pi k f_{\text{res}} + \phi_k)
\]

in which:

- The spectral grid is equally spaced with frequency resolution \( f_{\text{res}} = f_{\text{sampling}} / N \) and thus only contains spectral energy at a discrete number \( T \) of frequency bins \( k \) (with \( k \in \mathbb{Z} \), being the set containing all integers).
- The amplitude \( \alpha_k \) is, in this work, chosen constant for each of the excited bins.
- The phase \( \phi_k \) is randomly chosen from a uniform distribution on \([0, 2\pi]\) to obtain a signal with Gaussian-like properties.

**Band-pass Multisine**

\[
    a_{\text{in}}(t) = \sum_{k=-T/2}^{T/2} \alpha_k \cos(2\pi (f_0 + k f_{\text{res}}) t + \phi_k)
\]

(3.5)

- These frequency bins are placed around a center frequency \( f_0 \) that, in general, can be placed anywhere, even inbetween the spectral grid. For purposes of this work, and for ease of the relevant calculations, it is chosen to limit this center frequency to also be part of this set (i.e. \( f_0 = k_0 f_{\text{res}} \) with \( k_0 \in \mathbb{Z} \), with \( \mathbb{Z} \) being the set containing all integers).
- Both \( \alpha_k \) and \( \phi_k \) are chosen similar as for the low-pass multisine.

In most cases, and especially for the band-pass multisine, it is much more intuitive to work with the average root mean square (rms) power \( P_{\text{average,dBm}} \) of the multisine instead of choosing the amplitude individually at each of the excited bins. Conversion between the input wave’s average power and the flat amplitude spectrum required at each bin is easily done, as such:
3.2. The BLA

\[
P_{\text{average,dBm}} = 10 \log_{10}\left(\frac{|a_{in}(t)|^2}{1\,\text{mW}}\right) = 10 \log_{10}\left(\frac{\alpha^2(T + 1)}{2}\right) + 30
\]

where the power wave definitions introduced in Chapter 2 ([Kuro 65]) are respected and the resulting amplitude at each spectral bin is a constant value \( \alpha \).

In the case of DPS transmitters and PAs in general, the limiting factor is not the average power \( P_{\text{average,dBm}} \), but the peak power \( P_{\text{peak,dBm}} \) instead. Violating this upper power threshold results in significant non-linear compression as is evident in all of the Power Amplifier (PA) gain metrics shown in previous chapters. Average and peak power are related to each other using the Peak-to-Average Power Ratio (PAPR), as defined in Chapter 1, giving following relation:

\[
P_{\text{peak,dBm}} = P_{\text{average,dBm}} + \text{PAPR}
\]

Due to the stochastic nature of the RPM, the PAPR (and as a result \( P_{\text{peak,dBm}} \)) becomes stochastic as well. Thus, consecutive phase realizations of the multisine result in quite different peak values and great care has to be taken to make sure that the PAPR does not exceed the device’s available power back-off. In this work, phase realizations that go above this threshold are rejected and replaced by new realizations that do respect this upper limit. This operation is quite similar to so-called crest factor reduction techniques that are used in practical applications to reduce the PAPR to manageable levels as discussed in [Sper 04], among others. In both cases the members of the LSOP are restricted to only those instances that obey the upper bound.

Choosing the \( P_{\text{average,dBm}} \) (and the bin amplitude \( \alpha \) as a result) is done on a trial-and-error basis, reducing or increasing the average power until at least a significant number of the phase realizations passes the test (> 99%). A more substantiated choice can be made with the help of extreme value theory [Cast 88] and the resulting calculations would yield actual percentiles for the PAPR distribution. However, such a technique was not implemented/exploited in this work.

**EXAMPLE 3A - BLA of a Class-B PA**

For demonstrating the BLA, the previously designed Class-B PA (in Section 2.3), is now to be excited with a band-pass RPM and the resulting BLA is extracted. For this purpose the supply voltage is left fixed at 28V and multiple average power levels are used to examine the variation of the BLA depending on the LSOP. The maximum allowed peak power for the Class-B PA is 22dBm as significant compression sets in if one would go beyond this point. Further properties for the example’s input wave are chosen as given in Table 3.1.
3. Introduction to the Theoretical Framework

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 )</td>
<td>2140 MHz</td>
</tr>
<tr>
<td>( f_{\text{res}} )</td>
<td>500 KHz</td>
</tr>
<tr>
<td>( T + 1 )</td>
<td>21</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>( P_{\text{average}, \text{dBm}} )</td>
<td>([-6, 14]) dBm</td>
</tr>
<tr>
<td>( P_{\text{peak}, \text{dBm}} )</td>
<td>22 dBm</td>
</tr>
<tr>
<td>( R )</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3.1.: Properties of the chosen input RPM (\( A_{\text{in}} \)) applied at the device's source port for BLA extraction purposes.

A maximum average power metric of 14 dBm is employed as this results in a significant amount (about 99%) of phase realizations that pass the peak test (the PAPR only exceeds the 8 dB mark occasionally). The bandwidth of the multisine is chosen as 10 MHz as this a common bandwidth employed in practical applications. Estimation using this particular multisine is likely not representable for any practical application as the number of tones (21) is quite small which was mainly done to significantly reduce simulation time for this demonstrational example. As discussed in Chapter 2, the circuit is simulated using Advanced Design System (ADS)'s Envelope simulator and uses identical settings as introduced there. The resulting spectra of the input & output waves are shown in Figure 3.4. As would be expected the output wave consists of an amplified version of the input spectral bins together with significant non-linear skirts. The blue and yellow colored spectral bins, the difference in color not having any importance for the current discussion, are bins that lie in-between the excited spectral grid and are proof that this system is indeed a member of the PISPO system class, as would be required for the BLA framework to be applicable.

Estimated BLA results are shown in Figure 3.5 for both amplitude levels. As is evident the amplitude of the BLA is rather flat and the most significant difference between both estimates is a small phase offset.
Figure 3.4.: Frequency-domain representation of a single phase realization of the supply voltage $V_{\text{supply}}$ (fixed at 28 V), the input wave $A_{\text{in}}$ and the output wave $B_{\text{out}}$ of the simulated Class-B PA, consisting of 4 periods. Spectral content at the output only appears on the input excitation grid ($\times$) and never on any of the sub-harmonics (all other colors) which, in the case of a system belonging to the Wiener class, is solely populated by the simulation noise floor.
Figure 3.5.: BLA estimation results are shown for both full input amplitude (■) as well as for one tenth of the input amplitude (▲) both extracted using the exact same set of 16 phase realizations. The standard deviation $\sigma_{\text{BLA}}$ of both estimates is depicted using × markers in the same color as their respective mean value.
The out-of-band and extended BLA

Estimation of the BLA gives information on the inband behaviour of the transmitter, but doesn’t say anything on the linearized behaviour outside of this band. For modelling purposes it might also be necessary to get an estimate on the system’s behaviour outside of the main excitation band. This knowledge can be used, for example, to obtain a pre-distorsion model for the amplifier [Scho 17].

Obtaining the BLA outside of the excited inband frequency range is impossible without explicitly adding an additional excitation signal. For both simulations to converge towards the exact same BLA it is essential that the LSOP, before and after addition of this signal, remains unchanged. This gives rise to the introduction of the so-called tickler [Van 09], which is defined as:

**Tickler signal** A tickler signal is a signal that is small enough not to influence the LSOP at all and thus doesn’t modify the inband BLA in any way or form. At the same time, it is chosen sufficiently large to enable the extraction of the out-of-band BLA.

In practice there are most definitely limits on decreasing this tickler amplitude as this signal should preferably still remain above the noise floor, otherwise excessive averaging procedures would be required to recover the excitation signal’s response. This results in some kind of trade-off, in which an amplitude has to be chosen that doesn’t influence the LSOP, but is large enough to give an estimate that is distinguishable from the noise floor.

An additional hurdle exists in the form of the non-linear skirts as these currently fall on the exact same spectral bins as this incredibly small tickler. Again, excessive averaging would be required as these non-linear skirts are quite a lot bigger and drown out the much smaller tickler. This problem can be solved by exploiting the fact that the device belongs to the PISPO class of systems. As is always possible, the output wave can be written as:

\[ b_{\text{out}}(t) = f_{\text{NL}}\{a_{\text{in}}(t)\} \]

where \( f_{\text{NL}}(\square) \) can be any dynamic non-linear system that is still a member of the Wiener class. As a direct consequence, the spectral content of the output wave \( b_{\text{out}}(t) \) falls on the exact same spectral grid as the input wave. This is also verified in Figure 3.4 where the spectral bins in-between the excited grid solely consisted of contributions due to the simulation noise floor. These empty spectral bins can thus be used for other purposes as the device’s non-linearity is unable to generate any spectral content at these bins. To achieve this the spectral excitation grid is subdivided into **even** (= 2k) and **odd** (= 2k + 1) spectral bins which were already depicted in Figure 3.4 using the colors ■ and ■ respectively.
The updated input wave $a_{in}$ now consists of both an inband main excitation as well as a tickler contribution, in the following form:

$$a_{in}(t) = a_{in}^0(t) + a_{in}^{\text{tickler}}(t) = a_{in}^0(t) + \sum_{k=-\frac{T_{\text{tickler}}}{2}}^{\frac{\pi}{2}-1} \beta_k \cos\left(2\pi(f_0 + (2kM_{in} + 1)\frac{f_{res}}{2})t + \chi_k\right)$$

in which:

- $a_{in}^0(t)$ is the even inband RPM, as defined in Equation 3.5, and is responsible for setting the LSOP of the device.
- $a_{in}^{\text{tickler}}(t)$ is the odd out-of-band (tickler) RPM possessing following properties:
  - $M_{in}$ ($\in \mathbb{N}$, being the set of all natural numbers) is a modification factor for the excited spectral grid such that a larger bandwidth can be covered without using an excessive amount of spectral tones. Having a lesser amount of spectral tones allows for a higher amplitude per tone without modifying the LSOP and results in more accurate estimates at these tones.
  - The phases $\chi_k$ are randomly chosen from a uniform distribution in the interval $[0, 2\pi]$, similar as before.
  - The amplitude $\beta_k$ at each frequency bin is chosen small enough so that it does not influence the non-linear behavior and only ‘tickles’ the device around its LSOP which is set by the main multisine excitation.

The out-of-band BLA, obtained using the tickler response, as well as the inband BLA are considered part of the same model which, in this work, is collectively named the extended BLA. An additional requirement to correctly estimate this extended BLA is that at least 2 periods with length $\frac{1}{f_{res}}$ are measured, doubling the simulation time as a result.

**EXAMPLE 3B - Extended BLA of a Class-B PA**

As before, the Class-B PA with fixed supply voltage is used as a demonstrational example for estimating the extended BLA. The input inband RPM $a_{in}^0(t)$ has the exact same values as in Table 3.1, while the properties for the out-of-band tickler multisine $a_{in}^{\text{tickler}}(t)$ are given in Table 3.2.

Amplitude at each of the tickler’s spectral bins was empirically tuned in such a way that no additional intermodulation products were generated resulting from the tickler’s inclusion. This can either be checked on sight when looking at the resulting spectral content at the device’s output as is shown in Figure 3.6 or by comparing the overlap of the 95% uncertainty boundary of the inband BLA before and after adding the tickler to the input as is done in Figure 3.7.
3.2. The BLA

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>2140 MHz</td>
</tr>
<tr>
<td>$f_{res}$</td>
<td>500 KHz</td>
</tr>
<tr>
<td>$M_{in}$</td>
<td>8</td>
</tr>
<tr>
<td>$T_{tickler}$</td>
<td>40</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>160 MHz</td>
</tr>
<tr>
<td>$\beta$</td>
<td>${-107, -127}$ dB</td>
</tr>
<tr>
<td>$R$</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3.2.: Properties of the chosen input tickler RPM ($A_{in}^{tickler}$) applied at the device’s source port for out-of-band BLA extraction purposes.

Figure 3.6.: Frequency-domain representation of a single phase realization of the supply voltage $V_{supply}$ (fixed at 28 V), the input wave $A_{in}$ and the output wave $B_{out}$ of the simulated Class-B PA. Spectral content only appears on the even (×) and odd (▷) members of the spectral grid due to the PISPO property. Additionally, as a consequence of the extremely low amplitude values of the tickler, no additional nonlinear intermodulation products are generated.
3. Introduction to the Theoretical Framework

Figure 3.7.: Comparison of the inband BLA before (■) and after ( ■) adding the tickler tone gives an indication if the LSOP was modified by the tickler’s inclusion. In this case, the 95% uncertainty boundary regions have significant overlap giving a large indication that the LSOP is unmodified.

The relevant uncertainty boundary can be derived from the BLA’s standard deviation by using following formula (with $p = 0.95$) [Pint 12]:

$$R = \sqrt{-\ln(1 - p)} \sigma_{BLA} = 1.73 \sigma_{BLA}$$

In both cases the device’s response passes the test and the resulting estimated extended BLA, for both amplitude levels, is shown in Figure 3.8 together with its standard deviation. As before the BLA is flat inband and there’s little difference between both the high and lower amplitude estimates except for a phase offset. The out-of-band behaviour reveals that the amplitude (and thus the gain) drops which is as expected since the PA was only matched in a narrow band around the center frequency. Phase of the estimate also starts varying more significantly which can similarly be blamed on the narrowband match.

Evidently, the out-of-band BLA can be estimated accurately when employing the special grid techniques without any problem. Furthermore, the extended BLA is quite smooth and has similar standard deviation across all excited frequencies.
3.2. The BLA

Figure 3.8.: Extended BLA estimation results are shown for both full input amplitude (■) as well as for one tenth of the input amplitude (■) both extracted using the exact same set of 16 phase realizations. The standard deviation of both estimates is also depicted using △ markers in the same color as their respective mean value.
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**Parametrization of the BLA**

Previously defined frequency response functions are classified as non-parametric models, meaning that they are only defined at a specific set of frequencies, as given by the excitation grid, and thus have several deficiencies:

1. Number of datapoints is equal to the number of excited frequency bins, which can be quite high for small $f_{res}$.

2. Intermediate values are undefined and thus cannot be used directly.

Both problems can be solved by introduction of a parametric model which significantly reduces the number of necessary parameters to describe the extended BLA. In this work, the assumption will be made that any DPS transmitter’s extended BLA can be correctly modelled by a parametric rational form, defined as [Pint 12]:

\[
BLA(\vec{\Omega}_N(k), \vec{\Theta}) = \frac{\sum_{i=0}^{n_c} c_i(\vec{\Omega}_N(k))^i}{\sum_{i=0}^{n_d} d_i(\vec{\Omega}_N(k))^i}
\]

where:

- $\vec{\Theta}$ is a column vector of dimensions $[(n_c + 1) + (n_d + 1)] \times 1$ that contains the stacked parametric model parameters, in following order:

\[
\vec{\Theta} = \begin{bmatrix} \vec{\Theta}_c \\ \vec{\Theta}_d \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n_c} \\ d_0 \\ \vdots \\ d_{n_d} \end{bmatrix}
\]

- $\vec{\Omega}_N$ is a column vector of dimensions $[(T + 1) + T_{in}^{\text{tickler}}] \times 1$ that contains the normalized (to 1) and down-converted frequencies of the extended BLA:

\[
\vec{\Omega}_N = (\vec{\Omega} - j2\pi f_0)/\omega_{\text{max}}
\]

in which $\vec{\Omega}$ is the column vector of the same dimensions that contains all of the excited pulsatances, multiplied with $j$, of the extended BLA while $\omega_{\text{max}}$ is the maximum absolute pulsatance present in the down-converted pulsatance column vector.
Due to the frequency down-conversion, each of the model parameters \( c_x \) (and \( d_x \)) becomes a complex number. As a result, the estimated parametric model will be a normalized complex baseband equivalent of the actual model which can easily be determined by denormalization and frequency up-conversion of the estimation result. Estimation of the unknown model parameter vector \( \vec{\Theta} \) is most simply done with the help of a Total Least-Squares (TLS) estimation procedure [Pint 12]. This procedure requires the construction of the so-called Jacobian matrix that is defined in the following way:

\[
J_X = \sum_{i=0}^{n_x} [X \odot (\Omega_N)^{i}] \otimes (\bar{C}_i)^T
\]

where:

- \( X \) should be substituted by either \( A_{in} \) or \( B_{out} \) while \( n_x \) is respectively equal to either \( n_d \) or \( n_c \).
- \( \odot \) denotes the element-wise multiplication of two column vectors.
- \( (\square)^T \) denotes the transpose operator.
- \( \bar{C}_i \) is the so-called connection column vector with dimensions \((n_x + 1) \times 1\) and obeys following constraints:
  \[
  \bar{C}_i(k) = 1 \quad \forall k = i \\
  \bar{C}_i(k) = 0 \quad \forall k \neq i 
  \]  
  (3.7)
- \( \otimes \) is the kronecker product and places each column vector at its correct position in the eventual Jacobian matrix.

