DEEL I: WEERSTANDSNETWERKEN

1. Basiswetten netwerken
2. Éénpoorten
3. Superpositiestelling
4. Tweepoorten
5. Stellingen van Thévenin en Norton
6. Methode van de knooppuntpotentialen
7. Compensatiestelling

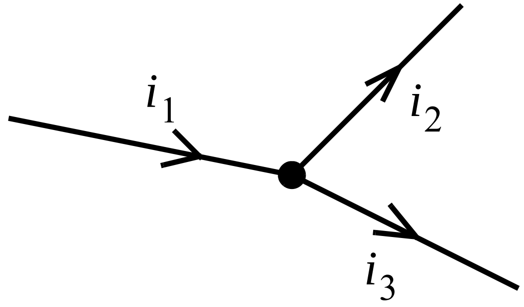
DEEL II: RLC-NETWERKEN

1. Sinusoidaal regimeantwoord
2. Overgangsverschijnselen - tijdsdomein
3. Overgangsverschijnselen - Laplace domein
4. Methode van de maasstromen

Rik Pintelon, Brussel, 24 september 2015
versie 19 oktober 2018

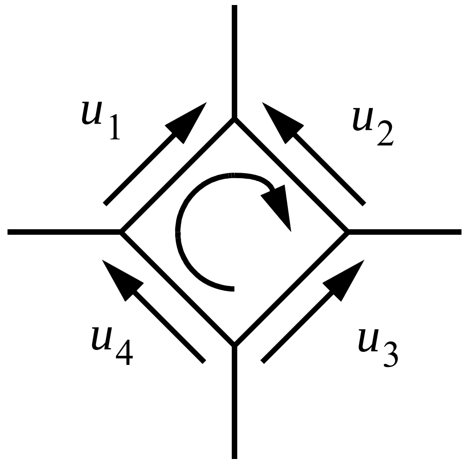
Basiswetten netwerken: KCL, KVL, en VAL

KCL



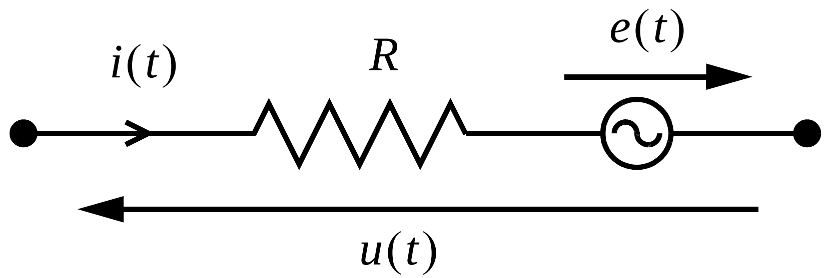
$$i_1 - i_2 - i_3 = 0$$

KVL



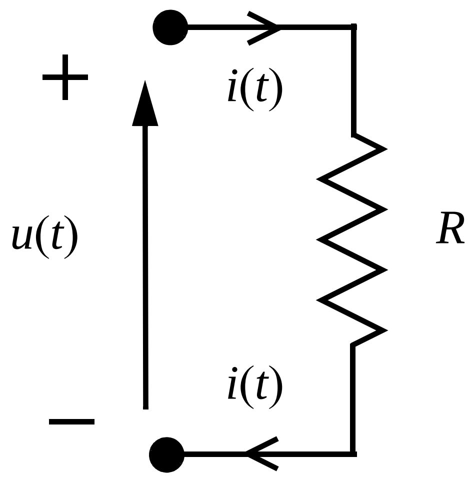
$$u_1 - u_2 - u_3 + u_4 = 0$$

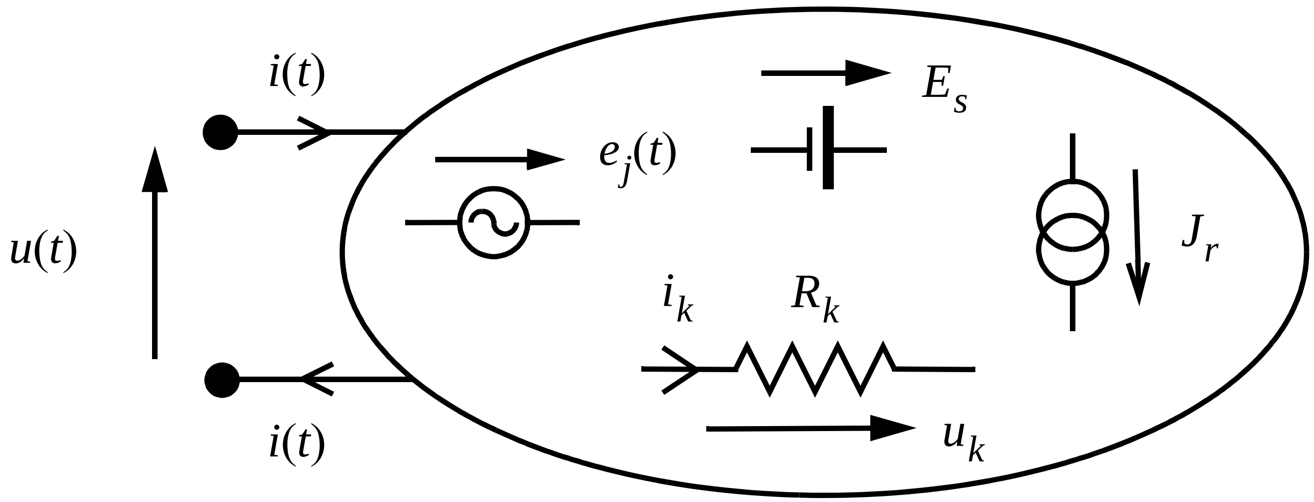
VAL



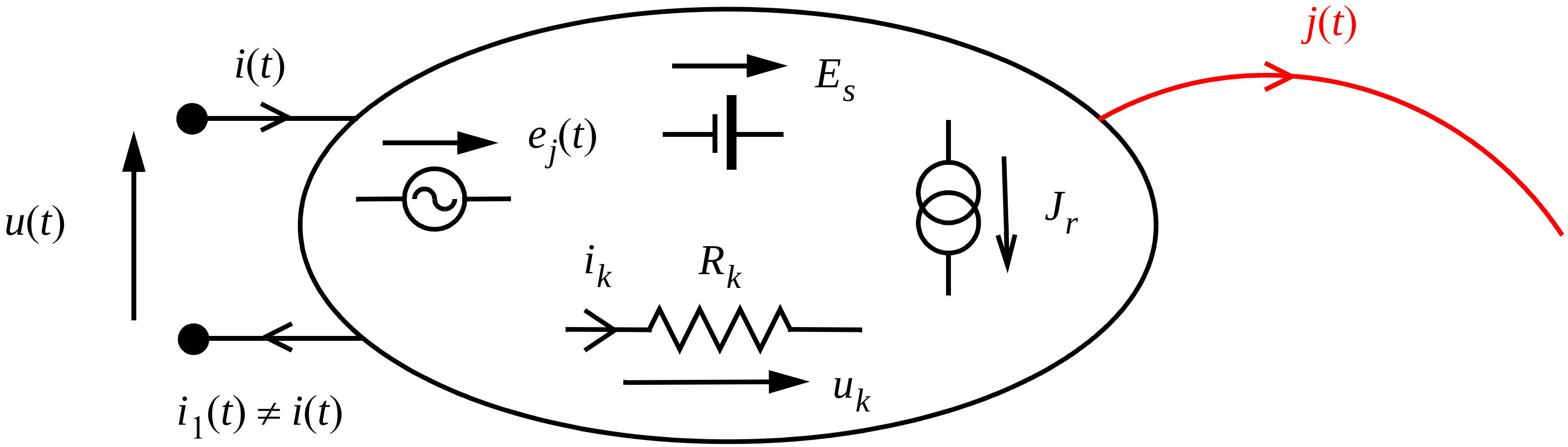
$$u(t) = Ri(t) - e(t)$$

Éénpoorten



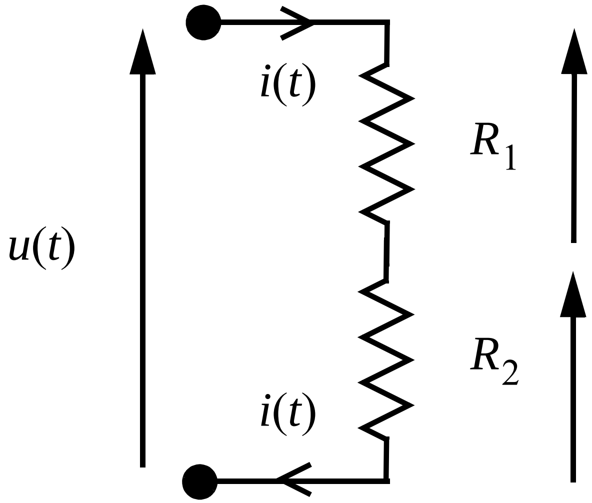


$i(t)$
E_s
$e_j(t)$
$u(t)$
J_r
i_k
R_k
$i(t)$
u_k



$i(t)$
$u(t)$
$e_j(t)$
E_s
J_r
i_k
R_k
u_k
$i_1(t) \neq i(t)$
$j(t)$

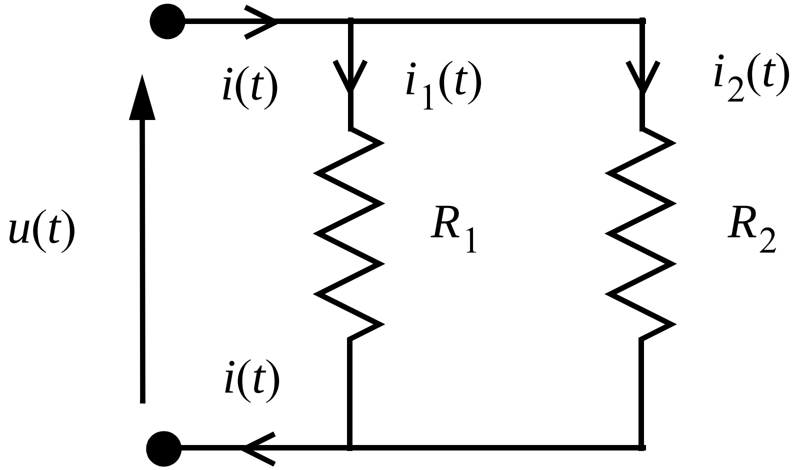
Wet spanningsdeler



$$u_1(t) = \frac{R_1}{R_1 + R_2} u(t)$$

$$u_2(t) = \frac{R_2}{R_1 + R_2} u(t)$$

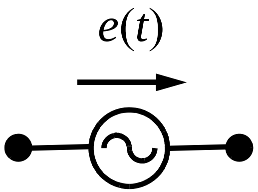
Wet stroomdeler



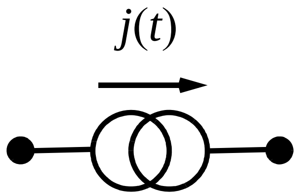
$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t)$$