For the clarity’s sake, the first two columns of the Jacobian input matrix are then to be constructed as follows:

\[
J_{A_{in}} = \begin{bmatrix}
  A_{in}(k_{min}) \\
  \vdots \\
  A_{in}(0) \\
  \vdots \\
  A_{in}(k_{max})
\end{bmatrix} \otimes [1 \cdots 0] + \begin{bmatrix}
  A_{in}(k_{min})\Omega_N(k_{min}) \\
  \vdots \\
  A_{in}(0)\Omega_N(0) \\
  \vdots \\
  A_{in}(k_{max})\Omega_N(k_{max})
\end{bmatrix} \otimes [0 \cdots 0] + \ldots
\]

with \( k_{min} \) and \( k_{max} \) being respectively either the minimum or maximum excited spectral bin in the reference input excitation.

Both of the resulting Jacobian matrices \( J_X \) have respective dimensions \([(T + 1) + T_{in}^{\text{tickler}}] \times (n_x + 1)\) and are stacked next to each other to give an overdetermined set of equations:
3. Introduction to the Theoretical Framework

\[
\begin{bmatrix}
J_{B_{\text{out}}} & -J_{A_{\text{in}}}
\end{bmatrix}
\begin{bmatrix}
\Theta_d \\
\Theta_c
\end{bmatrix} = J\Theta = \Delta \approx 0
\]

in which the column vector \(\Delta\) (with dimensions \([(T + 1) + T_{\text{tickler}}] \times 1\)) is the residual vector, containing a measure for the modelling errors [Pint 12].

In the case of multiple phase realizations, the resulting Jacobians for each and every one of these simulations/measurements has to be stacked on top of each other as they pertain to the exact same underlying BLA:

\[
J_{\text{stacked}} =
\begin{bmatrix}
J_{\text{real}=1} \\
J_{\text{real}=2} \\
\vdots \\
J_{\text{real}=R}
\end{bmatrix}
\]

The resulting stacked Jacobian \(J_{\text{stacked}}\) has dimensions \(R[(T + 1) + T_{\text{tickler}}] \times (n_c + n_d + 2)\) and has to be correctly transformed to a real matrix using the polymorphic equivalence [Pint 12], as such:

\[
J_{\text{real}} =
\begin{bmatrix}
\Re\{J_{\text{stacked}}\} & -\Im\{J_{\text{stacked}}\} \\
\Im\{J_{\text{stacked}}\} & \Re\{J_{\text{stacked}}\}
\end{bmatrix}
\]

(3.8)

This real matrix can be used in a minimization scheme to find an estimate for the unknown parameter vector, which is also transformed to the real vector \(\Theta_{\text{real}}\) by stacking the real and imaginary parts on top of each other. Linear Least-Squares (LS), the most simple LS estimation procedure in which the residual vector \(\Delta\) is directly minimized, has several disadvantages, such as the fact that it is neither efficient nor consistent, that make it a less desirable estimator to use in noisy circumstances as is the case here. To circumvent these issues, the Bootstrapped Total Least-Squares (BTLS) procedure discussed in [Van 92] can be implemented instead. This procedure starts from a Weighted Generalized Total Least-Squares (WGTLS) optimization problem of following form:

\[
\arg\min_{J_{\text{real}},\Theta_{\text{real}}} \left\| W(J_{\text{real}} - J_{\text{real}})C^{-1} \right\|_F^2 \\
\text{subject to } J_{\text{real}}\Theta_{\text{real}} = \vec{0} \text{ and } (\Theta_{\text{real}})^T\Theta_{\text{real}} = 1
\]

in which \(\|\|_F^2\) is the Frobenius norm, \(W\) is a left-weighting matrix that is iteratively updated for efficiency purposes and \(C\) is a right-weighting matrix that contains the column standard deviation of the matrix product \(WJ_{\text{real}}\).
3.2. The BLA

An estimate for the unknown parameter vector $\overrightarrow{\Theta_{\text{real}}}$ can be found by calculating the generalized singular value decomposition (GSVD) of the matrix pair $(WJ_{\text{real}}, C)$ [Pint 98, Peum 18]. A clear disadvantage of this method is that it requires prior knowledge on the standard deviation to construct the right-weighting matrix $C$. Consequently, an estimation procedure is required to take care of this prerequisite. In this particular case, the previously established non-parametric standard deviation $\sigma_{\text{BLA}}$ can be exploited for this exact purpose.

Both nominator and denominator model orders of the rational form have to be carefully chosen, high enough to accurately model all relevant model dynamics, but low enough to discourage any modelling of the noise and other irrelevant spectral content. To accurately deduce the correct model order, the Akaike Information Criterion (AIC) cost function is exploited [Pint 12]:

$$V_{\text{AIC}} = V_{\text{BTLS}}(1 + \frac{n_c + n_d + 2}{(T + 1) + T_{\text{tickler}}})$$

with:

$$V_{\text{BTLS}} = \sum_{k \in \text{excited bins}} \left| (W_{\Delta})^H C W_{\Delta} \right|^2$$

This allows for selecting the most plausible model order after sweeping through the entire model range. Moreover, great care is taken such that the eventual model is also stable by checking the estimated pole positions. Afterwards, independent simulation data, not used for estimation purposes, is used to validate the model’s veracity as to make sure that indeed, no over- or undermodelling is present in the parametric model by comparing the parametric model estimate, expected for the relevant validation input data, with the actual simulated output data [Pint 12].

An important side-note to the entire parametric framework discussed here is that the microstrip transmission lines employed for matching the PA introduce a small phase delay. Such a delay cannot be modelled by the rational form proposed in Equation 3.6 and would necessitate an extension of the parametric model by addition of some kind of delay term. This subject will be addressed in depth in Chapter 5 as this becomes an actual issue when moving to measurements. Luckily, in simulations, the amount of time delay introduced by the matching network’s transmission lines is negligible and can be safely ignored without any loss of generality.
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![Figure 3.9: Comparison between the non-parametric BLA (■) and the parametric model estimated using the BTLS procedure (■), both down-converted and normalized in frequency. The residuals of the parametric model as well as the standard deviation of the non-parametric estimate are shown as well and depicted using × markers of their respective colors.](image)

**EXAMPLE 3C - Parametric model of a Class-B PA**

The BTLS estimation procedure is now used to extract a parametric model by exploiting the standard deviation $\sigma_{BLA}$ obtained using the previously estimated non-parametric extended BLA of the Class-B PA. With the help of the AIC cost function the most plausible model order was found to be $n_c = 2$ and $n_d = 2$ and the resulting parametric model is depicted, alongside the non-parametric extended BLA, in Figure 3.9. The extended BLA shown here is the one extracted by using the input RPM with the largest amplitude ($P_{average, dBm} = 14 dBm$). To validate the model’s compatibility to the underlying data, both the residuals as the non-parametric standard deviation are plotted as well. As can be clearly seen their levels coincide and this indicates that the parametric model successfully captures all relevant dynamic behaviour.
3.2. The BLA

Figure 3.10: Evolution of the poles and zeros of the normalized parametric BLA in function of the device’s input amplitude. Used amplitude levels for the average input power $P_{\text{average, dBm}}$ were 4 (■), 8 (■), 11 (■), 12.75 (■) and 14dBm (■) (corresponding to respectively 0.1, 0.25, 0.5, 0.75 and 1 times the total input voltage).

Furthermore, the pole and zero locations are all situated in the left-half plane, as seen in Figure 3.10, and thus results in a stable minimum phase parametric model. Additionally, to examine the evolution of the pole and zero positions in function of the device’s input amplitude, the model estimation procedure was used to extract 5 parametric models at different amplitude levels, but with the exact same phase realizations.

The resulting pole-zero map is depicted in Figure 3.10 and verifies that the input power dependency of the parametric model is quite tame as was already evident in the previously estimated non-parametric models. An in-depth analysis of the pole-zero movement, including uncertainty bounds, is however outside the scope of this work.
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3.3. Harmonic Transfer Functions (HTFs)

While previous section dealt with linearizing non-linear devices, the model was still considered time-invariant. This subsection aims to discuss a possible way, one of many, to introduce time-variation into the (non-)parametric BLA framework. Such an extension is most definitely useful when any external parameter, that is not part of the input, modifies the behaviour of the device. For the purposes in this work, this parameter is always assumed to be observable and subject to the same signal assumptions as the input excitation (discretization & periodicity).

Before going any further, it is imperative that the nature of this time-variation is adequately cleared up. For a DPS transmitter, the time-variation most definitely pertains to the modulated supply voltage \( v_{\text{supply}}^0(t) \) and, due to the periodic nature of the input wave \( a_{\text{in}}^0(t) \), will be periodic as well.

This can be easily seen by analyzing all the necessary mathematical equations required to transform the input wave to the shaped supply voltage, as such:

\[
v_{\text{supply}}^0(t) = f_{\text{shaping}} \{ |a_{\text{in}}^0(t)| + j \mathcal{H} \{ a_{\text{in}}^0(t) \} \} \tag{3.9}
\]

in which:

- \( \mathcal{H} \{ \square \} \) is the Hilbert transform, an operator which calculates a 90° phase-shifted version of a real-valued signal (\( = \) the so-called harmonic conjugate) such that the complex sum gives an analytic signal.
- \( |\square| \) denotes the modulus (absolute value) operator.
- \( f_{\text{shaping}} \{ \square \} \) is a static non-linear shaping function.

For simplifying the discussion at hand, the necessary pre-compensation unit, for compensating the static and dynamic tracking errors of the DPS, as in Figure 2.9, is not taken into account. All of these operators obey the PISPO principle and thus not only ensure periodicity of the supply, but also the fact that the supply voltage’s spectrum will lie on the exact same spectral grid as the input wave. This is quite problematic, as the spectral content generated by the device’s non-linearity and the spectral content coming from the supply voltage’s modulated nature are indistinguishable at the transmitter’s output. Both fall on the exact same bins and, even worse, are generated by intermodulation products of the same spectrally pure input wave \( a_{\text{in}}^0(t) \).

To solve this issue, a unique tickler excitation is added to the supply voltage, such that the contributions due to variations in the supply voltage can be separated from those generated by the device’s non-linear behavior. As before, this new tickler is extremely small and thus has to be protected from being buried by the non-linear skirts lying on an even spectral grid.
In an attempt to easily pinpoint the contributions due to the input tickler \( a_{\text{in}}^{\text{tickler}}(t) \), which was previously placed on an odd grid, the spectral grid is again subdivided to accommodate for these additional contributions. As a result these new spectral tones are placed on the odd \((= 2k + 1)\) bins, while the total input excitation \( a_{\text{in}}(t) \) now lies on an even grid instead. As before, this even grid contains both the main excitation \( a_{\text{in}}^{0}(t) \), on even-even \((= 4k + 0)\) bins, and the input tickler excitation \( a_{\text{in}}^{\text{tickler}}(t) \), on odd-even \((= 4k + 2)\) bins. In effect nothing has changed for the input excitation except for the naming convention and the fact that spectral bins inbetween the original grid can now also contain spectral energy. Furthermore, a minimum of 4 periods is now required for correct estimation purposes.

This new supply tickler is added to the shaped supply voltage amounting to following total supply voltage:

\[
v_{\text{supply}}(t) = v_{\text{supply}}^{0}(t) + v_{\text{supply}}^{\text{tickler}}(t) = v_{\text{supply}}^{0}(t) + \sum_{k=0}^{\infty} \gamma_{k} \cos(2\pi((4kM_{\text{supply}} + 1)f_{\text{res}}/4)t + \psi_{k})
\]  

(3.10)

where:

- \( v_{\text{supply}}^{0}(t) \) is the supply voltage derived from the even-even inband RPM, as defined in Equation 3.9, and is thus, together with the inband excitation, responsible for setting the LSOP of the device.
- \( v_{\text{supply}}^{\text{tickler}}(t) \) is the odd supply tickler possessing similar properties as the input tickler and for similar reasons:
  - \( M_{\text{supply}} \) is a modification factor for the excited spectral grid such that a larger bandwidth can be covered.
  - The phases \( \psi_{k} \) are randomly chosen from a uniform distribution in the half-closed interval \([0, 2\pi]\).
  - The amplitude \( \gamma_{k} \) at each frequency bin is chosen as small as possible.

A fundamental assumption of this work is that this supply tickler is up-converted by the device’s inherent non-linearity and, consequently, appears in some way or form at the device’s output. This is quite a dangerous assumption, especially in the case of the Envelope Tracking (ET) technique as introduced in Chapter 1. Here the ET basic principle (Equation 1.8) declared that the output power is totally insensitive to any modulation on the supply voltage whatsoever. Thus for an ideal ET PA the supply tickler would not appear at the device’s output at all. Such an ideal device doesn’t exist (as discussed in the same chapter) and the choice of the Power Supply Sensitivity (PSS) boundary region ensures that at least some part of the tickler contribution gets transported to the device’s output.
3. Introduction to the Theoretical Framework

Figure 3.11.: Model structure proposal for a preliminary attempt at estimating the baseband dynamics of a DPS transmitter. For now the RF input-output path is assumed as a single complex quasi-static constant while the supply tickler is multiplied by a common part $BB$ and then filtered through their respective first-order HTFs before being up-converted by the device’s intrinsic non-linearity.

The resulting tickler contributions at the transmitter’s output get influenced by both the baseband (before up-conversion) and the RF dynamics (after up-conversion) of the DPS transmitter and thus contain, in part, a measure for the actual baseband dynamics of the device. In an effort to validate such behavior the so-called first-order HTFs [Loua 12, Sand 05] are estimated and compared with the actual voltage that is present at the internal node. This leads to the model structure shown in Figure 3.11 in which the internal supply voltage is up-converted to the output spectrum’s lower and upper sideband. HTFs of higher order, as well as the zeroth order HTF exist, but result in contributions around DC and harmonics of the carrier frequency which are not or difficult to observe at the device’s output and such are not taken into account.

Evidently such a model structure has limited useability when the input wave is an actual modulated RPM since the transfer function of the RF path, in this model, is assumed to be of a quasi-static nature. However, when the input wave is only a single sinusoid, the quasi-static constraint is valid and this simplified schematic is an accurate representation of the transmitter’s behavior.

This results in following expression for the complex baseband equivalent representation of the output wave $B_{\text{out}}(k)$:
3.3. HTFs

\[ B_{\text{out}}(k) = D_0 \left[ HTF_{+1}(j\omega_k)V_{\text{supply}}^{\text{int}}(k) \ast \Re\{A_{\text{in}}\} + jHTF_{-1}(j\omega_k)V_{\text{supply}}^{\text{int}}(k) \ast \Im\{A_{\text{in}}\} \right] \]

with \( V_{\text{supply}}^{\text{int}}(k) \) being the internal supply voltage that gets filtered by a common part, present in both branches, being the actual baseband dynamics \( BB \), as such:

\[ V_{\text{supply}}^{\text{int}}(k) = BB(j\omega_k)V_{\text{supply}}(k) \]

in which:

- \( \ast \) is the discrete convolution product and is depicted in its time-domain equivalent form in the model structure (as a multiplication).
- \( A_{\text{in}}(0) \) is a complex DC-value that represents the complex baseband equivalent of the single RF input tone at the center frequency \( f_0 \).
- \( D_0 \) is a complex constant that aims to describe the quasi-static gain of the transmitter’s RF input-output path.

The DC-component of the complex baseband equivalent output wave \( B_{\text{out}} \) contains a combination of the quasi-static gain of the RF path and the DC-contributions coming from the up-converted supply voltage, in the following form:

\[ B_{\text{in}}(0) = D_0 [HTF_{+1}(0)\Re\{A_{\text{in}}(0)\} + jHTF_{-1}(0)\Im\{A_{\text{in}}(0)\}] BB(0)V_{\text{supply}}(0) \]

To get rid of this equation’s inherent ambiguous nature, the DC-value of the real (complex conjugate) filter \( BB \), as well as the DC-values of the HTFs are all normalized to 1. Any gain of the supply voltage to the output is thus assumed to be solely caused by the quasi-static gain term, as such:

\[ B_{\text{in}}(0) = D_0 A_{\text{in}}(0)V_{\text{supply}}(0) \]

An immediate advantage of this method is that the previously discussed estimation procedures for the non-parametric BLA can be readily recycled without much modification. Taking into account the randomized phase component of the input wave, which is different for each phase realization, is necessary for being able to correctly average the different simulation results.
3. Introduction to the Theoretical Framework

<table>
<thead>
<tr>
<th>Parameter</th>
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</thead>
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<td>500 KHz</td>
</tr>
<tr>
<td>$M_{\text{supply}}$</td>
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</tr>
<tr>
<td>$T_{\text{tickler}}$</td>
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</tr>
<tr>
<td>Bandwidth</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>${-50, -70}$ dB</td>
</tr>
<tr>
<td>$R$</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3.3.: Properties of the chosen supply tickler RPM ($V_{\text{tickler}}^{\text{supply}}$) applied at the device’s supply port for checking time-varying behaviour.

**EXAMPLE 3D - Estimation of the first-order HTFs for a Class-B PA**

As before the Class-B PA simulation example is now used to validate the model assumptions. This time an additional supply tickler is added to the fixed DC supply value (28 V), as in Equation 3.10. The device’s input is excited with a single sinusoidal tone at the center frequency $f_0$ with a chosen input power of 14 dBm.

The properties of the new supply tickler are given in Table 3.3 and, as before, 16 phase realizations are simulated for both full and halved input power in an attempt to check for the presence of power-dependency in the device’s behavior.

To have some form of known baseband dynamics, a fourth-order Legendre-Papoulis output filter is added to the PA’s drain, as discussed in Chapter 2 (Figure 2.4). This output filter is designed with an output impedance $R_{\text{design}}$ of 1 Ω and is thus heavily mismatched with the actual baseband impedance $Z_{\text{PA}}$, as presented by the PA. For reducing simulation time, the Buck converter as well as its specific simulation components were disabled in the Ptolemy cosimulation testbed, see Figure 2.20. Furthermore, to speed up the PA’s Envelope simulation, the Backward Euler integration method is used and this, as is inherent for this integration method, results in significant damping of the output filter’s resonance peaks as well as some frequency warping of lesser interest.

As an important sidenote, the device is still very much being operated as a classic PA, because, while the supply voltage has now become a modulated signal, the supply tickler, due to its very nature, is unable to modify the device’s LSOP.

The resulting output spectrum, for a single phase realization, is depicted in Figure 3.12, and very clearly showcases the fact that the supply tickler gets up-converted to the device’s output. Additionally, it also proves that these up-converted contributions can only exist on odd spectral bins. Indeed, intermodulation products of an even-even bin (at the center frequency) with any odd bin always falls on other odd bins. Higher order intermodulation products lie below the simulation noise floor due to the tickler’s small amplitude.
Figure 3.12.: Frequency-domain representation of a single phase realization of the supply voltage $V_{\text{supply}}$ (DC-value + tickler), the input wave $A_{\text{in}}$ and the output wave $B_{\text{out}}$ of the simulated Class-B PA excited with a single sinusoid. Spectral content, at the output, only appears on the even center frequency $f_0$ ($\times$) and the odd spectral grid (○) due to frequency up-conversion of the supply tickler. The simulation noise floor is also shown and is at the same level for all spectral bins.
3. Introduction to the Theoretical Framework

As can be seen by comparing the flat amplitude spectrum of the supply tickler with its up-converted version, there are most definitely significant baseband dynamics present in this device as is to be expected. Amplitude differences between the spectral content appearing at the lower- and upper-sideband is minimal, but not negligible, for this particular example.

The non-parametric up-conversion products are depicted in Figure 3.13 for both input powers. As can be seen in these figures the power dependency of the up-conversion product is limited. A small dependency of the quasi-static gain factor $D_0$ is present, but this has no direct influence on the baseband dynamics. Some variation in the phase is also evident, but can safely be ignored for purposes of this example.

The estimated complex up-conversion product, denoted as $Z(j\omega_k)$ from now onwards, can be split into a sum of real (= complex conjugate) filters, in the following way:
3.3. HTFs

\[ X(j\omega_k) = \frac{(Z(j\omega_k) + Z(-j\omega_k))}{2} = BB(j\omega_k)HTF^{+1}(j\omega_k) \]

\[ Y(j\omega_k) = -j\frac{(Z(j\omega_k) - Z(-j\omega_k))}{2} = BB(j\omega_k)HTF^{-1}(j\omega_k) \]

Splitting the complex filter into an in-phase filter \( X(j\omega_k) \) and a quadrature part \( Y(j\omega_k) \) reveals that the baseband dynamics can only be found in the form of a product with their respective HTF. Unfortunately, the current framework is unable to split these products and cannot give a direct estimate. A possible solution for this problem might be found by estimating several up-conversion products \( Z(j\omega_k) \) at different center frequencies, as one could make the assumption that the resulting baseband dynamics would stay the same while the HTFs shift in frequency. Though such a procedure most likely modifies the device’s LSOP and thus the baseband dynamics as a consequence. Albeit the extent to which this would be a problem is not known since this particular experiment was not attempted in simulation.

In this particular simulation example, the actual baseband voltage arriving at the transistor’s extrinsic drain is known and can be compared with the estimation results. However, it is important to remember that the simulation result at this intermediate node might not yet be filtered by all of the dynamic behaviour of the real filter \( BB \). The resulting Frequency Response Function (FRF), obtained by division of the measured internal supply voltage \( V_{int}^{\text{supply}} \) by the reference supply voltage \( V_{\text{supply}} \), is shown in Figure 3.14 alongside the in-phase and quadrature part of the complex non-parametric up-conversion product \( Z(j\omega_k) \). The quasi-static component \( D_0 \), present at DC, is of lesser interest for the current discussion and was removed from these figures as a result.

Interestingly the in-phase component \( X(j\omega_k) \) gets quite close to the real baseband dynamics and only starts deviating at higher frequencies. However, as could erroneously be assumed, such behaviour is, in this particular case, not related to damping induced by the chosen integration method (= Backward Euler) of the simulation example since both in-phase component as the real baseband dynamics perceive the same damping. Meanwhile, the exact form of the baseband dynamics is still very much visible in the quadrature component giving a large indication that both non-parametric models get filtered by identical baseband dynamics before being up-converted.
Figure 3.14.: Comparison between the estimated in-phase (■) and quadrature (■) components of the HTFs and an approximation of the actual baseband dynamics (■).
Figure 3.15.: Frequency-domain representation of a single phase realization of the supply voltage $V_{\text{supply}}$ (DC + tickler), the input wave $A_{\text{in}}$ (inband + tickler) and the output wave $B_{\text{out}}$ of the Class-B PA excited with a single large-signal sinusoid. Spectral content only appears on the even frequency $f_0$ ($\times$), the odd spectral grid ($\bigcirc$) due to up-conversion of the supply tickler and the odd-even bins ($\triangleright$) belonging to the input-output response of the RF input tickler.

**EXAMPLE 3E - Input tickler influence on the HTFs estimate**

Both supply and input tickler have their own designated spectral grid and should, if behaving as proper tickler excitations, have minimal interference with each other. As an interesting additional example, the frequency-domain representation of the output spectrum, with both ticklers enabled, is shown in Figure 3.15. Unexpectedly, third-order non-linear contributions start emerging out of the noise floor at unexcited odd-even spectral bins. Such intermodulation products can only appear by combination of a sum of two even-even bins (the large sinusoid at center frequency) with an odd-even bin of the input tickler. Other combinations of bins, in particular those containing multiple tickler contributions, are assumed to be negligible due to the extremely low amplitude of the ticklers involved. Observation of such phenomena is an on-sight indicator that the tickler excitation might have to be reduced even more.
3.4. Conclusions

This chapter demonstrated a preliminary attempt at extraction of an approximate estimate of the baseband dynamics. Such a procedure makes the assumption that the RF input-output dynamics can be adequately modelled with a single quasi-static constant which is most definitely a dangerous assumption that needs to be checked. Nevertheless, the extracted complex up-conversion product already comes extremely close to the actual baseband dynamics without the usage of an overly complex model.

Future chapters aim to take the theoretical foundations discussed here and augment the preliminary extraction procedure to something that is usable in a practical environment. This includes taking into account that the device exhibits complex input-output RF dynamics as well as the fact that the baseband dynamics start having a dependency on the input power metric when actual modulated input excitations are involved.
4. Extraction of the Baseband Dynamics

Previous chapters introduced several important theoretical concepts and made a preliminary attempt at extraction of the baseband dynamics with the help of the Harmonic Transfer Functions (HTFs). This framework is now further build upon and expanded to solve some of the problems encountered, such as the fact that the previously defined model structure is unable to include the complex input-output dynamics of the Dynamic Power Supply (DPS) transmitter across any realistic High-Frequency (HF) bandwidth. Therefore, an initial part is solely dedicated to enhancing the quasi-static model structure to a more advanced Linear Parameter-Varying (LPV) framework (Section 4.1) and the requisite estimation procedure that comes with it (Section 4.2).

Eventually this leads to an identification method that is initially demonstrated on several Advanced Design System (ADS) simulation examples (Section 4.3) and ultimately, in the next chapter, on actual measurement data to verify the method’s validity in a practical environment.
4. Extraction of the Baseband Dynamics

Figure 4.1: Proposed LPV system in which an encapsulated Linear Time-Varying (LTV) system is connected to the outside scheduling parameter $v_{\text{supply}}$ using a series of Linear Time-Invariant (LTI) filters.

### 4.1. Linear Parameter-Varying (LPV) Models

This subsection aims to propose a model structure that combines the capabilities of the quasi-static model structure and the extended Best Linear Approximation (BLA), both defined separately in the previous chapter. In this work, the model structure as seen in Figure 4.1 is used in an attempt to get a more accurate model of the device’s baseband dynamics. This particular approach takes the estimated model parameters of the extended BLA and assumes that, due to the modulated nature of the supply voltage $v_{\text{supply}}$, they become periodically time-varying. As the actual supply voltage is a known and observable parameter this internal LTV model can be augmented with a series of LTI model blocks that relate the reference supply voltage with the actual time-varying parametric coefficients. This leads to an LPV system that internally consists of an LTV system as annotated in Figure 4.1 and discussed in [Goos 15b]. An important side-note is that the LTV models under discussion here are to be considered only linear in the ‘best’ linearized sense as they still make extensive use of the underlying extended BLA framework introduced in previous chapters.

The encapsulated LTV model can be written as a rational form, similar as in Equation 3.6, but this time in the form of an Ordinary Differential Equation (ODE) with time-varying model parameters, as such:

$$\sum_{i=0}^{n_c} c_i(t) \frac{d^i b_{\text{out}}(t)}{dt^i} = \sum_{i=0}^{n_d} d_i(t) \frac{d^i a_{\text{in}}(t)}{dt^i}$$
The resulting time-dependent model parameters are assumed to consist of a summation of a time-invariant and a parameter-varying part that is linearly dependent on the scheduling parameter \( v_{\text{supply}} \), like so:

\[
d_i(t) = D_i + (h_i * v_{\text{supply}})(t)
\]

\[
c_i(t) = C_i + (g_i * v_{\text{supply}})(t)
\]

where \( C_i \) and \( D_i \) are complex DC-constants that do not have any dependency on time whatsoever and \( * \) being the convolution operator as introduced in Equation 3.3. Due to the periodic nature of the modulated supply voltage, these time-dependent model parameters can be uniquely transformed to the frequency-domain by using the Discrete Fourier Transform (DFT) which results in following frequency-domain equations for \( d_i(t) \):

\[
D_i(0) = D_i + H_i(0)V_{\text{supply}}(0)
\]

\[
D_i(k) = H_i(j\omega_k)V_{\text{supply}}(k) \quad \forall k \neq 0 \land k \in \text{excitedbins}
\]

Identical equations can be found for the nominator’s coefficients \( c_i(t) \) by substituting \( H_i \) and \( D_i \) for \( G_i \) and \( C_i \) respectively. As in previous chapter, \( H_i \) is a non-conjugate complex parameter (\( H_i(j\omega_k) \neq H_i(-j\omega_k) \)) due to the fact that the baseband equivalent representation of the output wave sees different dynamics at the lower and upper sideband. Furthermore, each of the estimated complex model coefficients is assumed to consist of a product of two other LTI models, as such:

\[
H_i(j\omega_k) = BB(j\omega_k)HTF_{\pm H_i}(j\omega_k)
\]

(4.1)

where \( BB(j\omega_k) \) is a real (= complex conjugate) filter component that can be directly related to its namesake in Section 3.3 as it a measure for the device’s baseband dynamics. On the other hand, \( HTF_{\pm H_i} \) is a complex combination of both relevant first-order HTFs, in the following way:

\[
HTF_{\pm H_i}(j\omega_k) = HTF_{+ H_i}(j\omega_k) + jHTF_{- H_i}(j\omega_k)
\]

(4.2)

and serves as a means of up-converting the baseband signal \( V_{\text{supply}}^{\text{int}} \) to the in-phase and quadrature part of the Radio Frequency (RF) signal respectively. Rearrangement of the complex time-varying coefficients of the LPV system results in the model structure shown in Figure 4.2. The more complicated LPV model structure can thus be split in a parallel series of simple blocks reminiscent of the one shown in Figure 3.11.
4. Extraction of the Baseband Dynamics

Figure 4.2.: Rearrangement of the LPV system, depicted in Figure 4.1, reveals that this complicated model structure actually just consists of a parallel series of HTF model structures as discussed previously in Figure 3.11.

As before all supply-output branches go through the exact same baseband dynamics $BB$ and thus possess a common pole/zero set in their parametric representation. Not all of the estimated model parameters $d_i(t)$ and $c_i(t)$ are assumed to be time-varying as their upconversion to RF frequencies is entirely dependent on the magnitude of the HTFs. The actual presence of time-variation in each of the parametric model coefficients can be verified by plotting the respective time-domain transformed coefficient in function of the scheduling parameter $v_{\text{supply}}(t)$, as is discussed in [Goos 15a].

Estimation of each of these coefficients, as given in [Goos 15b], is based on a straight-forward linear Least-Squares (LS) extraction procedure. The estimated frequency-domain coefficients should then, preferably, be refined using a more extensive iterative Weighted Non-linear Least-Squares (WNLS) that can be proven to be consistent and close to asymptotically efficient [Loua 14]. In this work, a slightly different approach is taken in the form of a combination of the previously estimated time-invariant Bootstrapped Total Least-Squares (BTLS), which models the model coefficients, together with a modified linear LS estimation procedure (that captures the parameter-varying behaviour). Important to stress is that such a procedure is only possible because the tickler excitation, added to the supply voltage, does not influence the device’s Large-Signal Operating Point (LSOP).
4.1. LPV Models

Using multiple estimation procedures, one being responsible for capturing the non-linear behaviour and model order selection while the other aims to model the parameter-varying behaviour, has ample advantages:

- A Best Linear Time-Invariant (BLTI) approximation of the DPS transmitter is obtained using a procedure that is both efficient and consistent [Peum 19b], while the dynamics are captured in a broad bandwidth, due to the presence of the input tickler, minimizing the chance of erroneous model order selection.

- Extraction of the device’s parameter-varying behaviour, containing the baseband dynamics $BB$, is made possible using a linear LS procedure as the model’s non-linear behaviour has already been captured and linearized by the BTLS estimation procedure. This significantly reduces the probability that the cost function gets stuck in a local minimum as the obtained minimum for the BTLS, compensated during each iteration, is the global minimum.

As a result, the parameter-varying Jacobian matrix, for LS estimation purposes, is to be constructed using the exact same model order as the one found to be necessary for the parametric extended BLA. Since the linearized DC-values of these model coefficients have already been estimated, there’s no need to include them anywhere in the LS estimation. As a result, a multitude of rows and columns can be scrapped from the Jacobian matrix, simplifying the estimation procedure. Construction of the necessary Jacobian matrix is achieved using following pattern:

$$J_X = \sum_{i=0}^{n_c} [\text{Circulant}(\vec{X} \odot (\vec{\Omega}_N)^i)\text{diag}(V_{\text{supply}})] \otimes (\vec{C}_i)^T$$

in which:

- The vector $\vec{X}$ is to be replaced by either $A_{in}$ or $B_{out}$ while $n_c$ is respectively equal to either $n_d$ or $n_e$.

- $\vec{C}_i$ is the connection column vector, not to be confused with the $i^{th}$ denominator model coefficient $C_i(k)$, and has identical properties as discussed in previous chapter (Equation 3.7).

- The frequency vector $\vec{\Omega}_N$ and the Kronecker product operator $\otimes$ are defined identical as in Section 3.1.

- The operator $\text{Circulant}(\vec{Y})$ creates a circulant matrix, a special kind of Toeplitz matrix, starting from a column vector $\vec{Y}$ and is constructed as follows:
4. Extraction of the Baseband Dynamics

\[
\text{Circulant}(\vec{Y}) = \begin{bmatrix}
Y(T_{\text{supply}}^\text{tickler}) & \cdots & Y(0) & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots & \ddots & Y(0) \\
Y(N+1) & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & Y(N+1) & \cdots & Y(N+1 - T_{\text{supply}}^\text{tickler})
\end{bmatrix}
\]

where the original column vector \( \vec{Y} \) is, in this particular case, either \( B_{\text{out}} \odot (\vec{\Omega}_N)^i \) or a zero-padded, in a symmetrical fashion, version of \( A_{\text{in}} \odot (\vec{\Omega}_N)^i \).

- \( \text{diag}(\vec{Y}) \) transforms a column vector of dimension \((2T_{\text{supply}}^\text{tickler} + 1) \times 1\) to a matrix of dimension \((2T_{\text{supply}}^\text{tickler} + 1) \times (2T_{\text{supply}}^\text{tickler} + 1)\) by placing the vector along the matrix’s diagonal, as such:

\[
\text{diag}(\vec{Y}) = \begin{bmatrix}
Y(-T_{\text{supply}}^\text{tickler}) & 0 & \cdots & 0 \\
0 & Y(-T_{\text{supply}}^\text{tickler} + 1) & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & Y(T_{\text{supply}}^\text{tickler})
\end{bmatrix}
\]

Due to the nature of the parameter-variation the output spectrum only has spectral content on a discrete set of \( N + 1 \) bins and, as a consequence, the \( \text{Circulant}(\vec{Y}) \) matrix operator transforms a column vector with dimensions \((N + 1) \times 1\) to a matrix of dimensions \((N + 1) \times (2T_{\text{supply}}^\text{tickler} + 1)\).