$$i_2(t) = \frac{R_1}{R_1 + R_2} i(t)$$

Onafhankelijke bronnen



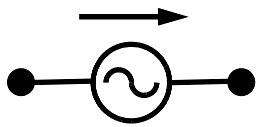
onafhankelijke
spanningsbron



onafhankelijke
stroombron

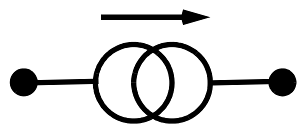
Afhankelijke (gestuurde) bronnen

$e(t) = Au(t)$



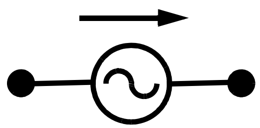
spanningsgestuurde
spanningsbron

$j(t) = \beta i(t)$



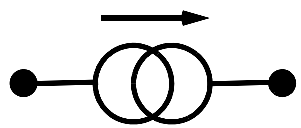
stroomgestuurde
stroombron

$e(t) = ri(t)$



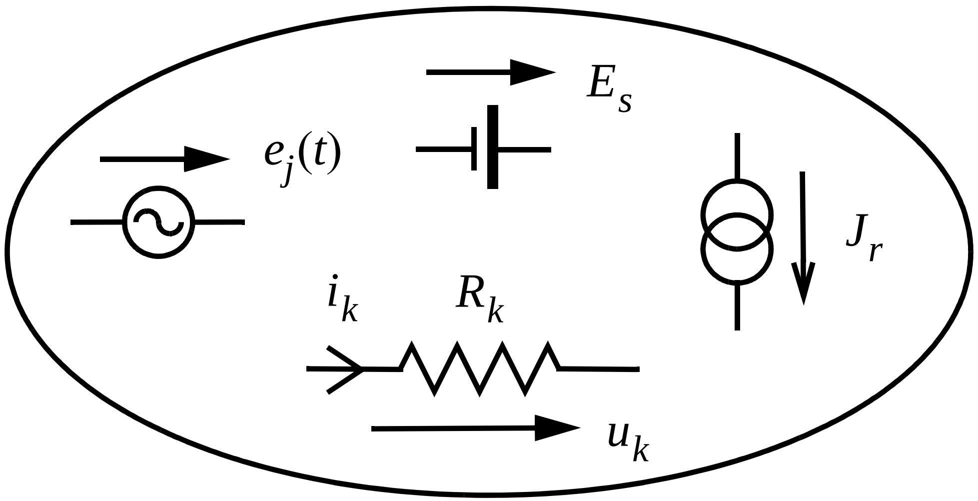
stroomgestuurde
spanningsbron

$j(t) = gu(t)$

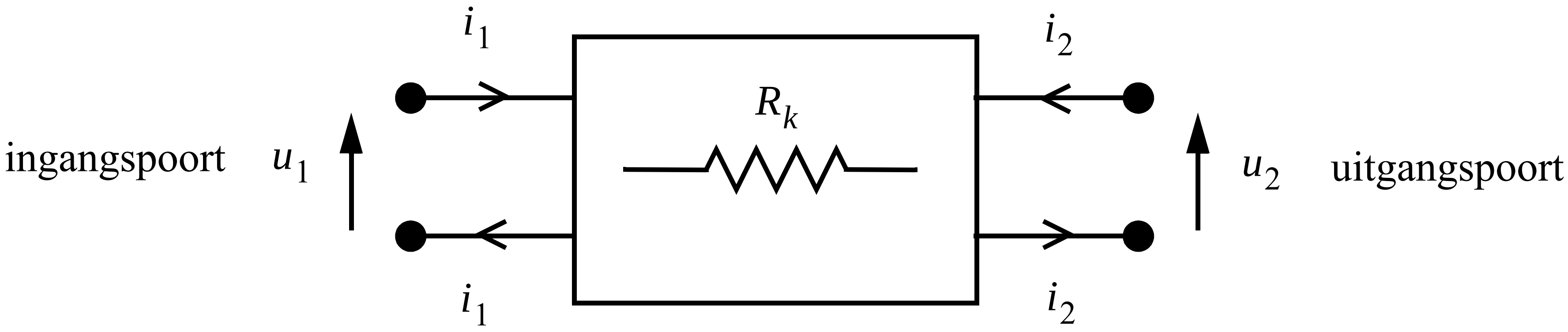


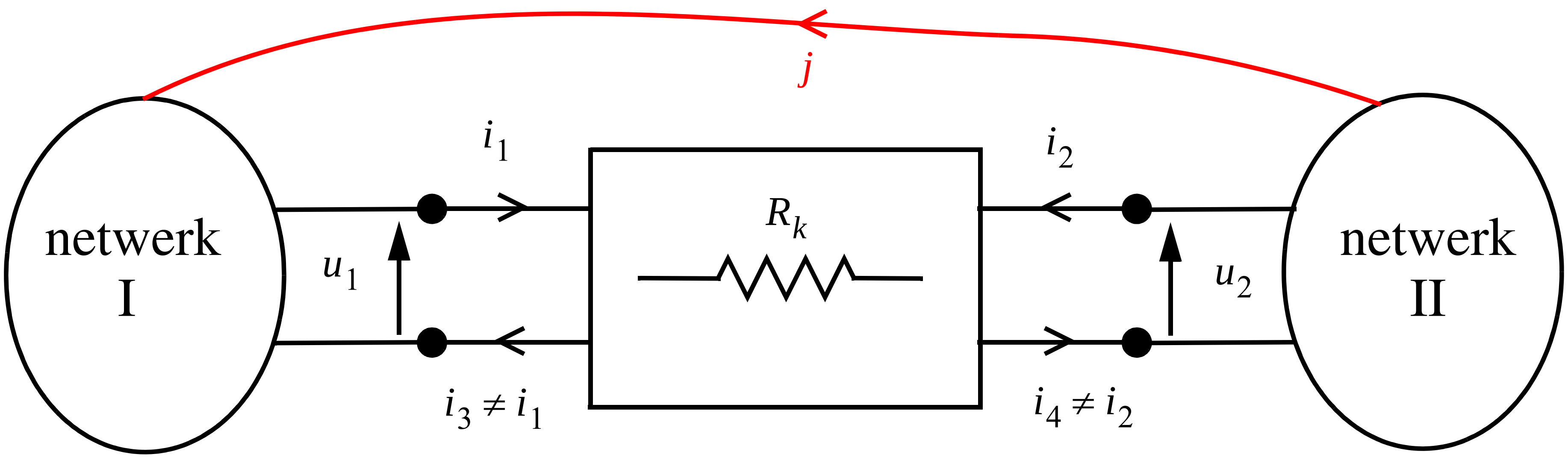
spanningsgestuurde
stroombron

Superpositiestelling

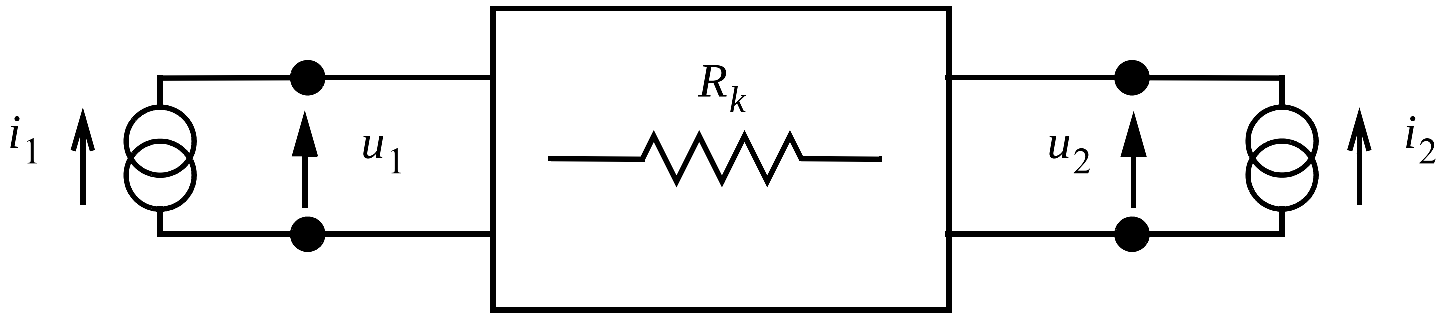


Tweepoorten

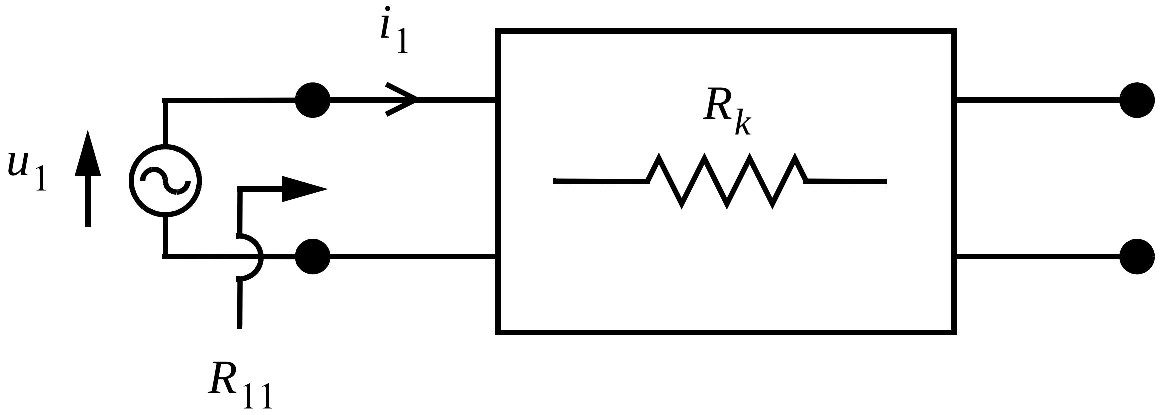



j
i1
i2
Rk
netwerk I
netwerk II
u1
u2
i3 ≠ i1
i4 ≠ i2

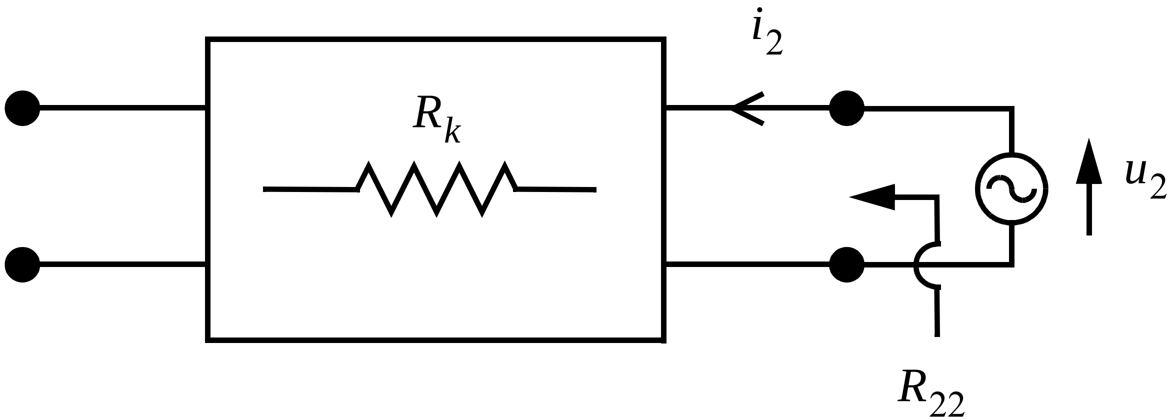
Weerstandsmatrix



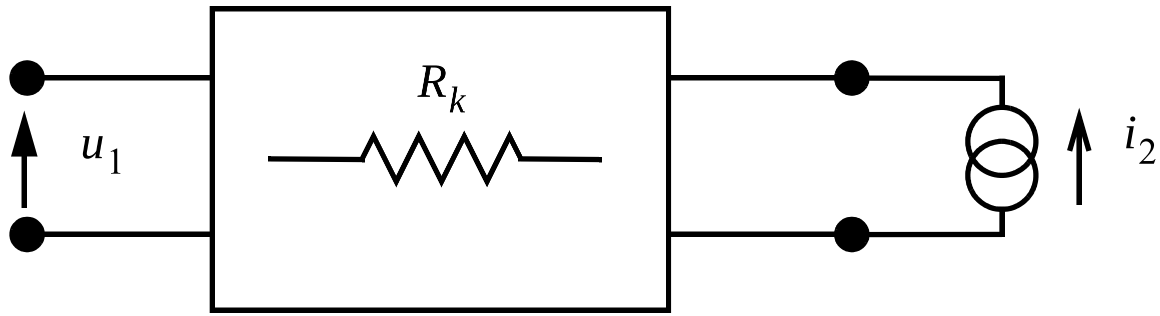
$$R_{11} = \left. \frac{u_1}{i_1} \right|_{i_2 = 0}$$