The exact amount and values of these \( N + 1 \) output excitation bins is readily found by taking the Kronecker sum of the input excitation bins with the spectral bins of the supply tickler excitation and removing any possible duplicates.

Important to note is that the input tickler is still present in both the input and output spectra, but is ignored for the purpose of this section as the relevant generated spectral content, up-conversion of the supply tickler around the input tickler bins, is a negligible contribution that lies far below the noise floor.

As before, both Jacobian matrices \( J_X \) are stacked next to each other to give an overdetermined set of equations:

\[
\begin{bmatrix}
J_{B_{\text{out}}} & -J_{A_{\text{in}}}
\end{bmatrix}
\begin{bmatrix}
\vec{\Theta}_d \\
\vec{\Theta}_c
\end{bmatrix} = J\vec{\Theta} = \vec{\Delta} \cong \vec{0}
\]
4.1. LPV Models

giving a jacobian matrix $\mathbf{J}$ with dimensions $(N + 1) \times [(2T_{\text{tickler supply}} + 1)(n_c + n_d + 2)]$ and $\vec{\Theta}$ is a column vector that contains the stacked frequency-domain converted model parameters in following form:

$$
\vec{\Theta}_d = \begin{bmatrix}
\vec{\Theta}_{d,0} \\
\vec{\Theta}_{d,1} \\
\vdots \\
\vec{\Theta}_{d,n_d}
\end{bmatrix}, \quad \vec{\Theta}_{d,i} = \begin{bmatrix}
H_i(-j\omega_{\text{res}}(4T_{\text{tickler supply}}M_{\text{supply}} + 1)) \\
\vdots \\
D_i + H_i(0) \\
\vdots \\
H_i(j2\pi(4T_{\text{tickler supply}}M_{\text{supply}} + 1)f_{\text{res}}/4)
\end{bmatrix}
$$

with the excited bins of the supply tickler being placed solely on an odd ($= 2k + 1$) grid as previously defined in Equation 3.10. Again, the column vector containing the denominator model coefficients $\vec{\Theta}_d$ is found by substitution of $D_i$ and $H_i$ by $C_i$ and $G_i$ respectively.

The previously estimated BTLS model coefficients, belonging to a BLTI approximation of the parameter-varying model, are then removed from both the column vector $\vec{\Theta}$ and the jacobian matrix. Such an operation is fairly elementary for the stacked column vector $\vec{\Theta}$ as it can easily be accomplished by removing the corresponding row belonging to each of the sums of DC-value contributions $(D_i + H_i(0))$. This is possible as the spectral contribution belonging to the DC-value of the tickler’s upconversion to the inband BLA was already included in the BLTI approximation. This means that the previously established DC-contributions $H_i(0)$ and $G_i(0)$ can be considered to be zero without any loss of generality, and were only added for completeness’ sake of the discussion at hand.

Removal of the relevant columns from the Jacobian matrix $\mathbf{J}$ is achieved by multiplying each of the Jacobian columns, corresponding to a DC-value row in the parameter column vector $\vec{\Theta}$, with their respective BTLS coefficient. Subsequently they are scrapped from the matrix and added (or substracted) to the column that corresponds to the row containing the $D_i + H_i(0)$ element in the parameter vector $\vec{\Theta}_{d,i}$, resulting in column vectors of following form:

$$
\mathbf{J} \begin{bmatrix} \vdots \ T_{\text{tickler supply}} + 1 \end{bmatrix} = \mathbf{B}_{\text{out}} \odot \sum_{i=0}^{n_c} \vec{C}_i(\vec{\Omega}_N(k))^i - \mathbf{A}_{\text{in}} \odot \sum_{i=0}^{n_d} \vec{D}_i(\vec{\Omega}_N(k))^i
$$

$$
\mathbf{J} \begin{bmatrix} \vdots \ T_{\text{tickler supply}} + 1 + p(2T_{\text{tickler supply}} + 1) \end{bmatrix} = \vec{0} \quad \forall n_c + n_d + 2 > p \geq 1
$$

where $\vec{0}$ is the all-zero column vector and $\mathbf{J} \begin{bmatrix} \vdots \ i \end{bmatrix}$ denotes the $i^{th}$ column of the Jacobian matrix.
All zero columns can be safely discarded while the column containing the weighted sum of BTLS model coefficients serves to capture the entire underlying LTI model component of the LPV model structure.

Additionally, the rows corresponding to the input in-band excitation grid are removed from the Jacobian since they have already been exploited in the BTLS estimation procedure and are of no further use for estimation purposes. The resulting curbed Jacobian matrix $J_{\text{curbed}}$ ends up with dimensions $(N - T) \times \left[ 2T_{\text{supply}}^\text{tickler}(n_c + n_d + 2) + 1 \right]$ where $T + 1$ is the number of excitation bins of the input Random Phase Multisine (RPM) $A_{in}^0$ as defined in Equation 3.5. As in previous chapter, multiple Jacobian matrices belonging to different phase realizations have to be stacked on top of each other to obtain a matrix of full rank. After transformation to a real matrix, using the isomorphic equivalence [Pint 12] as in Equation 3.8, a linear LS minimization scheme is used to obtain an estimate, as such:

$$\arg\min_{\Theta_{\text{real}}} \sum_{k=1}^{N-T} |\epsilon(k)|^2$$

subject to $(J_{\text{real}}\Theta_{\text{real}})(k) = \epsilon(k)$ and $\Theta_{\text{real}}(T_{\text{supply}}^\text{tickler} + 1) = 1$

A subsequent step then calculates the standard deviations $\sigma_{H_i}$ and $\sigma_{G_i}$ of the LS estimate by means of exploiting the residuals as discussed in [Pint 11].

After estimation and for model validation purposes the BTLS model coefficients have to be added back to the DC-values of their respective parameter-varying model coefficients. In hindsight, some theoretical issues remain unanswered on the nature of the standard deviation $\sigma_{H_i}$ (and $\sigma_{G_i}$) as the variance of the DC coefficients was assumed to be zero for purposes of the linear LS procedure which is most definitely not the case. As such, the variance obtained from the BTLS estimation procedure propagates and directly influences the standard deviation of the parameter-varying model parameters in some way or form. The mathematical derivation of this propagation mechanism is assumed to be of lesser importance and outside the scope of this work.

While the proposed model structure, in Figure 4.2, is theoretically sound, there are some additional side-constraints that have to be taken into account before attempting to apply the model on a simulation example. One such constraint is the issue of model ambiguity; i.e. the estimated model coefficients are not uniquely defined in the current framework. To address this model ambiguity, the parameter-dependency of the denominator model coefficients is removed entirely, thus the model vectors $\Theta_{c,i}$ only contain a single element, as such:

$$\Theta_{c,i} = [c_i]$$
Figure 4.3: Simplification of the LPV model structure of Figure 4.2 by artificially removing all parameter-dependency of the denominator model coefficients $c_i$ for reasons of ambiguity and prior knowledge on the underlying physical device.

Moreover, the first denominator coefficient $C_0$ is normalized to 1 as to give an explicitly defined input-output structure. As discussed before, the rows containing the DC-components are removed from the Jacobian matrix and thus the linear LS minimization scheme does in fact not contain any coefficients of the denominator. The resulting simplified model structure is seen in Figure 4.3 and can be identified using the same minimization scheme as before except for the fact that the denominator coefficients now remain strictly time-invariant.

The necessity of eliminating the time-variance of numerator coefficients is, in this case, a direct result of the excitation strategy. Due to the small size of the supply tickler $V_{\text{tickler supply}}$, the rational form of the LPV model breaks down and can be sufficiently approximated by its Taylor series equivalent. As such estimation of parameter-varying behaviour in both the numerator and denominator results in solutions that are not uniquely defined. Increasing the tickler’s amplitude would get rid of this ambiguity, but also goes directly against the constraints set upon the proposed excitation strategy i.e. having an additional excitation signal that does not influence the LSOP.
4. Extraction of the Baseband Dynamics

| \( A_{in}^{0} \) | \( f_{0} \) | 2140MHz |
| \( f_{res} \) | 100 KHz |
| \( R \) | 16 |
| \( T + 1 \) | 101 |
| Bandwidth | 10MHz |
| \( P_{average \, dBm} \) | 14dBm |
| \( P_{peak \, dBm} \) | 22dBm |

| \( A_{in}^{\text{tickler}} \) | \( M_{in} \) | 40 |
| \( T_{in}^{\text{tickler}} \) | 40 |
| Bandwidth | 160MHz |
| \( \beta \) | \(-90\)dB |

| \( V_{\text{tickler \, supply}} \) | \( M_{\text{supply}} \) | \{100, 10\} |
| \( T_{\text{supply}}^{\text{tickler}} \) | \{4, 40\} |
| Bandwidth | 40MHz |
| \( \gamma \) | \(-50\)dB |

Table 4.1.: Properties of the chosen excitation signals for validation purposes of the proposed LPV model structure.

**EXAMPLE 4A - Class-B Power Amplifier (PA) with modulated input signal**

In an attempt to validate the proposed parameter-varying framework, a classic Class-B PA is now to be excited by a modulated RPM with a bandwidth of 10MHz. As in previous example, the supply voltage consists of a large fixed DC-value (= 28V) around which a supply tickler is placed. To extract the extended parametric BLA, an input tickler is added to the input wave as defined in Section 3.3. Both excitations at the device’s input, in-band and out-of-band RPMs as well as the properties of the supply tickler are given in Table 4.1. As is evident, the number of spectral tones in the input RPM \( A_{in}^{0} \) has been vastly increased in comparison to previous simulation examples, which only employed 21 bins, to a much larger total of 101 spectral tones.
4.1. LPV Models

The resulting input RPM more closely resembles the stochastic properties of an actual modern communication signal, as discussed in Chapter 3.

In an initial simulation, the amount of supply tickler tones is chosen extremely low to allow for visual examination of the up-converted contributions of the supply tickler to the output spectrum. Afterwards a simulation with a finer excitation grid is used to obtain a much smoother estimate of the device’s baseband dynamics that can then be employed for pre-compensation purposes. Both experiments are repeated 16 times, each time changing the phase realizations of all relevant multisine signals. The excitation bandwidth of the supply tickler was capped at 40MHz as frequencies exceeding this threshold are far above the cut-off frequency of the Legendre-Papoulis filter and thus contain no useful information on the baseband dynamics.

As discussed in Chapter 2, the simulation framework employs ADS’ Ptolemy co-simulation and uses Envelope simulation to simulate the classic Class-B PA. To greatly decrease the simulation time only the output filter of the Buck converter is implemented and simulated as part of the Envelope simulation. Furthermore, the Envelope simulator has a sampling frequency $f_{\text{sampling}}^{\text{ENV}}$ of 500MHz and uses the Backward Euler integration method to save simulation time. Evidently, such a choice of integration method leads to issues related to artificial damping and, to a lesser extent, frequency warping, but these are considered unimportant for the current goal: finding out if the proposed framework has any practical merit at all.

The resulting simulated spectra for the sparsely chosen tickler grid are shown in Figure 4.4 and clearly show the parameter-varying nature of the device. Due to up-conversion of the supply tickler, the odd bins at the output are populated by mixing products, one for each of the supply tickler bins. These spectra are, in a first step, used to estimate the extended input-output BLA and its constituent parametric order, and secondly to verify the newly proposed LPV model structure by identifying the parameter-varying model coefficients. As the Large-Signal Operating Point (LSOP) is identical to the one used in previous simulation examples, the parametric model order is already known ($n_c = 2$ and $n_d = 2$). As a result, the necessary LTI model coefficients are readily estimated using the BTLS identification procedure.

In a next step, the newly introduced LS minimization scheme is employed to extract the parameter-varying components. As the grid is quite sparse, the three LPV model coefficients to be estimated, namely $H_0$, $H_1$ and $H_2$, are equally sparse and thus difficult to use in direct comparison with the actual baseband dynamics $BB$. As a consequence, they will not be shown here in favor of the more denser grid used in the second experiment.
Figure 4.4: Frequency-domain representation of a single phase realization of the supply voltage $V_{\text{supply}}$ (DC-value + tickler), the input wave $A_{\text{in}}$ and the output wave $B_{\text{out}}$ of the simulated Class-B PA excited with a supply tickler with a sparse excitation grid. Spectral content at the output appears on the even-even bins (■), the odd-even bins (■) and at odd bins (■) due to up-conversion of the supply tickler.
Figure 4.5.: Model output residuals (■) when employing a sparse grid for the supply tickler, obtained by substracting the simulated output (■) from the estimation result obtained with the help of the proposed model structure (■) for one randomly chosen phase realization.

The model fit is easily validated by calculation of the output residuals, as depicted in Figure 4.5. The even-even bins, except for those belonging to the inband excitation, are not shown in this figure as the proposed LPV model structure is unable to model any non-linear behaviour. As can be seen, the output residuals, obtained by substraction of the expected output spectrum with the actually simulated spectra, are on the same level as the output noise which indicates a good model fit.

In a second simulation, the number of spectral bins in the supply tickler is increased to an adequate number as to allow easy comparison with the baseband dynamics $BB$. After some trial-and-error, a total of about 40 bins was presumed sufficient for purposes of this simulation example. Similarly as for the sparse grid, the input, output and scheduling spectra are depicted in Figure 4.6 and show that the odd bins of the output spectra are again solely populated by up-converted mixing products, except in a much less structured fashion as before. It can be noted that the low-pass characteristics of the Legendre-Papoulis output filter can still be clearly observed in the overall shape of the mixing products.
Figure 4.6.: Frequency-domain representation of a single phase realization of the supply voltage \( V_{\text{supply}} \) (DC-value + tickler), the input wave \( A_{\text{in}} \) and the output wave \( B_{\text{out}} \) of the simulated Class-B PA excited with a supply tickler with a fine excitation grid. As before, spectral content at the output appears on the even-even bins (■), the odd-even bins (■) and at odd spectral bins (■) due to up-conversion of the supply tickler around each of the main excitation spectral lines.
4.1. LPV Models

Figure 4.7.: Estimation results for the dynamic model coefficients $H_0$, $H_1$ and $H_2$ alongside their respective standard deviation depicted using $\times$ of the exact same color.

Estimation results of the LPV model coefficients, together with their respective standard deviations, are depicted in Figure 4.7. Although higher order model coefficients are quite noisy and behave erratically in function of the frequency, all model coefficients share similar shape. This is a clear indication that they indeed might share a common pole/zero set as assumed by the main model. For extraction purposes of the baseband dynamics, $H_0$ is chosen as the prime extraction candidate as this model coefficient behaves quite smoothly in frequency and has the lowest standard deviation.
4. Extraction of the Baseband Dynamics

Figure 4.8.: Model output residuals ( ■ ) when employing a sparse grid for the supply tickler, obtained by subtracting the simulated output ( ■ ) from the expected result predicted with the help of the proposed model structure ( ■ ) for one randomly chosen phase realization.

The output model residuals can again be calculated for model validation purposes and the results are shown in Figure 4.8 alongside the expected and simulated output spectra. Unfortunately, the model residuals do not graze the noise floor as before, but are still sufficiently low to serve as an indication of validity. A possible reason for the lesser model validity, as indicated by the model residuals, could be that there’s a violation of the tickler condition due to the high number of tones which introduces high-order mixing products that also fall on odd bins and cannot be modelled by the proposed framework. Decreasing the level of the tickler tones even more was however not an option as the parameter-varying skirts would simply start disappearing under the noise floor in that case.

As this is a simulation example, access to the internal supply voltage terminal $V_{\text{int}}^{\text{supply}}$ is actually possible without any additional complications. This means that the baseband dynamics can be calculated in this particular case by a simple frequency-domain division of the internal voltage with the reference supply voltage. The resulting dynamic Frequency Response Function (FRF) $BB$ is shown in Figure 4.9 alongside the in-phase and quadrature parts of $H_0$. Even without explicitly removing any of the influence of the up-conversion models $HTF_{+H_0}$ and $HTF_{-H_0}$, the estimated model coefficient already comes quite close to the wanted baseband dynamics $BB$. 

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Figure 4.9.: Comparison of the in-phase (■) and quadrature (■) component of the 0th model coefficient $H_0$ with the baseband dynamics obtained by using the internal voltage node $V_{\text{int}}^{\text{supply}}$ (■).
In conclusion, the simulation examples discussed in this section indicate that the proposed LPV model structure, together with the excitation strategy, can be used to obtain a rough estimate for the unknown supply dynamics without that much of a problem.

In a next step this rough estimate can then be used to extract a more accurate estimate. According to the main model assumption, the resulting estimated LPV model coefficients are products of the wanted baseband dynamics and a complex up-conversion FRF and be seperated in some way. This can potentially be done by using a parametrization approach, in which a Z-domain model of both the in-phase and quadrature components of the estimated model coefficient is estimated. Poles and zeros common to both models most likely belong to the underlying baseband dynamics, while poles and zeros unique to each model are part of the complex up-conversion FRF instead. Evidently such an arrangement does not take into account the possibilities of:

- Pole/zero cancellations between the baseband dynamics and one or both of the up-conversion FRF.
- Common pole/zero configurations between the in-phase and quadrature parts of the up-conversion FRF that are wrongly identified as belonging to the baseband dynamics.