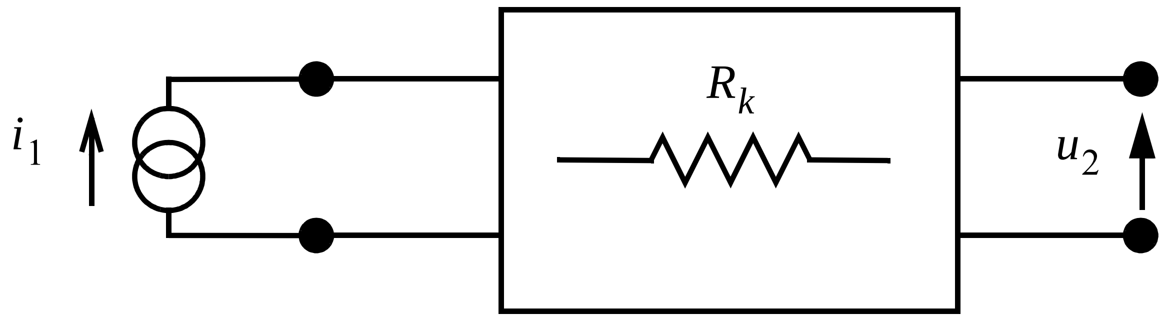
$$R_{22} = \left.\frac{u_2}{i_2}\right|_{i_1 = 0}$$

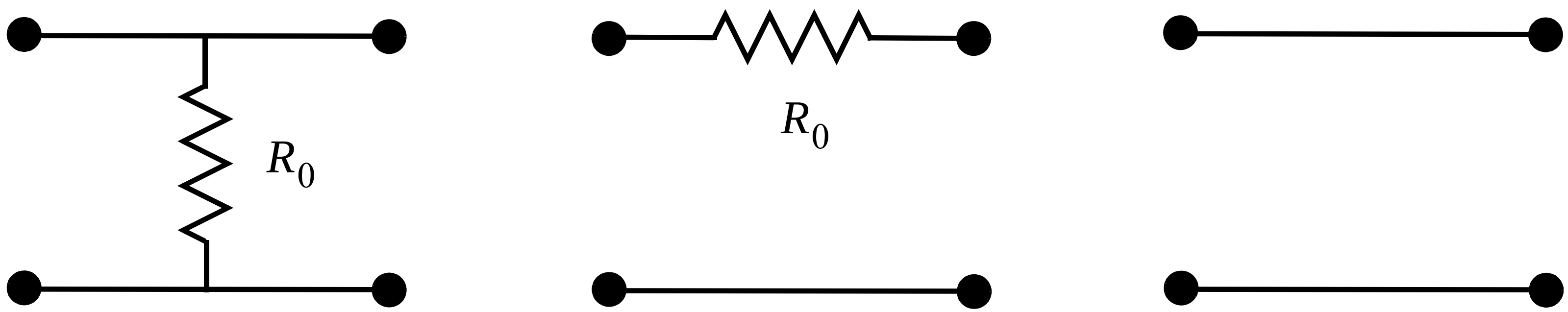


$$R_{12} = \left. \frac{u_1}{i_2} \right|_{i_1 = 0}$$



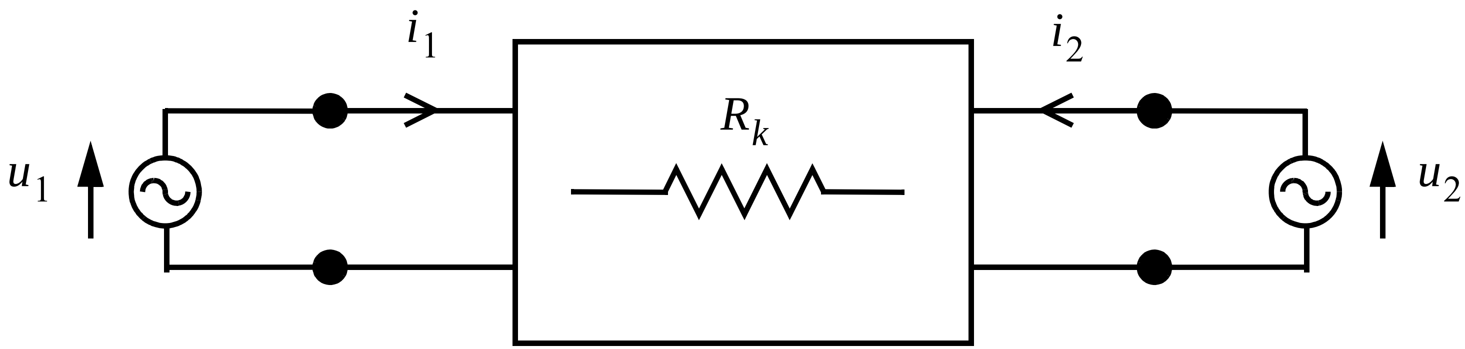
$$R_{21} = \left.\frac{u_2}{i_1}\right|_{i_2 = 0}$$



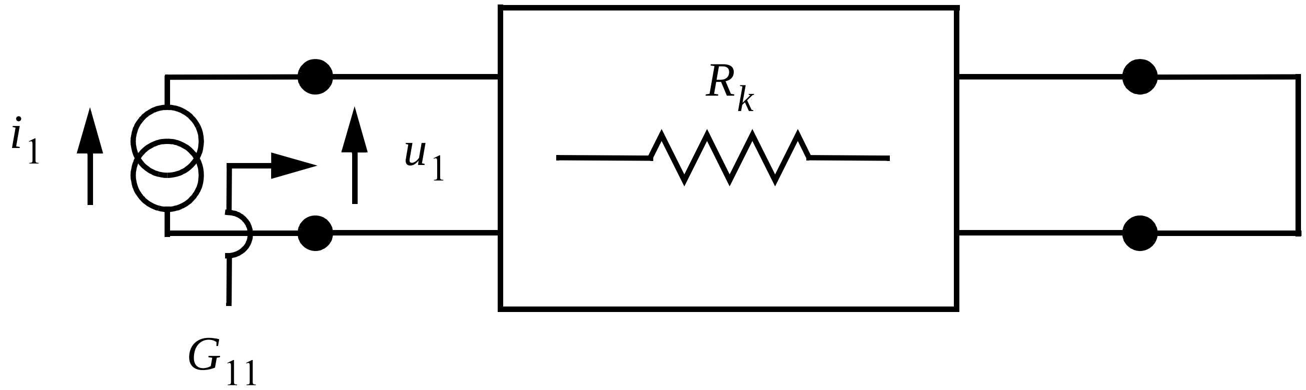


(a) (b) (c)

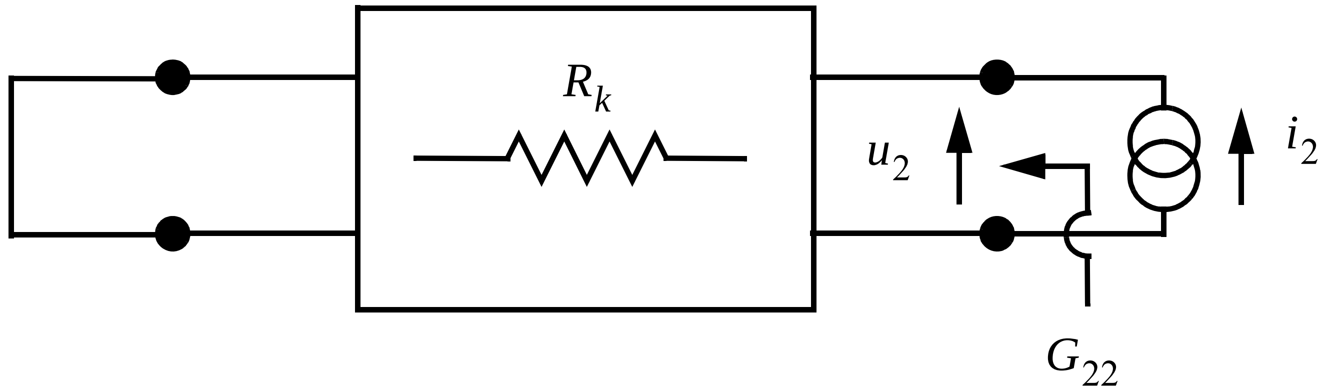
Conductantiematrix



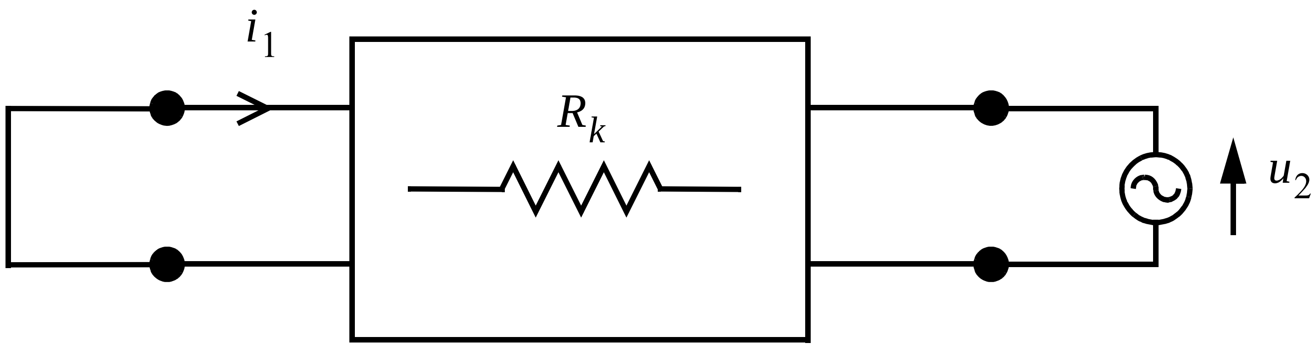
$$G_{11} = \left.\frac{i_1}{u_1}\right|_{u_2 = 0}$$



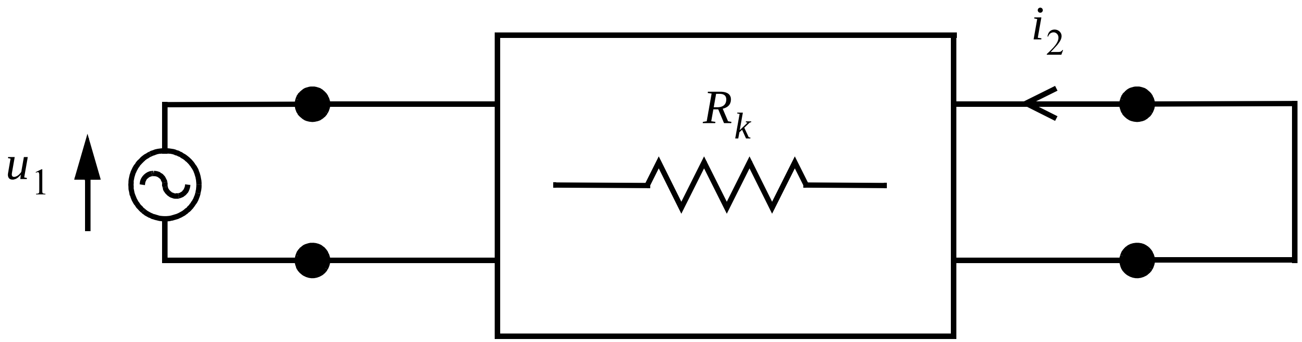
$$G_{22} = \left. \frac{i_2}{u_2} \right|_{u_1 = 0}$$

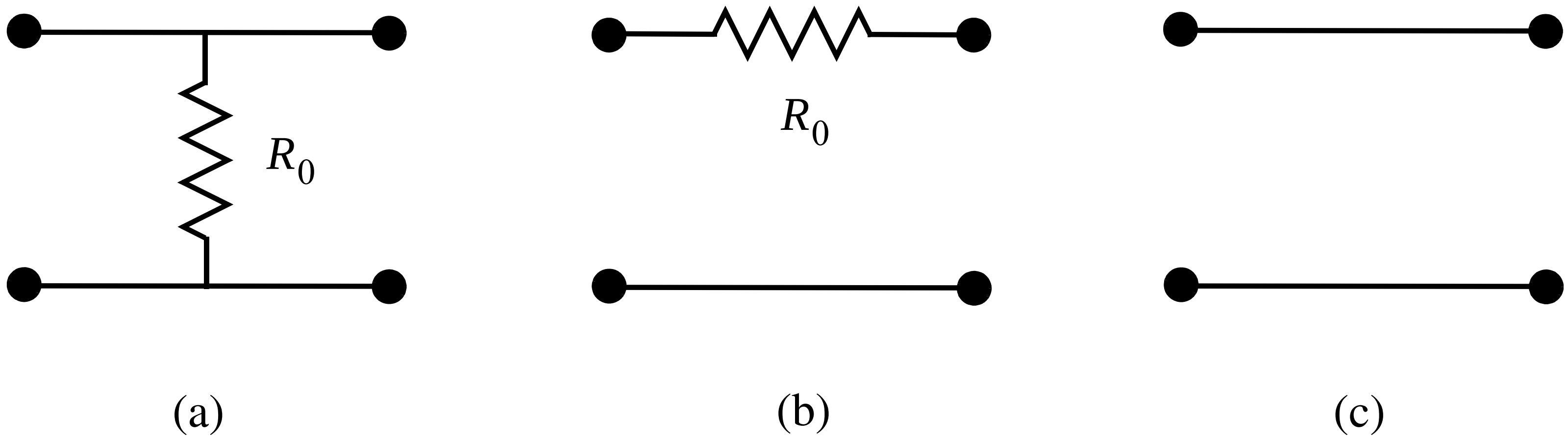


$$G_{12} = \left.\frac{i_1}{u_2}\right|_{u_1 = 0}$$



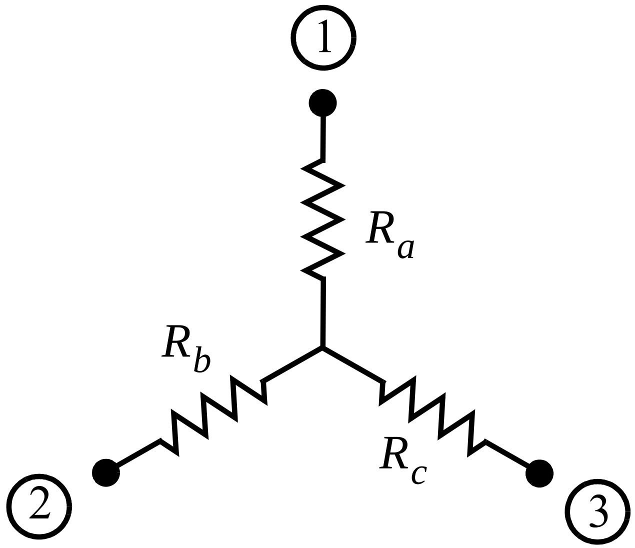
$$G_{21} = \left.\frac{i_2}{u_1}\right|_{u_2 = 0}$$

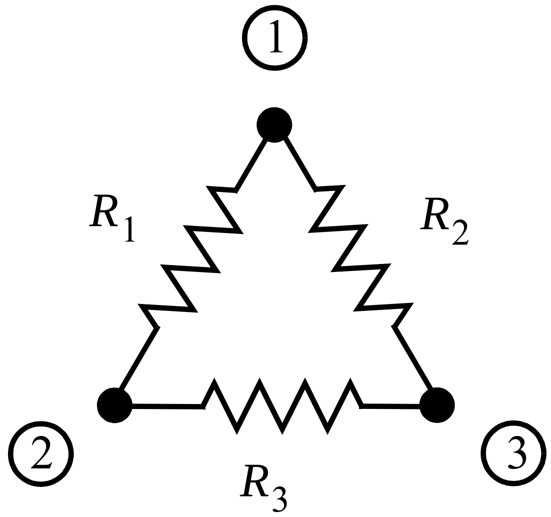




R_0
R_0
(a)
(b)
(c)