Due to time constraints this additional step in the identification framework was not studied in full detail.

As a result, from this point onwards, following assumptions are made:

- The actual influence of the complex up-conversion FRF is considered negligible and the in-phase component is used as an approximate version of the baseband dynamics.
- Due to the low-pass nature of the baseband dynamics, the inverted pre-compensation LTI filter increases in function of the frequency. As a result, the filter needs to be constrained in amplitude as to avoid amplifying noise in the baseband dynamics’s stopband. Suppression of this unwanted behaviour is done with the help of a fourth-order Linkwitz–Riley crossover filter structure [Link 78].
4.2. Identification Procedure

The complete identification procedure for estimating the device’s baseband dynamics consists of following steps:

1. Perform multiple measurements using the proposed excitation strategy and random phase realizations of the involved multisines at each consecutive measurement as proposed in Section 3.2.
2. Select the parametric model order of the (non-parametric) extended BLA.
3. Extract the parameter-varying model coefficients using a linear LS procedure, as in 4.1.
4. Estimation of the baseband dynamics of the LPV model structure using the methods as discussed in Section 4.1.
5. Choose a pre-compensation model using the obtained dynamics and validate its effectiveness by acquiring additional measurement data on the now pre-compensated device.
6. If significant changes in the baseband dynamics are detected due to changes in the Large-Signal Operating Point (LSOP), go back to step 1 and repeat.

Changing the transmitter’s LSOP by pre-compensating the supply voltage was impossible until now due to the fact that the spectrum was void of any content except for the supply tickler which cannot influence the operating point as per its definition. In the case of an actual DPS transmitter, the pre-compensation unit not only modifies the supply tickler, but also changes the shaped input envelope $v_{\text{supply}}(t)$ which might have ample impact on the LSOP. The resulting identification procedure becomes an iterative procedure that, in the best case scenario, converges to a product of consequent pre-compensation filters that, together with the current baseband behaviour, equates to an all-pass filter, as so:

$$1 = BB_{\text{iter}}^{n_{\text{iter}}} (j\omega_k) \prod_{p=0}^{n_{\text{iter}}} \left( PC_{\text{iter}=p} (j\omega_k) \right)^{-1} \quad \forall k \in \text{excited bins}$$

where $n_{\text{iter}}$ is the number of iterations necessary to reach convergence, $BB_{\text{iter}}^{n_{\text{iter}}}$ denotes the current baseband dynamics defined by the LSOP set by the product of pre-compensation filters $(PC_{\text{iter}=p})^{-1}$ that were estimated and implemented in the previous steps.
4. Extraction of the Baseband Dynamics

| \( A_{in}^0 \) | \( f_0 \) | 2140 MHz |
| \( f_{res} \) | 200 KHz |
| \( R \) | 16 |
| \( T + 1 \) | 51 |
| Bandwidth | 10 MHz |
| \( P_{\text{average}, dBm} \) | 14 dBm |
| \( P_{\text{peak}, dBm} \) | 22 dBm |

| \( A_{in}^{\text{tickler}} \) | \( M_{\text{in}} \) | 40 |
| \( T_{\text{tickler}} \) | 20 |
| Bandwidth | 160 MHz |
| \( \beta \) | −90 dB |

| \( V_{\text{tickler supply}} \) | \( M_{\text{supply}} \) | 20 |
| \( T_{\text{tickler supply}} \) | 10 |
| Bandwidth | 40 MHz |
| \( \gamma \) | −50 dB |

Table 4.2.: Properties of the chosen excitation signals for identification of the hybrid combination DPS transmitter’s baseband dynamics.

4.3. Application to DPS transmitters

Until now the supply voltage solely consisted of a DC-value and a supply tickler. From this section onwards the actual shaped envelope is applied at the supply node as is required for settings the correct LSOP for the DPS transmitter. As a final demonstration for this chapter’s purposes, the proposed identification and modelling techniques are now to be applied on the hybrid combination DPS transmitter as designed in Chapter 2.
4.3. Application to DPS transmitters

Figure 4.10.: Frequency-domain representation of a single phase realization of the supply voltage $V_{\text{supply}}$ (shaped envelope + tickler), the input wave $A_{\text{in}}$ and the output wave $B_{\text{out}}$ of the simulated hybrid combination DPS transmitter. As before, spectral content at the output appears on the even-even bins (■), the odd-even bins (■) and at the odd spectral bins due to up-conversion of the supply tickler around each of the main excitation spectral lines (■).

**EXAMPLE 4C - Modulated Baseband Dynamics of a hybrid DPS transmitter with 17.5 dB constant gain**

To demonstrate the proposed identification framework on a more realistic example, the hybrid combination DPS is simulated and pre-compensated. The chosen signal properties are shown in Table 4.2 and, similarly as before, the sampling frequency $f_{\text{ENV sampling}}$ has a value of 500MHz.

The resulting simulated scheduling, input and output spectra are shown in Figure 4.10 for a single realization. As can be seen in this figure, the supply voltage now contains the shaped envelope alongside the DC-value and supply tickler. The output spectrum retains the previously established advantages even with addition of the shaped envelope signal; i.e. spectral contributions of relevant input signals can be separated on sight.
Figure 4.11.: Comparison of both the wanted ideal shaped envelope (■) and internal baseband voltage (□) at the transistor’s extrinsic drain in function of the reference HF input signal envelope.

Figure 4.11 shows a comparison of the wanted shaped envelope, to be applied at the transistor’s drain, and the baseband that actually arrives at the device’s drain, both in function of the envelope of the reference HF input signal. As is evident from the displayed curves, there is considerable dynamic deviation from the wanted Static Non-linearity (SNL) behavior that is to be removed with the help of an active pre-compensation mechanism.
4.3. Application to DPS transmitters

Figure 4.12.: Estimation results for the dynamic model coefficients $H_0$, $H_1$, and $H_2$ alongside their respective standard deviation depicted using $\times$ of the exact same color.

The estimated LPV model coefficients are shown in Figure 4.12 and exhibit a similar outline as in Figure 4.7. The standard deviation has the same order of magnitude, which indicates that the identification method has no estimation issues when using a modulated supply voltage instead of a fixed DC-value.

As discussed in Section 4.1, the inphase component of the 0th order model coefficient is now chosen as the candidate for pre-compensation. As a result this model coefficient has to be inverted and parametrized in the Z-domain. This parametric model is estimated using the same model selection procedure as for the extended BLA (but modified to the Z-domain) and the resulting best model order is found to be of order $[4, 4]$. The model was then inverted and stabilized by reflecting the single unstable high-frequency pole back into the unit circle. Any deviation from the wanted phase characteristic caused by this stabilization is found to be out of the frequency band of interest. The resulting stabilized Z-domain filter characteristic is shown in Figure 4.13 alongside the magnitude and phase of the original non-inverted dynamics (both normalized to 80MHz for estimation purposes).
4. Extraction of the Baseband Dynamics

Figure 4.13.: Comparison of the initial Z-domain non-inverted parameteric model (■), the inverted stabilized model (■) and the model with constrained amplitude characteristic (■).

One additional issue is the amplitude response of the stabilized model which gets increasingly higher due to the lowpass behaviour of the original baseband dynamics and has to be bounded as to stop the simulation noise from becoming amplified. In this work, this done by exploiting a fourth-order Linkwitz-Riley cross-over architecture [Link 78]. This architecture consists of two parallel branches of two cascaded 2\textsuperscript{nd}-order lowpass Butterworth filters, in one branch, and two cascaded 2\textsuperscript{nd}-order highpass Butterworth filters with the same cutoff frequency in the other branch. The designed stabilized discrete filter is placed behind the cascaded lowpass filters and the same cutoff frequency is chosen for all Butterworth filters in such a way that the total model gain never exceeds 20dB (chosen arbitrarily).

As seen in Figure 4.13, the resulting constrained Z-domain filter introduces considerable linear phase delay that also has to be added to the RF input path. In this case the additional input delay was found to be around 39 samples (= 78ns) by linear least-squares estimation of the resulting linear phase deviation between the wanted phase characteristic and the phase characteristic as displayed by the constrained filter.
4.3. Application to DPS transmitters

For validation purposes, the DPS transmitter is now to be simulated again with a different random set of realizations. All of the settings for the excitation signals and the simulators remains the same, except for the addition of the discrete filter in the supply path and the input delay in the RF input path.

The resulting LPV model coefficients are shown in Figure 4.14 and all have much flatter appearance in both amplitude and phase. Estimated model coefficients have similar levels of standard deviation as before.
4. Extraction of the Baseband Dynamics

Figure 4.15. Comparison of the pre-compensated baseband dynamics (■) with the inphase (■) and quadrature (■) components of the pre-compensated 0th-order model coefficient.

Figure 4.15 compares the pre-compensated baseband dynamics, obtained by dividing the spectra of the internal Low-Frequency (LF) drain voltage with the reference supply tickler at the excited tickler bins, with the inphase and quadrature components of $H_0$. As can be seen in this figure, there’s some amplitude deviation evident at higher baseband frequencies. This is either caused by the fact that the internal LF drain voltage is extrinsically measured and thus does not contain some part of the intrinsic parasitics or caused by the fact that the complex model coefficient contains part of the up-conversion dynamics which were not removed from the estimate. Part of this up-conversion characteristic is also still present in the phase of both the in-phase and quadrature component.
4.3. Application to DPS transmitters

Figure 4.16.: Comparison of both the wanted ideal shaped envelope (■) and internal baseband voltage (■), in the case of pre-compensation, at the transistor’s extrinsic drain in function of the reference HF input signal envelope.

After pre-compensation, the internal baseband voltage is again compared with the wanted ideal shaped envelope in Figure 4.16. The internal baseband voltage has to be delayed with the added input delay (39 samples) to properly time align this voltage waveform with the reference envelope signal. The resulting internal baseband voltage lies almost perfectly on the wanted characteristic, indicating that the pre-compensation is succesful. Some small deviation is present at lower envelope values, caused by the envelope dropping to zero at certain time-instances. The internal baseband voltage is unable to track those drops due to limitations in the slew rate (caused by parasitics).
4.4. Conclusions

This chapter combined the non-linear and time-varying identification frameworks introduced in the previous chapter. In effect, the, previously assumed, static model coefficients of the extended BLA were made parameter-dependent on the supply tickler. In theory, this would allow us to extract the baseband dynamics of the transmitter without having to measure the baseband voltage at the device’s intrinsic drain terminal. To validate this assumption, the baseband dynamics of a Class-B PA with fixed supply were estimated and the results were found to be promising. Afterwards, a step-by-step identification procedure was proposed to estimate and pre-compensate the baseband dynamics of any DPS transmitter. The proposed method was tested on the hybrid combination DPS transmitter and enabled successful pre-compensation and removal of the supply dynamics from the device. Some amplitude deviation was evident in the pre-compensated baseband dynamics, but this could be explained either by missing parasitics in the baseband drain voltage used for validation or the inability of the current method to split the complex up-conversion (baseband to RF) FRF from the actual baseband dynamics. Nevertheless, comparing the wanted ideal shaped envelope with the voltage at the internal node shows significant improvement before and after the pre-compensation.
5. Measurements & Synchronization Issues

While previous chapters demonstrated the proposed methods and algorithms on simulations using behavioral models, actual measurements are preferred to validate the entire framework. To showcase the versality of the method, two different measurement setups, each with a different set of instruments, are employed and discussed in detail. The Device-Under-Test (DUT) is a Dynamic Power Supply (DPS) transmitter of the hybrid variety, enforcing a constant joint transfer function on the device. As the transmitter is of a different technology, namely Indium gallium phosphide (InGaP), than the device used in previous chapter, which was a Gallium Nitride (GaN) transistor, the eventual conclusions will be slightly different. However, it is shown that the identification methods are technology agnostic in nature.

As is to be expected, some additional tinkering is required to deal with issues solely encountered with measurements and this is the subject of this chapter’s first section. These issues relate to both the identification procedure and the actual measurements as a whole, where acquiring synchronized measurement data is a more dire problem than was initially expected. Subsequently, some effort is spent on explaining the root causes of these problems and the solutions recommended and employed in this work.
5. Measurements & Synchronization Issues

Figure 5.1.: In practice, the Linear Parameter-Varying (LPV) model structure is encapsulated by linear delay terms, dominated by the transmission lines that connect the DUT with the measurement instruments.

5.1. Molding Theory to Practice

The procedures discussed in previous chapters work on simulations and now have to be adapted to deal with issues specific to measurements. In effect, the two main issues that are encountered both relate to timing and delay between the measurement instruments, among themselves, and the DUT. A first section deals with the presence of time delays, which are systematic delays introduced by the presence of the transmission lines’s excessive length that are identical for every successive measurement. Afterwards, a second section aims to tackle issues that are of a more stochastic nature, as the instruments have to be adequately synchronized among themselves such that identical excitation signals result in identical measurement results regardless of the unobservable internal states of the measurement instruments.

Handling time delay

In simulations discussed previously, the time delay is negligible and thus doesn’t have to be addressed by the model structure. Measurements have no such benefit and the time delay needs to be correctly taken into account otherwise the parameter-varying framework would fail. To accommodate the presence of delays in the model structure, the assumption is made that all delays can be grouped together and separated from the underlying LPV model as in Figure 5.1. The rational form introduced in Equation 3.6 is now augmented with an additional delay component, as such:
5.1. Molding Theory to Practice

\begin{equation}
BLA(\mathbf{\Omega}_N(k), \mathbf{\Theta}) = \sum_{i=0}^{n_c} c_i(\mathbf{\Omega}_N(k))^i e^{-j(\tau_{in} + \tau_{out})} \mathbf{\Omega}_N(k)
\end{equation}

(5.1)

with \(\tau_{in}\) and \(\tau_{out}\) being the respective input and output delay that encapsulate the LPV model structure as shown in Figure 5.1.

The assumption that these delays are separable from the LPV model is giving weight by the fact that the larger part of these delays can be blamed on the physical length of the connections of the DUT with the measurement instruments. In most cases these are long coaxial cables that have negligible dynamics by design. As a result, the actual Radio Frequency (RF) input and output, as well as the supply, that excite the encapsulated LPV model structure are linear phase shifted versions of the measured ones, which can be written as:

\begin{align*}
A_{\text{LPV}}^{\text{in}}(k) &= e^{-j\tau_{in}} \mathbf{\Omega}_N(k) A_{\text{in}}(k) \\
B_{\text{LPV}}^{\text{out}}(k) &= e^{j\tau_{out}} \mathbf{\Omega}_N(k) B_{\text{out}}(k)
\end{align*}

These equations can be easily simplified by adding an additional delay block \((e^{j\tau_{in}} \mathbf{\Omega}_N(k))\) to all of the measured data, resulting in the following modified equations:

\begin{align*}
A_{\text{LPV}}^{\text{in}}(k) &= A_{\text{in}}(k) \\
B_{\text{LPV}}^{\text{out}}(k) &= e^{j(\tau_{in} + \tau_{out})} \mathbf{\Omega}_N(k) B_{\text{out}}(k)
\end{align*}

in which case only the relative input-output delay \((\tau_{in} + \tau_{out})\) needs to be known, but still requires an estimation procedure in some way or form.

The extended non-parametric Best Linear Approximation (BLA), introduced in previous chapter, has no issues with incorporating a delay term in its Frequency Response Function (FRF) due to its non-parametric nature. When transforming this non-parametric model to a parametric one, the total input-output delay has to be estimated alongside the necessary model order. In this work, this is achieved through an additional delay selection that is added on top of the more classical model order selection procedure. As discussed in previous chapter, the Akaike Information Criterion (AIC) is used to find the ideal model order, but is now faced with the issue of an unknown delay term that has to be removed from the non-parametric estimate before AIC can be succesfully applied.
5. Measurements & Synchronization Issues

As such, the unknown delay value is swept to different values and AIC is applied on the delay compensated non-parametric estimate, resulting in a sort of 3-dimensional cost function. The point at which this delay-dependent AIC is minimum, in a local sense, is selected as the most likely candidate for the rational form with delay as given in Equation 5.1 and depicted in Figure 5.2. Moreover, this procedure is facilitated by the fact that:

1. The actual model order necessary to accurately model the extended BLA will be quite low. This is mostly by design, as Power Amplifier (PA) devices are created to have smoothly-varying gain metrics in function of the frequency.

2. The linearized part of the non-linear phase of the extended BLA can be used as a starting value for the delay sweep to avoid checking for model compatibility for delay values that are excessively far from this initial value.

3. The cost function behaves smoothly in function of the delay value, avoiding situations in which the delay sweeping interval can be decreased indefinitely.

After estimating the total input-output delay, the output wave can be phase shifted to get the actual wave \( (b_{\text{LPV}}^{\text{out}}(k)) \) that is present at the output of the LPV model.