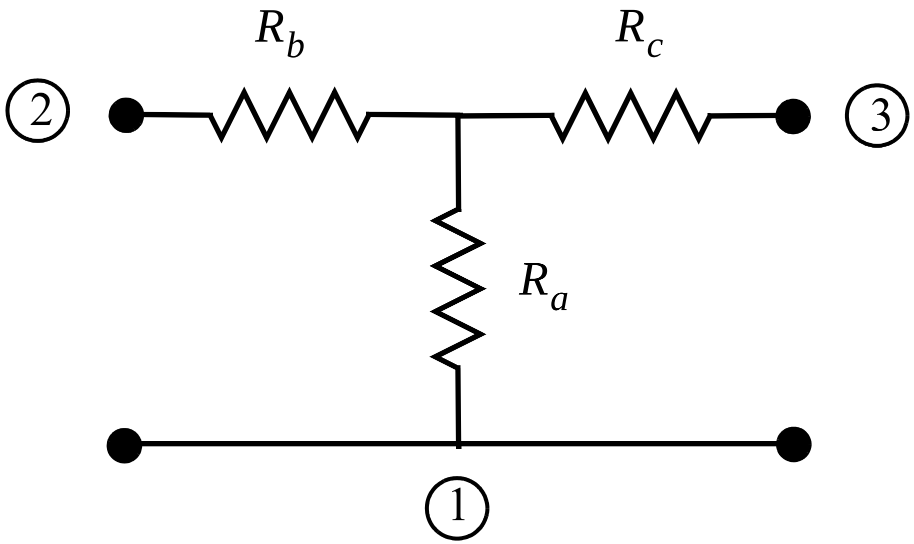
Ster-driehoek transformatie



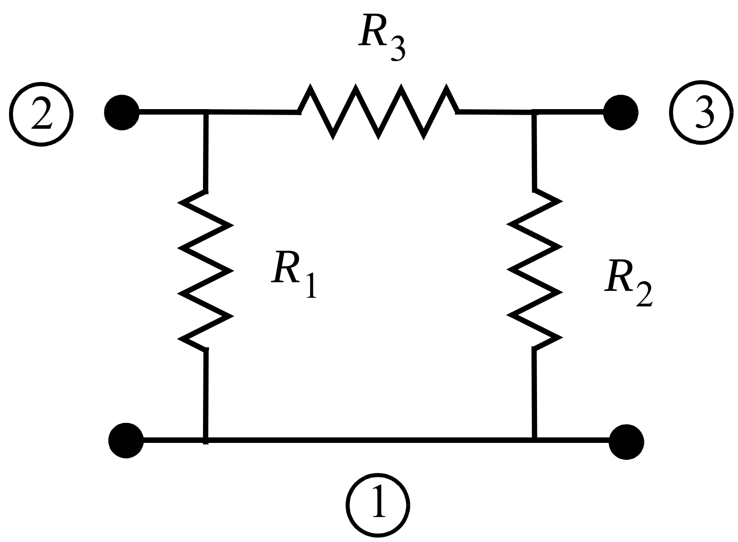


$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3},\quad R_b = \frac{R_1 R_3}{R_1 + R_2 + R_3},\quad R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_1 = R_a + R_b + \frac{R_a R_b}{R_c},\quad R_2 = R_a + R_c + \frac{R_a R_c}{R_b},\quad R_3 = R_b + R_c + \frac{R_b R_c}{R_a}$$