In contrast the supply voltage is, initially, modelled with the help of a non-parametric FRF and can be left in its delayed form without invalidating the LPV model structure. As with the input and output wave, an additional linear delay block is added to the measured supply voltage, as such:

\[
V_{\text{LPV}}^{\text{supply}}(k) = e^{j(\tau_{\text{in}} - \tau_{\text{supply}})\Omega(k)} V_{\text{supply}}(k)
\]

where \( (\tau_{\text{in}} - \tau_{\text{supply}}) \) is the exact linear delay that, in the pre-compensation procedure, has to be added to the supply port to synchronize both the RF and baseband envelope at the device’s intrinsic node. Thus this modified identification procedure directly gives the necessary delay between supply and input port that has to be compensated. Hence, no additional delay sweep measurements are required for time alignment between envelopes as is the case in other works [Wang 14].
5.1. Molding Theory to Practice

Synchronization issues

Actual measurements require a more cautious approach on the aspect of synchronization. In simulations, as has been the case until now, the whole issue of synchronization is taken care of by the simulator and is of minimal concern to the user. Unfortunately, in measurements there’s no such luxury and synchronization needs to be properly taken into account.

In general a distinction is made between synchronization that stems from systematic differences in clock frequency and those due to semi-random stochastic differences in phase of the different measurement instruments. Luckily, both frequency and phase synchronization, as they are called, can be solved by sharing clocks and/or sampling frequencies between instruments, in following ways:

• **Frequency synchronization**
  
  High-frequency sampling and clock frequencies are synthesized by upconverting an internal clock reference. When multiple independent measurement devices have to work together and useable measurement data is preferred, it is generally not a good idea to let every device fend for itself. Small deviations between the frequency of these devices’ internal clock references, mostly implemented in the form of crystal oscillators, lead to big deviations when these reference frequencies are upconverted to the desired frequency. An easy solution is to dedicate one clock reference as the master and to distribute it to all other measurement devices. The eventual clock or sampling frequency is then derived from the same reference clock and does not deviate in frequency between instruments.

• **Phase synchronization**
  
  While sharing a reference clock between instruments assures that all instruments have identical timebases, the phase of the eventual synthesized sampling or clock frequency is dependent on the internal states of the device. These internal states are unknown and the exact phase at each consequent measurement can change, even when exciting the measurement setup with identical excitation signals. A possible solution is to share the same sampling signal between all instruments as this assures that the phase difference is always a fixed value, regardless of the device’s unobservable state. This is not always possible as the instruments in the measurement setup might not use sampling signals with the same frequency. In that case, a different solution needs to be found to assure phase synchronization.

As is evident, phase synchronization is the more difficult problem to overcome, most definitely when both baseband and RF measurements instruments are employed, as both are measuring entirely different frequency bands.
To showcase the exact issues with phase synchronization, a simple unsynchronized measurement of the DUT is used as a demonstrational example. Frequency synchronization in this example was easily assured by sharing the 10MHz reference clock between all instruments, but no further efforts were made to actually guarantee phase synchronization.

For this particular measurement the PA device is excited with a single sinusoid at the Local Oscillator (LO) frequency, which is 836MHz in this case. On the baseband side, a supply tickler \(v_{\text{tickler}}\) is added alongside the device’s supply voltage. The exact properties of these excitation signals are of lesser interest in this demonstrational example as they are quite irrelevant to the synchronization issue at hand. Applying these excitations gives, as is expected, an amplified sinusoid at the device’s output together with an up-converted supply tickler due to the device’s inherent non-linear behaviour and the resulting measured output spectrum is shown in Figure 5.3 alongside both inputs.

Different phase realizations, a total of 16, are measured and the resulting Harmonic Transfer Functions (HTFs) are estimated individually for each. The resulting FRFs, easily derived by dividing the up-converted tickler by the reference supply tickler and the phase of the input sinusoid, are shown in Figure 5.4. As is evident in the phase, the estimated HTFs have a different, seemingly randomly varying, linear delay term. Averaging the different realization results would result in an erroneous estimate that progressively worsens for increasing (absolute) frequencies.

Interestingly, the linear delay term that is, in a semi-stochastic way, added to the phase estimate lies in the sample interval \([-0.5, 0.5]\). Meaning that this unsynchronized measurement setup only succeeds in synchronizing the instruments up to a single sample. Additionally, there are only a limited amount of internal states of the measurement instrument and thus the phase estimate’s stochastic delay term doesn’t vary continually and can only fall on a limited number of positions.

As a result, it is, in this simple example, possible to assume that one of these states gives the ‘correct’ phase estimate and to remove the additional delay term from all other estimates. This is quite easy to achieve and results in measurement data that can be averaged. However, such a thing is only possible when working with these simple excitations as removing this stochastic term is not as evident when working with modulated input and scheduling signals. Applying a time-varying supply voltage and a modulated bandpass input to a DPS transmitter in which the relative phase changes in a discrete semi-random fashion could result in changes of the Large-Signal Operating Point (LSOP) over different phase realizations, something which has to be avoided for achieving persistent consecutive measurement experiments. In conclusion, removing the stochastic delay term in this fashion is not assumed to be a valid option for extracting the device’s baseband dynamics.
Figure 5.3.: Frequency-domain representation of a single phase realization of the supply voltage \( V_{\text{supply}} \) (DC-value + tickler), the input wave \( A_{\text{in}} \) and the output wave \( B_{\text{out}} \) of the measured PA excited with a single sinusoid. Spectral content, at the output, only appears on the center frequency (×) and the odd spectral grid (○) due to frequency up-conversion of the supply tickler. The measurement noise floor is also shown and contains spectral content at all possible frequency bins.
Figure 5.4.: Estimation of the upper and lower sideband HTF, for multiple realizations of the supply tickler signal, gives different phase slopes due to non-existent phase synchronization between the baseband and RF measurement instruments. Variations in amplitude, at higher frequencies, are due to other causes that are discussed in Section 5.4.
5.2. Modified Identification Procedure

The identification procedure for estimating the baseband dynamics for DPS transmitters, as introduced in previous chapter, has to be slightly modified to accommodate for the issues discussed previously:

1. Perform multiple measurements using the proposed excitation strategy and change the phase realizations of the involved multisines at each consecutive measurement.

2. Perform a parametric model order selection, including a delay estimation, of the (non-parametric) extended BLA using the Bootstrapped Total Least-Squares (BTLS) estimation procedure [Peum 18].

3. Extract the parameter-varying model coefficients using a linear Least-Squares (LS) procedure, as in Section 4.1.

4. Estimate the baseband dynamics of the LPV model structure using the methods as discussed in Section 4.1.

5. Choice of pre-compensation model using the obtained dynamics (including delay) & validation of its effectiveness by acquiring additional measurement data on the now pre-compensated device.

6. Detect changes in the baseband dynamics caused by changes in the Large-Signal Operating Point (LSOP). If significant go back to step 1 and repeat, otherwise the identification procedure is finished.

5.3. Measurement Setup

The proposed techniques are now to be tested on a real device. For this purpose, the LM3290-91 Evaluation Board was used as the DUT for verifying the identification procedure in practice. This transmitter implements the hybrid combination technique of the so-called “constant gain transfer function” variety, as was discussed in depth in Section 1.9. In this case, a constant joint transfer function of 28.4dB is adopted. The necessary shaping function, for achieving this metric, was already extracted by Texas Instruments and is shown in Figure 5.5.

This particular transmitter is designed to accommodate an LTE signal at E-UTRA Band 5. The center frequency $f_0$ for this particular band is 836MHz while allowed channel bandwidths are 1.4, 5 and 10MHz. The bandwidth of the Random Phase Multisine (RPM) has to be chosen as to respect these frequency constraints. A depiction of the measurement board, with annotations added to position the most important device components, is shown in Figure 5.6.
5. Measurements & Synchronization Issues

Figure 5.5.: Shaping function that gives the relationship between the instantaneous input power $P_{in}$ at $RF_{in}$, and the DC supply voltage $V_{supply}$ required to obtain a constant joint transfer function $[McCu 15]$ of 28.4 dB for all relevant input powers.

Figure 5.6.: LM3290-91 Evaluation Board used a proof-of-concept for the proposed identification procedure, consisting of (A) SKY77621 Multimode Multiband Power Amplifier Module, (B) Hybrid Supply Modulator, combining a Boost converter and a linear regulator, (C) RF Input Port, (D) Differential Envelope Input Port & (E) RF output port.
5.3. Measurement Setup

Figure 5.7.: Schematic of classic parallel hybrid DPS in which a linear amplifier and a Switch-Mode Power Amplifier (SMPA) are placed in parallel to achieve the efficiency and bandwidth requirements. The linear amplifier acts as a voltage source while the DC-DC converter behaves as a current-controlled DC-current source [Wang 14].

Of particular interest is the DPS, a circuit that, in this case, combines the capabilities of both a DC-DC Boost converter (LM3290) and a linear amplification device (LM3291) to create a highly efficient broadband hybrid supply modulator, able to accurately generate the required modulated supply voltage. In this case, the outputs of both devices are connected in parallel, as is depicted in Figure 5.7, in a so-called “parallel hybrid” arrangement. The linear amplification device receives the wanted envelope signal and controls the DC-DC converter by means of a current sensor. Operation of this device can be described as follows:

1. The envelope signal, as given by $E_s$, ramps up and the linear amplifier starts to supply a current to the power amplifier terminal to meet this voltage requirement.

2. At a certain point, a current threshold is crossed that toggles the hysteresis comparator and the DC-DC converter starts to output a DC-current.

3. Current generated by the linear amplification device, which has low power efficiency, drops and only higher frequency spectral content needs to be assured by the linear amplifier.

4. The reference signal $E_s$ drops and current through the sensing resistor becomes negative. After crossing the negative current threshold, the hysteresis comparator is toggled off and DC-converter stops supplying any current.
5. Measurements & Synchronization Issues

Figure 5.8: RLC-snubber placed in-between the DPS and the PA as a compensation mechanism for the baseband dynamics as well as to provide noise shaping for the generated supply voltage [Texa 14].

Connection of the outputs of both components requires a filter structure that restricts lower and higher frequency spectral content, relative to the baseband frequency range, from leaking through the opposite element. In case of the DC-DC converter, the output filter can serve this exact purpose. For example, in Chapter 2 a fourth-order Legendre-Papoulis filter is added as part of the proposed Buck converter to remove any switching frequency harmonics as well as to remove any up-converted envelope signals, appearing around the aforementioned switching harmonics, from the DPS output. This output filter can doubly serve as a means to easily combine both linear and switched-mode amplifier in the required parallel arrangement.

Unfortunately, any mismatch between the PA’s baseband impedance $Z_{PA}$ and the DPS’s total impedance $Z_{DPS}$ results in the appearance of both unwanted amplitude ripple and non-linear phase on the baseband dynamics. Likewise, a possible frequency region exists in which the switched stage is “handing over” the amplification to the linear stage [Minn 09]. A possible way to avoid these unwanted dynamics is to add an additional passive compensation network, generally denoted as a snubber circuit, that serves as a means to mend the mismatched impedance environment, as discussed in depth in Chapter 2. In this particular case, an RLC-snubber network was already added by Texas Instruments in-between the DPS’s output and the PA’s supply port, as seen in Figure 5.8.
This DPS transmitter’s baseband dynamics are now to be measured in a synchronized fashion. Two different setups are used to validate the proposed identification method and showcase the method’s independence from the exact set of measurement devices. Initially, the way in which the synchronization problems are tackled, inherent to the measurement instruments used, is discussed.
5. Measurements & Synchronization Issues

Measurement setup - Chalmers

In this measurement setup, depicted in Figure 5.10, both the RF input and the base-band envelope are generated by the same instrument (M8190 Arbitrary Waveform Generator (AWG)), thus ensuring that both frequency and phase synchronization are guaranteed without any additional work. In contrast, the RF output is measured with a N9030 PXA. To allow for frequency synchronization between instruments, the 10MHz reference clock of the M8190 PXA is exported and connected to the N9030 PXA. No further efforts were made to enforce phase synchronization between both devices and the resulting measured output delay term varies semi-randomly depending on the internal clock states of the N9030 PXA’s sampling clock. Thus in the case of this measurement setup, the linear phase characteristics of the measured output voltage have to be normalized for each realization. The wanted extended BLA can then be obtained by averaging out the normalized spectra for each realization.

In a similar fashion the RF signal generator’s LO could not be shared with the RF signal analyzer, thus the LO’s phase is different between both instruments and needs to be removed from the measurements, since it depends on the unobservable state of the instrument’s internal oscillator. This inability to share the LO stems from the fact that the M8190 AWG does not involve any LO whatsoever and instead works with a very high sampling frequency (chosen as 4.8GHz). As a result, this measurement setup doesn’t have the ability to measure the correct absolute phase of the extended BLA’s complex coefficients.

A photograph showing all measurement instruments as well as a simplified schematic are shown in Figure 5.9 and Figure 5.10 respectively. Important devices, other than the measurement instruments, include:

- **LZY-2X+ Ultra-Linear RF Amplifier**, used for amplifying the generated RF input signal to the required power range at the DPS transmitter’s input.
- **8487A Power Sensor**, used for power calibration purposes of the generated RF input signal. The power sensor’s operational principle is based on thermal measurements and is thus suitable for accurate power calibration of modulated signals (in contrast to a power sensor that uses a diode).

Phase calibration was not done for this measurement setup and the inevitable phase error, appearing at the RF output, was considered part of the DUT.

User control of the entire measurement setup was made possible using a Matlab script, which was also responsible for processing the acquired measurement data coming from the N9030 PXA.
5.3. Measurement Setup

Figure 5.9: Photograph of the Chalmers measurement setup with relevant instruments annotated, which are: (A) M8190 AWG, (B) LZY-2X+ Ultra-Linear RF Amplifier, (C) LM3290-91 Evaluation Board, (D) N9030 PXA & (E) 8487A Power Sensor.
5. Measurements & Synchronization Issues

Figure 5.10: Simplified schematic of the Chalmers measurement setup as depicted in Figure 5.9. Frequency synchronization is assured by sharing the 10MHz among instruments, while phase synchronization is inherently assured by the fact that the input signal as well as the scheduling are generated by the exact same instrument. No phase synchronization was enforced between analyzer and generator. Dashed lines denote connections that are not measured, but are solely required for synchronization purposes. In this particular setup only the output voltage $RF_{out}$ is measured while the reference spectra of the input signals ($E_s$ & $RF_{in}$) are used for estimation purposes.
Measurement setup - National Instruments

Generation and acquisition of the RF in- and output is accomplished with the aid of a PXIe-5646R Vector Signal Transceiver (VST) and employs the maximal device bandwidth of 200MHz. Synchronization of the instruments LO, as required for correct estimation of the phase of the extended BLA’s complex model coefficients, is ensured by using a master-slave arrangement between the generator and analyzer components of the device. A photograph of the complete measurement set-up, with its relevant instruments annotated, is shown in Figure 5.11. A simplified schematic, showing the connections required for synchronization, is depicted in Figure 5.12.

To generate the required modulated supply voltage the differential channels of a PXIe-5451 AWG are exploited. Properly achieving phase synchronization between the RF signals and their low-frequency counterpart, as required for the method at hand, is made possible by routing the internal FPGA clock of the VST, which operates at a frequency of 250MHz, to the output reference of the VST. Supplying this signal to the external clock input of the AWG, and using this external excitation as the instrument’s sample clock, results in a device that is phase synchronized with the VST.

The LM3290-91 Evaluation Board has an additional connection that can be used to measure the internal node voltage, the voltage in-between the DPS’s output and the PA’s node. This node can be used to validate the estimated baseband dynamics. The voltage at this node is measured with a PXIe-5162 Oscilloscope that also needs to be properly synchronized with the other instruments. The sampling clock of this device cannot be replaced by the FPGA’s internal clock and thus will be measured instead. Phase synchronization can then be enforced in post-processing by taking the measured phase of this internal clock and compensating the measured internal voltage by this phase component. Identically as for the previous measurement setup, a 8487A Power Sensor is used to calibrate the RF generator’s modulated power.

Lastly, all instruments are frequency synchronized by using the 10MHz backplane reference clock of the PXI chassis.

Generation of the relevant input signals and control of the measurement instruments is done by using a custom LabVIEW VI. Lastly, a Matlab script is employed for data-processing and for creation of the relevant pre-compensation models.
5. Measurements & Synchronization Issues

Figure 5.11: Photograph of the National Instruments measurement setup with relevant instruments annotated, which are: (A) PXIe-5646R Vector Signal Transceiver, (B) PXIe-5451 AWG, (C) LM3290-91 Evaluation Board, (D) PXIe-5162 Oscilloscope, and (E) 8487A Power Sensor.
5.3. Measurement Setup

Figure 5.12.: Simplified schematic of the National Instruments measurement setup as depicted in Figure 5.11. Frequency synchronization is assured by sharing the 10MHz reference clock of the backplane among instruments, while phase synchronization is assured between all instruments by either sharing or measuring the 250MHz FPGA sampling clock of the RF transceiver. Dashed lines denote connections that are not measured, but are solely required for synchronization purposes. For estimation purposes only the output spectra \( RF_{out} \) is measured while for validation purposes, the internal voltage node \( V_{int} \) is acquired using a scope.
5. Measurements & Synchronization Issues

\[
\begin{array}{ll}
A_0^0 & \\
\hline
f_0 & 836\text{MHz} \\
f_{\text{res}} & 50\text{KHz} \\
T+1 & \{1, 29, 101, 201\} \\
\text{Bandwidth} & \{0, 1.4, 5, 10\} \text{MHz} \\
P_{\text{average}, \text{dBm}} & -5\text{dBm} \\
\end{array}
\]

\[
\begin{array}{ll}
A_{\text{tickler}} & \\
\hline
M_{\text{in}} & 360 \\
\gamma_{\text{tickler}} & 20 \\
\text{Bandwidth} & 120\text{MHz} \\
\beta & -90\text{dB} \\
\end{array}
\]

\[
\begin{array}{ll}
V_{\text{tickler}} & \\
\hline
M_{\text{supply}} & 120 \\
\gamma_{\text{tickler}} & 39 \\
\text{Bandwidth} & 60\text{MHz} \\
\gamma & -50\text{dB} \\
\end{array}
\]

Table 5.1.: Chosen properties of the input multisine \(A_0^0\), the input tickler \(A_{\text{in}}^{\text{tickler}}\) and the supply tickler \(V_{\text{tickler}}^{\text{supply}}\) used for measurements of the \textit{LM3290-91 Evaluation Board}. Both \(f_0\) and \(f_{\text{res}}\) are identical for all excitation signals, while the bandwidth (and the number of tones as result) is modified over different experiments.