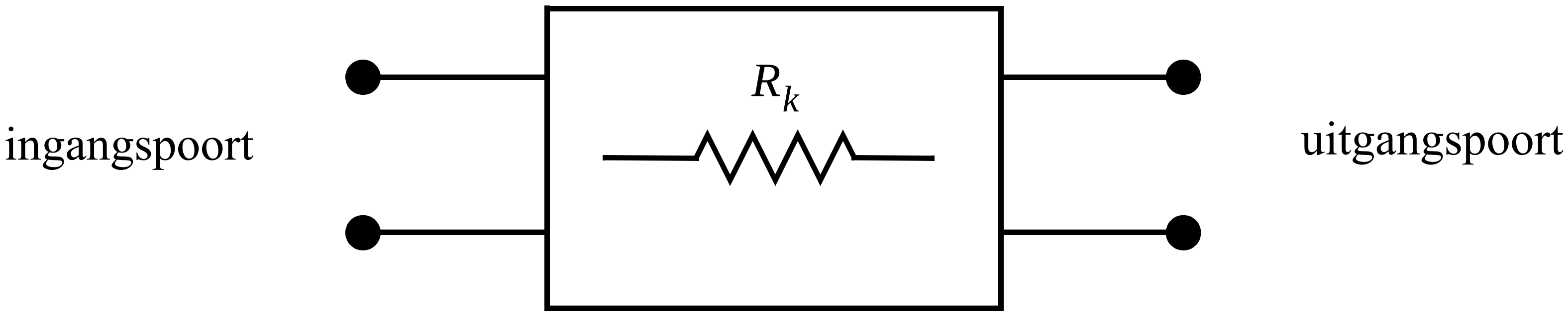


R_b
R_c
2
3
R_a
1

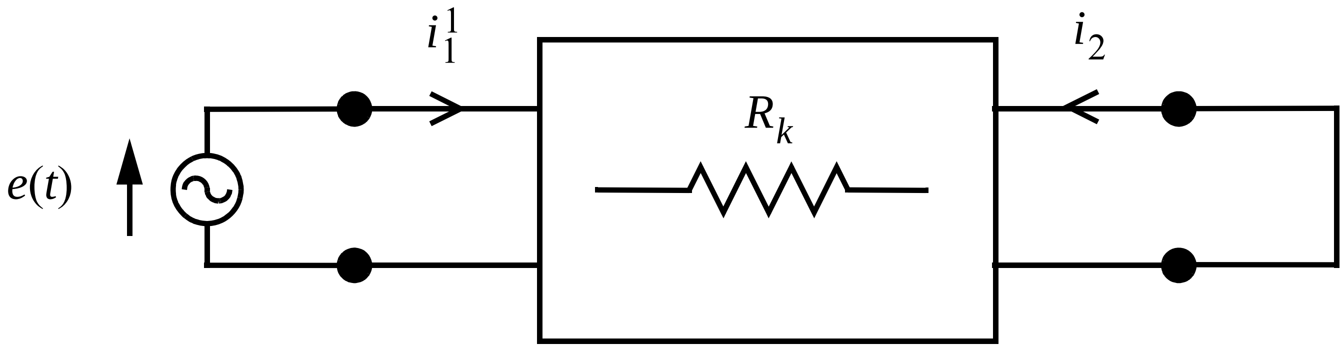


R_3
2
3
R_1
R_2
1

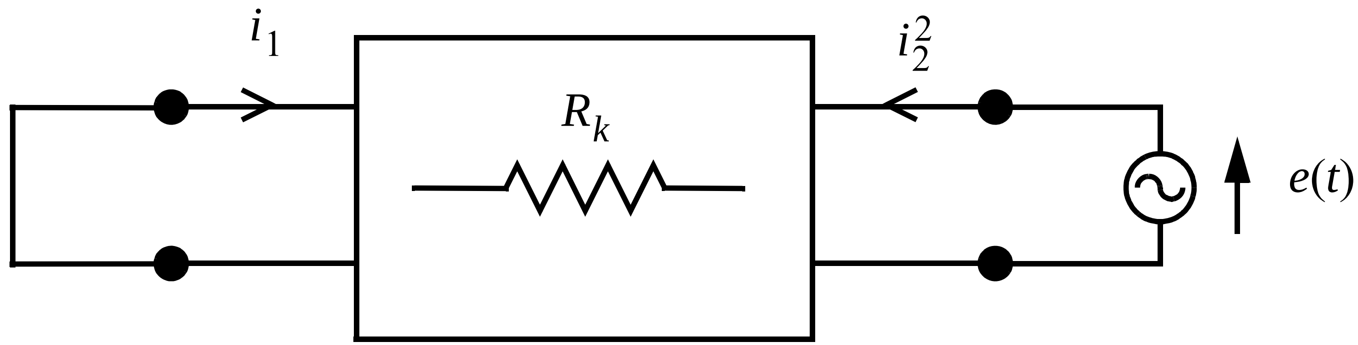
Reciprociteit



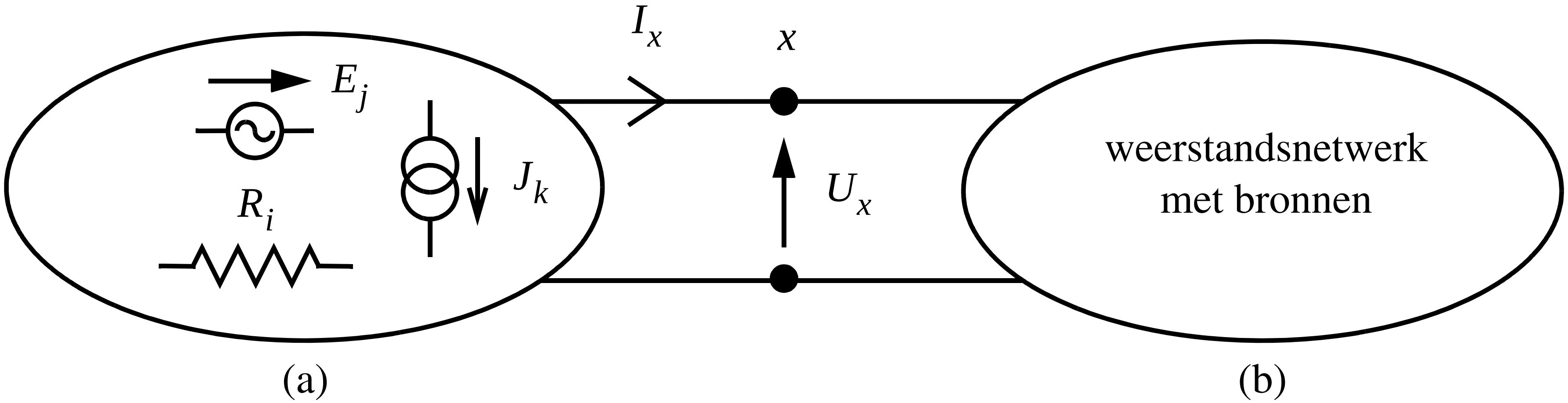
Experiment 1

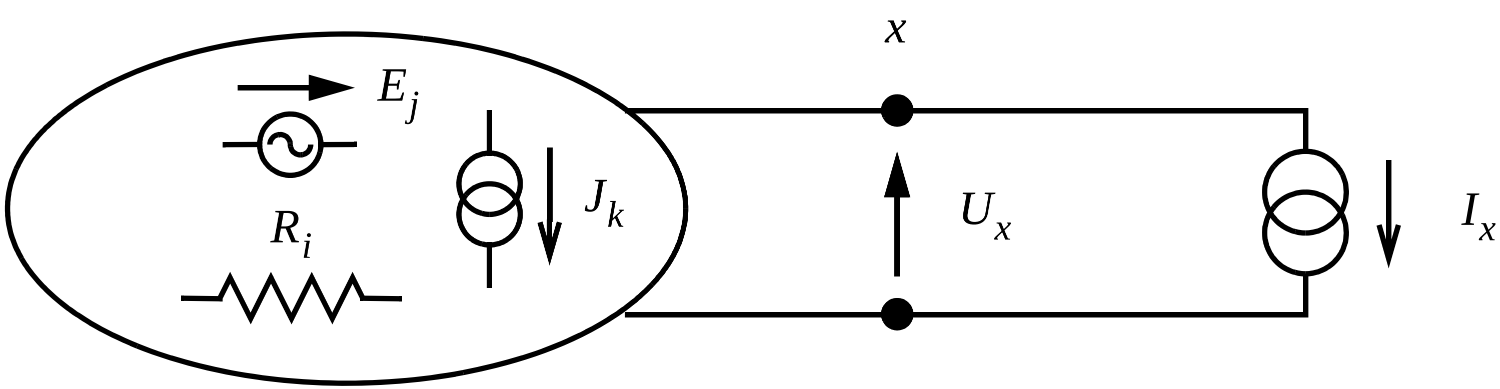


Experiment 2



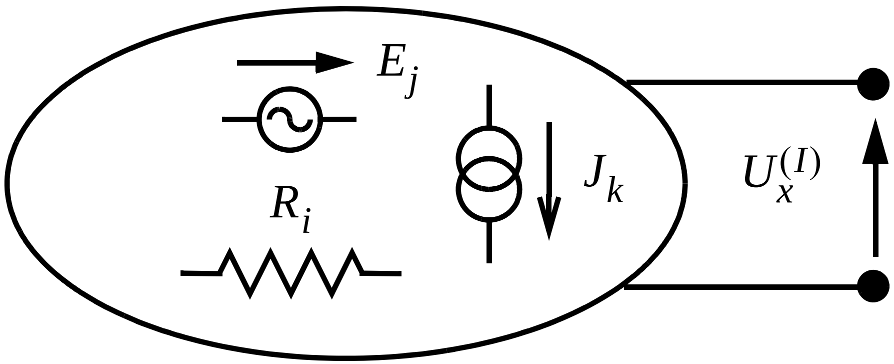
Stelling van Thévenin



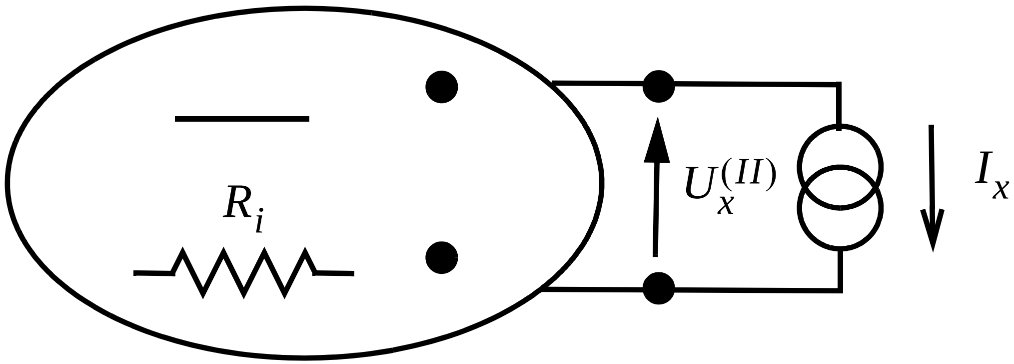


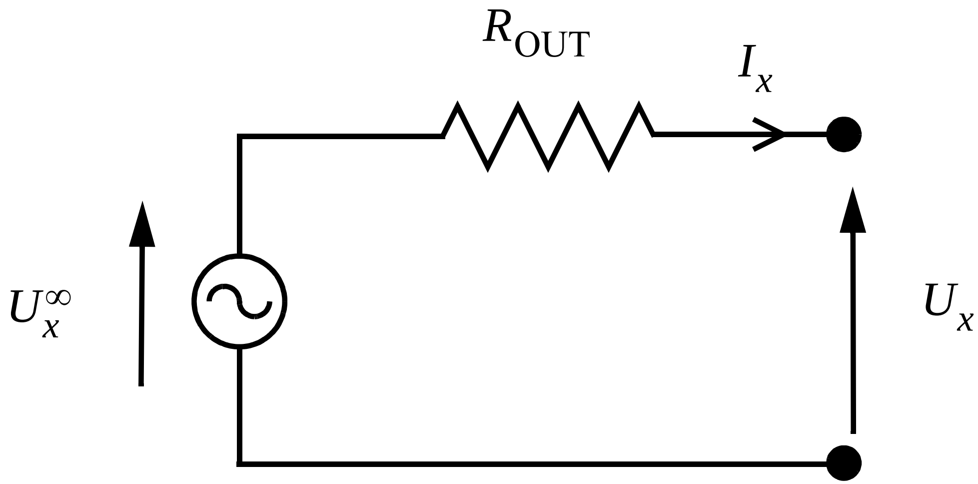
(a)