### 5.4. Measurement Results

For demonstration purposes the baseband dynamics of the previously established DUT are estimated using the modified identification procedure. Both measurement setups aim to extract the exact same dynamics and differ only in the nature of the exploited measurement equipment. However, this chapter uses the measurement results achieved using the second setup (National Instruments) as a reference and discusses the differences with the first setup whenever appropriate.
5.4. Measurement Results

**Experiment Design**

All relevant properties of the chosen input and supply excitations are listed in Table 5.1. A total of 16 phase realizations (modifying both the random phase of the ticklers as the input RPM simultaneously) is measured for different bandwidths of the RF input signal. To get rid of the system’s transient response a total of 40 periods is acquired for each realization and the first half of these periods is discarded. The remaining 20 periods, only containing the steady-state response as verified using the substraction method of consecutive periods in the time-domain [Pint 12], are averaged in time to suppress the measurement noise. The bandwidth of the supply tickler is chosen as 60MHz as to capture as much dynamic behaviour as possible and get an idea of the general trend of the dynamics that are to be compensated. All the while, the bandwidth of the input tickler is set at 120MHz to cover a similar range as the up-converted supply tickler and, as such, to get an accurate estimate of the out-of-band BLA in this region. Amplitudes of both tickler excitations were tuned in such a way that they had marginal influence on the device’s LSOP, but were still sufficiently above the noise floor.

The resulting spectra for the measurements with RF bandwidths of 0 and 5MHz are shown in Figures 5.13 and 5.14 respectively. As is evident in both output spectra, the resulting baseband dynamics of this measurement example don’t decrease as much in amplitude for higher frequencies. Hence, they have a much broader baseband bandwidth than the Buck converter used in previous chapter for the simulation example. This is quite logical as the DUT makes use of a linear regulator which has a much wider frequency range as a Buck converter at the cost of a lower efficiency. Unfortunately the exact efficiency lost could not be compared since the modulated supply current could not be measured.

To ease of the burden of estimating the linear delay component, the input tickler grid was modified to increase the amount of tones positioned around the center frequency \( f_0 \). Having an increasing amount of tones close together greatly simplifies the estimation of the initial value of the linear delay term. Estimation of a linear delay is only known on an integer multiple of \( 2\pi \). Having a smaller spectral distance (in frequency) between the excitation tones decreases the degree of ambiguity. This is especially useful when working with only a single sinusoid as is the case for the 0MHz bandwidth measurement.

As is verified by these figures, the excitation signals elicit the intended response and the spectral content can be easily separated between different contributions, even on sight.
Figure 5.13.: Resulting spectra for the measurement example when excited by a single sinusoid. Shown are the supply voltage $V_{\text{supply}}$ (DC-value + tickler), the input wave $A_{\text{in}}$ (single tone + tickler) and the output wave $B_{\text{out}}$ of the measured PA. Spectral content, at the output, only appears on the 836MHz center frequency ($\times$), the odd-even spectral grid due to amplification of the input tickler ($\triangleright$) and the odd spectral grid ($\circ$) caused by frequency up-conversion of the supply tickler. The measurement noise floor is also shown and contains spectral content at all possible frequency bins.
5.4. Measurement Results

Figure 5.14.: Resulting spectra for the measurement example when excited by bandpass multisine with a bandwidth of 5MHz. Shown are the supply voltage $V_{\text{supply}}$ (DC-value + tickler), the input wave $A_{\text{in}}$ (single tone + tickler) and the output wave $B_{\text{out}}$ of the measured PA. Spectral content, at the output appears on the even-even grid due to both linear and non-linear amplification of the input RPM ($\times$), the odd-even spectral grid due to amplification of the input tickler (▷) and the odd spectral grid (⊙) caused by frequency up-conversion of the odd supply tickler. Spectral bins containing the measurement noise floor are similarly shown at all possible bins.
5. Measurements & Synchronization Issues

![Image of Figure 5.15: Estimated extended non-parametric BLAs for all measured input bandwidths, namely 0MHz (■), 1.4MHz (■), 5MHz (■) and 10MHz (■). A significant part of the linear delay term (= 1035.94ns) was removed for showing the underlying non-linear phase metric.]

Measurement Results

Estimation of the extended BLA

In the first step of the extraction procedure the non-parametric extended BLA has to be estimated. The estimation results, for 16 measured phase realizations, are shown in Figure 5.15 alongside the standard deviations $\sigma_{\text{BLA}}$ for each of the chosen excitation bandwidths. Initial LS estimation of the first-order phase component revealed that the total linear delay component is about $-1037.46 \pm 0.07\,\text{ns}$ and the averaged value of this delay was removed in Figure 5.15 such that the non-linear part of the delay could be adequately inspected on sight.

As discussed before, this total delay component contains the pure delay, caused by both the length of the connections, the programmatic delays in the LabVIEW script and the linear part of the phase present in the actual system dynamics belonging to the poles/zeros of the underlying rational form. However, the extremely high value of this delay component is an unexpected result.
When solely looking at the length of all the transmission lines used in the RF path, the expected pure delay would be about $\sim 9.14\,\text{ns}$ ($= 6\,\text{feet at 836MHz}$). This is further affirmed by the Chalmers measurement setup which indeed has a delay component close to this value. The cause of this excessive delay is found to be due to the existence of a fixed time mismatch between the reference input signal $A_{\text{in}}$, used in the estimation procedure, and the actual excitation signal generated by the VST. While this digital delay makes it impossible to derive the absolute input-output delay from the measurements, this term is not required for the matter at hand as the synchronization procedure only requires the relative input-supply delay, which can still be derived without having access to the absolute delay.

As is required for correct estimation of the non-parametric extended BLA, the inclusion of the input tickler is not allowed to significantly modify the device’s LSOP in any way or form. This condition is verified and validated using the methods discussed in Chapter 3 for all measured input bandwidths.

As seen in Figure 5.15, the extended BLA’s standard deviation is highest for the measurement results obtained using the 0MHz bandwidth. This is quite a strange observation as this particular measurement should, from a logical standpoint, possess the highest spectral purity of all since no non-linear spectral regrowth is present in this case. The exact same phenomena is also observable in the amplitude estimate of the HTFs used for demonstration purposes of the phase synchronization issue as previously depicted in Figure 5.4. Adding phase synchronization to the measurement setup, by definition, only removes the semi-random stochastic contribution on the phase estimate and shouldn’t be able to modify the amplitude in any way or form. Depicted in Figure 5.16 are the resulting HTFs estimated separately for each phase realization and compared between the NI and Chalmers setup, with the phase synchronization enabled. Results as obtained by the Chalmers setup have the exact same uncertainty across all frequencies except for the bins right next to the DC-component. This can be easily explained by the lack of proper LO synchronization. Albeit having the exact same form, the estimates given by the NI setup deviate significantly at higher frequencies and in a more noticeable fashion at the lower side-band.

The actual cause of these stochastic deviations is unfortunately unknown, but is found to be an unique issue to the NI setup. As this problem appears in both the input-output RF path, as observed in the extended BLA, as well as in the supply-output path it cannot be directly blamed on synchronization problems that are specific to the supply baseband path. Possible candidates are the stability of the LO or even the sampling frequency clock of the VST. However, the issue is not present when working with excitations that possess a more realistic signal bandwidth and was, as a result, not investigated further.
5. Measurements & Synchronization Issues

Figure 5.16: Comparison between the estimated upper and lower sideband HTFs obtained by using the Chalmers and NI setup for multiple realizations of the supply tickler signal. Uncertainty of the amplitude and phase estimates of the NI setup greatly increases when going to higher frequencies, even when the setup is properly synchronized in phase. As before, a large portion of the linear delay component is removed to be able to showcase the underlying non-linear phase characteristics, −477 ns in the case of the NI setup and +15 ns for the Chalmers setup.

The large portion of the linear delay component is removed to be able to showcase the underlying non-linear phase characteristics, −477 ns in the case of the NI setup and +15 ns for the Chalmers setup.

![Graphs showing frequency and phase comparison between Chalmers and NI setups.](image-url)
Nonwithstanding this issue, all measurements result in non-parametric BLAs of similar dynamics which seems to support the conclusion that the choice of bandwidth, in the chosen range, has only minimal influence on this device's LSOP.

**Parametrization of the non-parametric estimate**

The next step concerns the parametrization of the non-parametric estimate using the BTLS procedure, as discussed in Chapter 3. As all non-parametric extended BLAs have the same form, it is evident that their respective parametric model orders most likely coincide. Additionally, special care is taken to assure that the chosen model order results in a stable and minimum-phase model.

As discussed before, both the linear delay component and the underlying dynamics have to be properly separated from each other to keep the required parametric model order under control. Evidently the parametric model order will be quite low as the device's input-output RF dynamics are quite calm in the excited band. The modified AIC model selection procedure, proposed previously, gives a linear delay term of $1035.94\text{ns}$ and a parametric model order of $n_c = 1$ and $n_d = 2$ in which both $n_c$ and $n_d$ are defined as in Equation 5.1. As hypothesized before and now properly verified, all experiments, even though having different bandwidths, result in parametric models of the same order.

Both the non-parametric extended BLA and the estimated parametric model are shown in Figure 5.17 for the $10\text{MHz}$ bandwidth experiment. Also depicted are the model residuals and the non-parametric standard deviation which are of comparable amplitude, giving an indication that all dynamic behaviour is indeed properly captured by the parametric model. The linear delay term, common to all measurements and independent from the baseband dynamics, was removed from the figure as to be able to show the underlying non-linear phase dynamics.

**Extraction of the baseband dynamics**

The exact same procedure as in previous chapter is now exploited to find an estimate for the parameter-varying behaviour of the zeros of the parametric model while the poles are assumed time-invariant. The resulting estimates, again for the $10\text{MHz}$ bandwidth experiment, are shown in Figure 5.18 together with their respective standard deviations. The model coefficients clearly exhibit parameter-varying behaviour, although the dynamics of the 1st model coefficient are quite flat in frequency. Contrary to what is expected, the estimated dynamics do not conform to a low-pass filter characteristic even when increasing the supply tickler’s bandwidth to the maximum possible bandwidth of $100\text{MHz}$ (as constrained by the $200\text{MHz}$ sampling clock) which was done in a separate experiment.
5. Measurements & Synchronization Issues

Figure 5.17: Comparison between the non-parametric BLA (■) and the parametric model of the 10MHz bandwidth experiment estimated using the BTLS procedure (■), both down-converted and normalized in frequency. The residuals of the parametric model as well as the standard deviation of the non-parametric estimate are shown as well and depicted using × markers of their respective colors.
Figure 5.18.: Estimated parameter-varying model coefficients $H_0$ (■) and $H_1$ (■) shown together with their respective standard deviations depicted using × markers of the same color. A significant part of the linear delay term (around $-558\,\text{ns}$) was removed in both estimates to be able to showcase the underlying non-linear phase dynamics.

Since the assumption still holds that the static pre-compensation unit already takes care of removing any discrepancy in the static supply voltage component (= DC-value), the complex DC-value of the estimates holds no further information and is removed as a result. The resulting DC-deprived dynamics are further normalized by taking the average value between the two spectral bins with lowest frequencies ($-f_{\text{res}}/4$ and $+f_{\text{res}}/4$) and dividing all spectral bins by this factor. As discussed in Section 3.3, the complex model coefficient can be split in both an in-phase and a quadrature part, as depicted in Figure 5.19. The in-phase component belonging to the $0^{\text{th}}$ model coefficient $H_0$ can be proposed as a first rudimentary candidate for the actual unknown baseband dynamics $BB$ of the measured DPS transmitter. The in-phase component doesn’t contain any wild behaviour, such as resonances, nor does it display any significant roll-off in amplitude, meaning that the potential increase in both efficiency and linearity that can be achieved by pre-compensation, in this case, mainly derives from the linear delay component.
5. Measurements & Synchronization Issues

Figure 5.19: Comparison between the complex baseband dynamics of the 0th model coefficient $H_0$ (■) and its in-phase component (■) as a first rudimentary candidate for the actual baseband dynamics. A significant part of the linear delay (around $-558\text{ns}$) was removed to reveal the underlying non-linear phase.

In an attempt to validate the proposed framework, the derived in-phase component is directly inverted and used as a pre-compensation model without any further parametrization steps. Although it should be pointed out that this is only possible due to the absence of any explicit resonance behaviour as otherwise there could be a fairly large discrepancy between the non-parametric resonance frequency and the one found by employing the parametric identification procedure. Thus, the excited bins of the in-phase component are inverted while bins that fall in-between the tickler grid are derived from the known estimation set using piecewise cubic interpolation. Outside the tickler’s excited band, the non-parametric pre-compensation unit is set to unity as to avoid influencing the modulated supply voltage at these frequencies. Due to the large tickler bandwidth, in comparison to the envelope bandwidth, the spectral energy at these frequencies is also close to the noise level, as seen in Figure 5.14, and pre-compensation at these bins is of lesser importance.
5.4. Measurement Results

Figure 5.20.: Successful pre-compensation results in an in-phase component with characteristics close to all-pass behavior. Shown are the estimated complex coefficient $H_0$ alongside its in-phase component. In this case, only a small part of the linear delay (0.15 ns) has to be removed, since the principal part is taken care of by the pre-compensation.

Baseband dynamics (and non-parametric pre-compensation models as a consequence) extracted for the experiments that used different excitation bandwidths give largely similar results and are not plotted in favor of the more noisier FRFs belonging to the 10 MHz estimate.

In hindsight, normalization of the FRFs by using the lowest positive and negative spectral bins might not have been the best choice as higher uncertainties are present at these bins, most likely due to phase noise derived from the measurement setup’s LO. A better way to tackle this problem would be to leave the complex gain as such and only normalize the baseband dynamics after parametrization, as was done in previous chapter. Nevertheless, the proposed non-parametric pre-compensation model is assumed sufficient for purposes of this chapter.

As a result, the proposed pre-compensation model is implemented as part of the LabVIEW VI, as schematically depicted in Figure 2.9, and subsequent measurements are aimed at validating the pre-compensated device.
Validation Results of the Pre-Compensated Device

In an effort to check the veracity of the extracted baseband dynamics, three different validation procedures are presented independently from each other.

Re-identification of the baseband dynamics

After successful pre-compensation, the in-phase component of the 0th model coefficient $H_0$ should resemble an all-pass behaviour since the reference supply voltage now arrives without any dynamic modification at the device’s intrinsic node. As such, additional measurements were acquired using the same experiment design as given in Table 5.1 and the entire LPV estimation procedure was repeated on the new dataset. Estimated model orders are found to be the same as before pre-compensation and the resulting 0th model coefficient $H_0$ alongside its in-phase component are depicted in Figure 5.20. As seen in this figure the in-phase estimate now quite closely resembles all-pass behaviour as a significant part of baseband dynamics has been successfully removed.

Some part of the baseband dynamics remains in the form of a linear delay component with a value of 0.15 ns (corresponding to a sample delay of 0.04 Sa), but this amount was assumed low enough to conclude that the pre-compensation procedure was successful. In case this delay would still be considered too high, repeating the identification procedure and concatenating both of the estimated baseband dynamics would most likely improve the results.

Method performance versus delay compensation

As mentioned before, the state-of-the-art solely attempts to pre-compensate the linear delay term by tuning the relative delay between both baseband and RF branches [Wang 14]. As such, an effort is made to compare the performance of the state-of-the-art with the proposed pre-compensation procedure by calculating the Normalized Mean Square Error (NMSE) for the pre-compensated device as well as for an entire range of linear delay values without any dynamic pre-compensation model. Delay tuning was done in discrete steps equal to an integer multiple of a single sample (=4 ns), mainly for convenience’s sake as applying subsample delays requires a more intricate implementation. A multitude of experiments, all using the exact same set of 16 multisine realizations to allow for a fair comparison, were repeated with different delay values as to find the value that minimized the NMSE. Both tickler contributions were removed from the input excitations as the baseband dynamics didn’t need to be estimated during validation.
Figure 5.21.: Additional delay is added to the supply path and swept in steps of 1 samples (4ns) to find an optimal value of the NMSE (■). Both the original uncompensated value (■) and the value obtained with the proposed pre-compensation method (■) are singular fixed values that are repeated for ease of comparison.

Shown in Figure 5.21 are the calculated NMSE for each of the different chosen delay values as well the original error value without any compensation and the one found for the proposed pre-compensation framework. As can be seen in this figure, the minimized values for the delay sweep procedure and the proposed procedure pretty much coincide. One could make the claim that the minimum is slightly lower for the proposed framework (as seen in Table 5.2), but this lower value could most likely be reached by the tuning procedure when switching to subsample delay steps. This is more or less expected as the dynamic behaviour of the baseband is quite calm and doesn’t exhibit any resonant behaviour that could seriously degrade the device’s linearity metric.

The exact same procedure was repeated for the other excitation bandwidths and the resulting minima are listed in Table 5.2. As is evident from these values, similar conclusions as for the 10MHz bandwidth can be derived for the other excitation bandwidths.
5. Measurements & Synchronization Issues

<table>
<thead>
<tr>
<th>BW (MHz)</th>
<th>original</th>
<th>sweep min.</th>
<th>pre-comp.</th>
</tr>
</thead>
<tbody>
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<td>0.6529</td>
<td>0.6505</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>10</td>
<td>3.8562</td>
<td>0.8333</td>
<td>0.8092</td>
</tr>
</tbody>
</table>

Table 5.2.: Comparison between the original non-compensated NMSE, the minimum value for the delay sweep procedure and the errors resulting from the proposed pre-compensation procedure for different excitation bandwidths.

**Comparison with the internal node’s measured voltage**

The estimated baseband dynamics can also be directly compared with the measured voltage at the internal node $V_{PA}$ which is positioned in-between the DPS’s output and the input to the PA’s supply connection as defined in Figure 5.8. It is important to state that the voltage at this internal node could be moderately different from the actual voltage that arrives at the intrinsic drain and, moreover, cannot give any information on the required relative delay between both the RF and the baseband path ($\tau_{\text{in}} - \tau_{\text{supply}}$). Nevertheless, comparison between the estimated baseband dynamics and the voltage at this node should at least give an approximative indication on the veracity of the estimation result, excluding the relative linear delay term.

In contrast to the other acquired measurement data, the internal voltage is measured with an oscilloscope which, in this particular setup, cannot be synchronized to the other instruments. Therefore the 250MHz FPGA sampling clock of the VST is measured on the second channel of the scope. The post-processing synchronization procedure then entails taking the relative phase between the phase of this spectral tone and the 1.25GHz sampling clock of the scope itself. This relative phase is then converted to a linear delay term and this delay is removed from the measured internal supply voltage. Extra care is taken to deal with any phase ambiguity as this linear delay term is only defined on an integer number of the FPGA sampling clock period (= 4ns). The necessary physical connections to accomplish this feat are shown in Figure 5.12 for the National Instruments measurement setup. As for all other instruments, frequency synchronisation of the oscilloscope is accomplished by using the 10MHz reference clock of the backplane. In the case of the Chalmers measurement setup this internal voltage node was not measured in any way or form so no comparison between setups is possible on this aspect.
5.4. Measurement Results

Figure 5.22.: Comparison between the in-phase component of the 0th model coefficient $H_0$ (■) and the baseband dynamics found at the internal node (■). A significant part of the linear delay (again around $-558\,\text{ns}$) was removed from the in-phase part of the model coefficient to reveal the underlying non-linear phase.
The resulting synchronized internal voltage is divided, in the frequency domain, by the input and the averaged result, again for 16 multisine realizations, and is shown in Figure 5.22 alongside the previously estimated baseband dynamics. As seen in this figure, both amplitude and phase are quite close to one another giving a clear indication that the proposed procedure gives a correct estimation result. The result found using the proposed identification framework is a bit noisier which can be explained by the fact that there are, unavoidably, some stochastic contributions of the output spectrum’s spectral regrowth that fall on the odd bins. Evidently the linear delay term is somewhat different, but the underlying non-linear phase component is very much the same.

As can be seen this figure, the in-phase component’s amplitude level is slightly higher at higher frequencies. This is an unexpected result as one would assume that the parasitic elements, present along the supply path, behave as low-pass elements since they are capacitive in nature. Thus draining even more energy from the supply signal and dragging the amplitude even more downwards at those high frequencies. A possible explanation could be that the dynamics at higher frequencies, in this case, actually increase due to a very wide resonance peak or another possible explanation could be that an estimation bias might be present at higher frequencies.
5.5. Conclusions

This chapter discussed the application of the proposed baseband dynamics extraction procedure on an actual physical device. This did not go without a hitch as severe synchronization problems were encountered that first had to be properly addressed. In the end, two different measurement setups were successfully used to acquire both estimation and validation data in a synchronized fashion.

Afterwards, the actual identification procedure could be employed, notwithstanding some modifications on the subject of linear delay term estimation that had to be included in the parametrization of the extended BLA. This also showcases a first potential weakness of the method as it is quite difficult to discern which part of the linear delay component is caused by an actual transmission line delay and which part is due to the pole/zero behaviour contained in a rational form. Additionally, this is in the assumption that splitting the delays in this manner is a valid assumption. Nevertheless, the procedure is found to deliver a model for the baseband dynamics that performs quite well when put to the test at the hand of several validation methods.

Unfortunately, the DUT did not exhibit any of the more extreme dynamic behaviour that the proposed identification procedure attempts to reconcile, such as resonant behaviour of the supply path, which is a legitimate concern in modern DPS designs [Pela 15, Hass 12]. In hindsight, it might have been a better indicator of success if some parasitic components would have been added to secure additional resonant behaviour. A preliminary attempt at adding these parasitics, in the form of a parallel capacitor, was made, but did not provide any significant dynamic deviation, most likely due to the extremely low output impedance of the DPS that behaves close to ideal at the frequencies of interest.
6. Conclusions and Future Work

This last chapter is dedicated to summarizing the main contributions, as discussed in this work, to the modelling and design techniques of Dynamic Power Supply (DPS) transmitters. It elaborates more specifically on the identification procedure for the extraction of the baseband dynamics and its application on actual measurements. Afterwards, some possible future augmentations to the identification framework are proposed.
6. Conclusions and Future Work

Conclusions

The main contributions, as considered in this work, can be summarized as such:

• **Proposal of a theoretical framework based on both the Best Linear Approximation (BLA) and the Linear Parameter-Varying (LPV) modelling techniques**
  
  Due to the operational principles of the DPS transmitter, modelling techniques from both the time-invariant and time-varying domains had to be combined. This endeavour was successfully achieved in Chapters 3 and 4 by making the parameter coefficients of the extended BLA, linearly dependent on an external and independent parameter, namely the supply voltage.

• **Application of the framework for extraction and pre-compensation of a DPS transmitter’s baseband dynamics**
  
  Additional considerations had to be made as to make the theoretical framework applicable to the domain of DPS transmitters. Such considerations range from the use of a special multisine grid to the introduction of tickler signals. Similarly, the nature of the extracted parameter-varying model coefficients and its role in the transmitter as an active pre-compensation mechanism for the baseband dynamics was discussed in Chapter 4.

• **Implementation & validation of the proposed framework in both simulation and measurements**
  
  The proposed identification method was successfully tested on both a simulation testbed and measurements on an actual DPS transmitter.

  – **Simulation**
    
    An advanced simulation testbed was designed in Advanced Design System (ADS) as to combine both DPS and Power Amplifier (PA) in a Ptolemy co-simulation which joins a transient simulation and an Radio Frequency (RF) envelope simulation using an overall dataflow simulation. Afterwards, this testbed was successfully used to validate the framework on a simulation example in Chapter 4.

  – **Measurements**
    
    Considerable effort was first spent on making sure that the measurements were phase-synchronized across all units of the instrument. Such synchronization is more difficult to achieve than the more common frequency synchronization and required additional attention. Further extensions to the model were made to accommodate the excessive linear delay at the input, output and scheduling interfaces.

    Actual measurement results successfully validated the method’s versatility on multiple measurement setups in Chapter 5.
Proposals for future work

The proposed identification framework, and the subject matter in itself, poses several issues that went unsolved during this work:

- Increasing the number of design parameters for the actual DPS as to solve some of the issues with the aforementioned Ptolemy co-simulation testbed (i.e. the Buck converter has issue tracking small variations) and to make the value of the output filter’s design impedance a less critical choice. A possible way to achieve this would be the introduction of a different (or more enhanced) DPS architecture.

- Separation of the baseband dynamics from the complex up-conversion Frequency Response Function (FRF), either by pole/zero splitting as proposed in Section 4.1 or in another way.

- Application of the identification framework on the Class-E PA and the specific modifications that would be required to make the framework applicable to the Polar modulation (PM) variant of DPS transmitters.

Alongside these unsolved issues, some potentially important extensions to the framework are also proposed as potential candidates for future work.

Model extension to Multiple-Input Multiple-Output (MIMO)

In the entirety of this work, all DPS transmitters were assumed to behave in a Single-Input Single-Output (SISO) fashion. This assumption is quite acceptable when the device’s input and output ports are perfectly matched for all relevant input powers and frequencies. However, the very nature of power amplifier design, as discussed in Chapter 1, already violates this basic assumption by principle of loadline matching alone. Depending on the severity of this mismatch, present at both the device’s input and the output, there might be a considerable dissonance between results acquired with the SISO framework and the actual behavioural principles of the device. Potentially, these disagreements could be solved by moving to a framework that takes these mismatches into account.

In a future step, the previously established LPV framework could be augmented to a more device-friendly MIMO environment. This would mean upgrading the previously extracted extended SISO BLA to an extended MIMO BLA, as seen in Figure 6.1, as well as introducing a persistent MIMO excitation matrix. For demonstration’s sake, an augmented MIMO version of the Class-B PA, as designed in Chapter 2, is excited with a two-dimensional excitation matrix and the resulting input and output spectra for both experiments is shown in Figure 6.2.
The required excitation matrix $\mathbf{A}$ is a so-called Full Orthogonal Multisine (FOMS) matrix which aims to excite the system in a persistent and well-conditioned manner [Pint 12]:

$$
\mathbf{A} = \begin{bmatrix}
A_{\text{in}}^{\text{exp}=1} & A_{\text{in}}^{\text{exp}=2} \\
A_{\text{out}}^{\text{exp}=1} & A_{\text{out}}^{\text{exp}=2}
\end{bmatrix} = \begin{bmatrix}
A_k e^{j\alpha_k} & A_k e^{j\alpha_k} \\
B_k e^{j\beta_k} & B_k e^{j\beta_k}
\end{bmatrix} \begin{bmatrix}
e^{j\phi_k} & e^{j\phi_k}\\1 & -1
\end{bmatrix}
$$

As in the case of SISO, the even-even and odd-even bins are populated by the main input multisine and input tickler respectively at both input and output spectra. Non-linear skirts appear around the output reflected wave ($B_{\text{in}}$) and around the input reflected wave ($B_{\text{in}}$) at the transmitter’s source terminal. Interestingly, due to input matching, one would expect the input reflected wave to be perfectly matched at the center frequency. As can be seen in the spectra of both the input tickler as the main input multisine this matching minimum lies at a slightly lower frequency, most likely due to a large-signal loadline matching trade-off.

**Model Extension towards both supply & bias modulation**

Until now only the supply voltage was considered as a time-varying scheduling parameter. In the case of a Class-A Envelope Tracking (ET) amplifier, both the supply and the bias voltage become time-varying as discussed in Section 1.7. In that case, the parametric model coefficients of the extended BLA have to be extended with an additional input variable, as such:

$$
\sum_{i=0}^{n_c} c_i(v_{\text{supply}}(t), v_{\text{bias}}(t)) b^{(i)}_{\text{out}} = \sum_{i=0}^{n_d} d_i(v_{\text{supply}}(t), v_{\text{bias}}(t)) a^{(i)}_{\text{in}}
$$
(a) Input excitation spectra for extraction of the MIMO extended BLA. The main input multisine (■) is solely applied at the transmitter’s source ($A_{\text{in}}$) and sets the device’s Large-Signal Operating Point (LSOP), while the input tickler (■) is applied at both the device’s source and load ($A_{\text{in}}$ and $A_{\text{out}}$).

(b) Output spectra at both the transmitter’s source ($B_{\text{in}}$) and load ($B_{\text{out}}$). As in the case of SISO, non-linear intermodulation skirts of the main input excitation appear at even-even bins (■), while the response to the High-Frequency (HF) input tickler only appears at the odd-even bins (■).

Figure 6.2.: Input and output spectra for extraction purposes of the extended MIMO BLA as applied to the Class-B simulation example. As in the case of SISO, the main input multisine is only applied at the transmitter’s input and its response solely appears at even-even bins (■). An additional input tickler is applied at both the transmitter’s input and output at odd-even (■), while the odd bins (■) only contain simulation noise and reserved for the supply tickler (not present in this simulation).
6. Conclusions and Future Work

The parametric model coefficients of the extended BLA now behave as the output of a Multiple-Input Single-Output (MISO) system and multiple consecutive experiments (at least 2) are necessary to make the modelling problem identifiable. As a result, the necessary filter for pre-compensation purposes is to be upgraded to a MIMO filter with the reference supply and bias voltages as inputs, and the pre-filtered supply and bias (to be applied as an input to either the supply or bias DPS respectively) as outputs.

Evidently, the inclusion of the bias voltage as an additional time-varying coefficient can also be considered in the case of a MIMO framework.

Integration of Large-Signal Stability Verification methods

As discussed in Section 2.4, DPS transmitters have potential issues with baseband instabilities at the supply and bias DC-feeding lines. In the case of a classic PA this potential source of instabilities is suppressed by the DC-blocking capacitor which shorts all possible baseband ripples before it can get out of control. An important design requirement for the DPS transmitter is the removal of this very DC-blocking capacitor as this component would also, indiscriminately, short the modulated supply and bias voltages.

Any generated baseband ripple can be suppressed by having an output filter with a sufficiently low output impedance, but choosing this value too low brings with it its own share of issues such as the presence of large resonance peaks in the baseband dynamics due to impedance mismatch (see Section 2.4).

To make an informed decision on the matter at hand, an identification method has to be introduced that is able to verify the large-signal stability of the simulated circuit. Such methods are readily available in the literature [Coom 17b] and would, at first glance, require little modification to be properly integrated in the proposed identification framework.

Model extension towards inclusion of input power-dependency

As seen in Figure 6.3, the estimated 0th model coefficient exhibits both amplitude and phase variations when the transmitter’s input power is changed. These deviations are primarily caused by static power-varying behaviour of the PA’s supply impedance $Z_{PA}$. Changing the transmitter’s input power during operation might be preferred as to adapt to temporal operational conditions, such as, for example, the distance to the receiver. Employing a range of pre-compensation models, one for each input power, is a possible way to address this additional challenge, but has several disadvantages.
Figure 6.3.: Estimated 0th complex model coefficients of measurement data employing different input power levels, namely −20 (■), −15 (■), −10 (■) and −5dBm average input power. (■). The exact same set of 16 phase realizations with a bandwidth of 5MHz is used for all measurements.

A potentially better solution is to include the input power as an additional parameter into the pre-compensation model as this allows the model to be used at a continuous range of input powers, potentially even at those at which it was not estimated. Furthermore this solution reduces the number of model parameters, as only a single (power-dependent) model is required. Such an endeavour requires the introduction of the so-called macromodelling technique as to include the input power as a static parameter into the model equations [Ferr 10].
A. List of Publications

Journal publications


Conference publications


Acknowledgements

This work almost didn’t see the light of day, but after some toiling and sufferances, and with a slight delay, was realized nonetheless. None of this would’ve been possible without the aid of a select group of people and their tolerance to listen to my lamentations.

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Wireless signals are all around us and make it possible to, for example, browse the internet on your smartphone. Most of these signals are transmitted by antennas installed on radio masts and towers all around us. Due to the abundance of these towers, it is important that these signals are transmitted in an efficient manner; i.e. if you supply the tower with power, you expect the same amount of power to be transmitted by the antennas. Unfortunately, a large part of the supplied power is wasted. The main contributor to bad power efficiency is the so-called PA, a device that transforms the small “whispering” signal outputted by a previous stage to a much larger “roaring” output so your smartphone can “hear” the antenna from much farther away.

Many techniques exist that aim to improve power efficiency of the PA, but this work solely focuses on the group of so-called DPS transmitters. These are PAs that have a changing power supply which only provides enough energy for the amplification and, theoretically, none for wasting purposes.

All these PAs work in the assumption that this power supply actually supplies the correct amount of power at the right time instance. If the power supply is a little bit too early or too late and supplies an insufficient amount of power then the communication signal may become corrupted. On the other hand, if too much power is supplied, the power is again wasted. Thus, the relationship between both the power supply’s output value and the communication signal to be amplified needs to be closely monitored.

The main goal of this work is to find a method to identify and rectify any and all issues involving the (mis)alignment of both signals. This is done by taking the power supply’s input and modifying it in such a way (i.e. delaying, advancing, amplifying and/or decreasing the power supply) that the output signal of the power supply arrives at both the perfect time and with the perfect amplitude level, resulting in an overall more energy efficient transmitter.