Experiment I



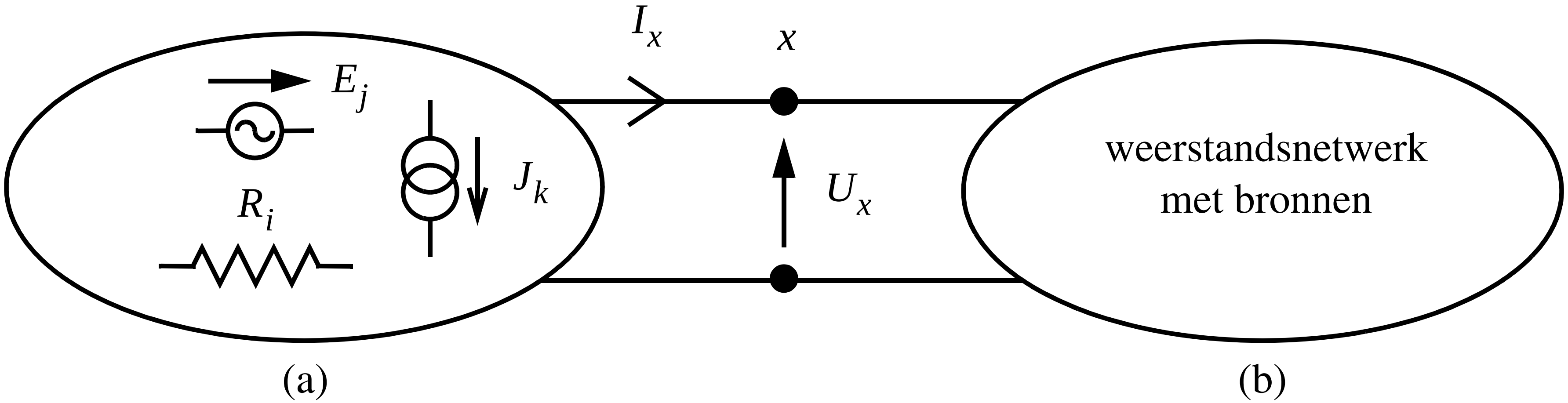
Experiment II

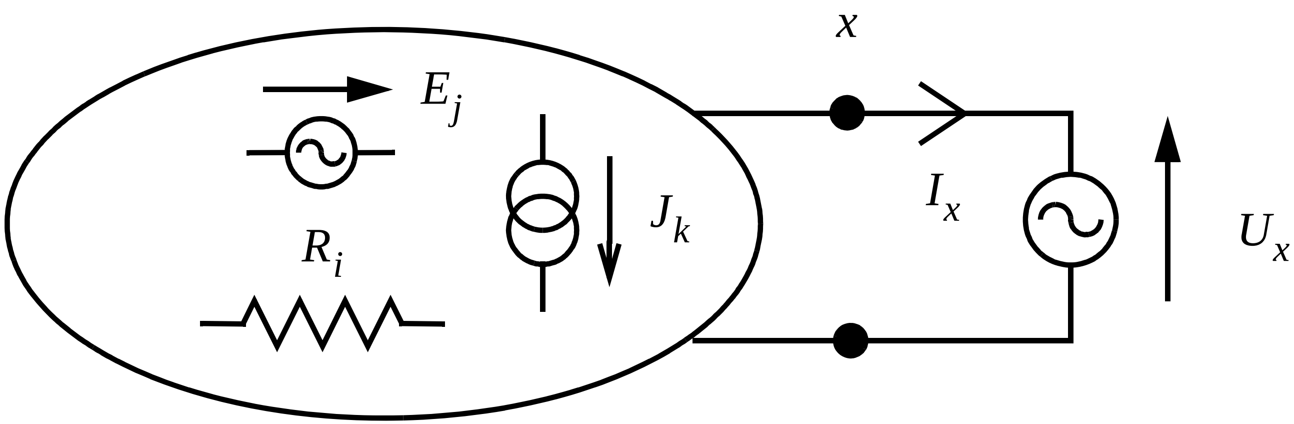




R_{OUT}
I_x
U_x^{∞}
U_x

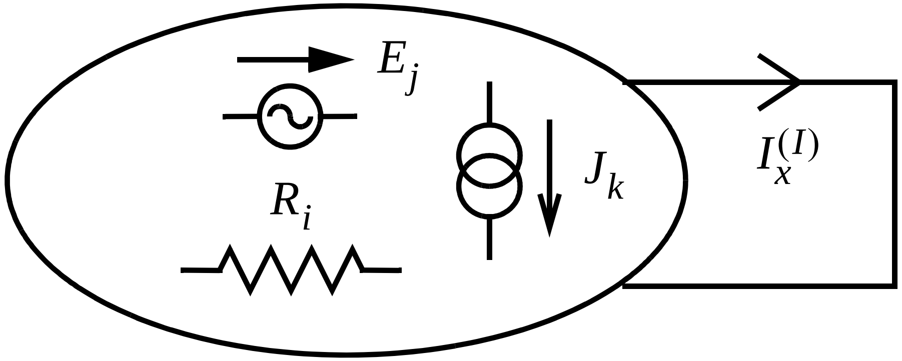
Stelling van Norton



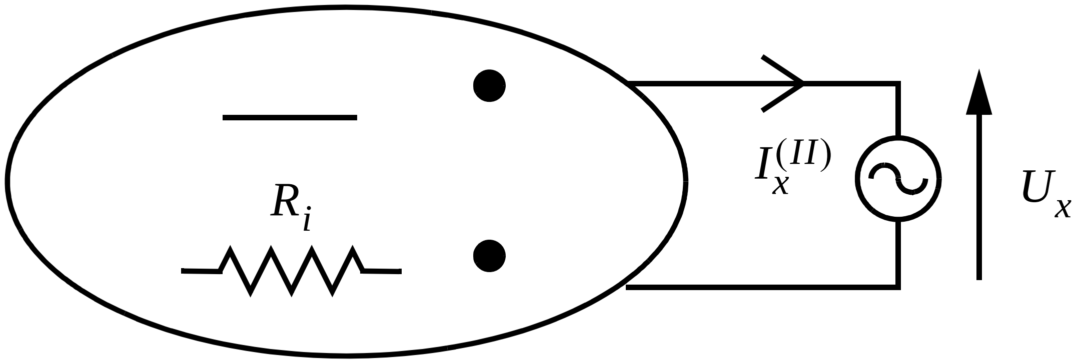


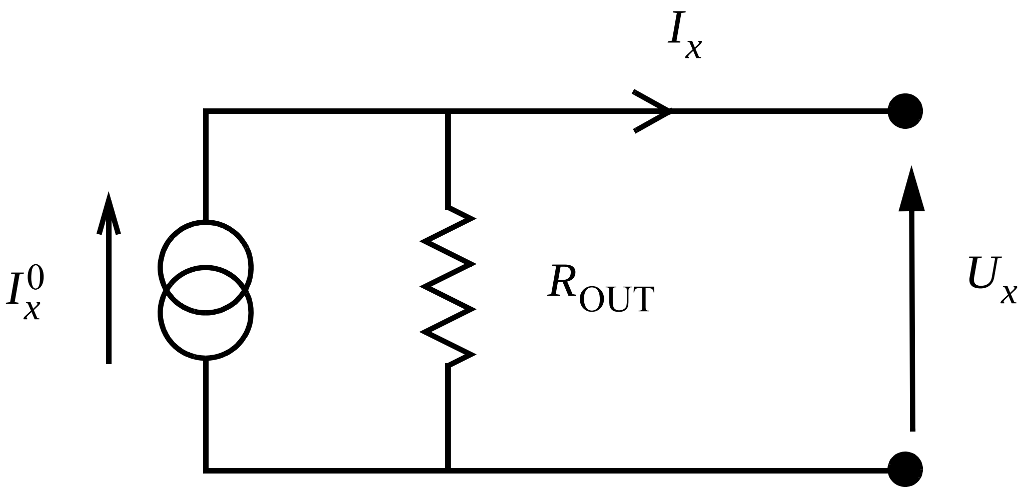
(a)

Experiment I



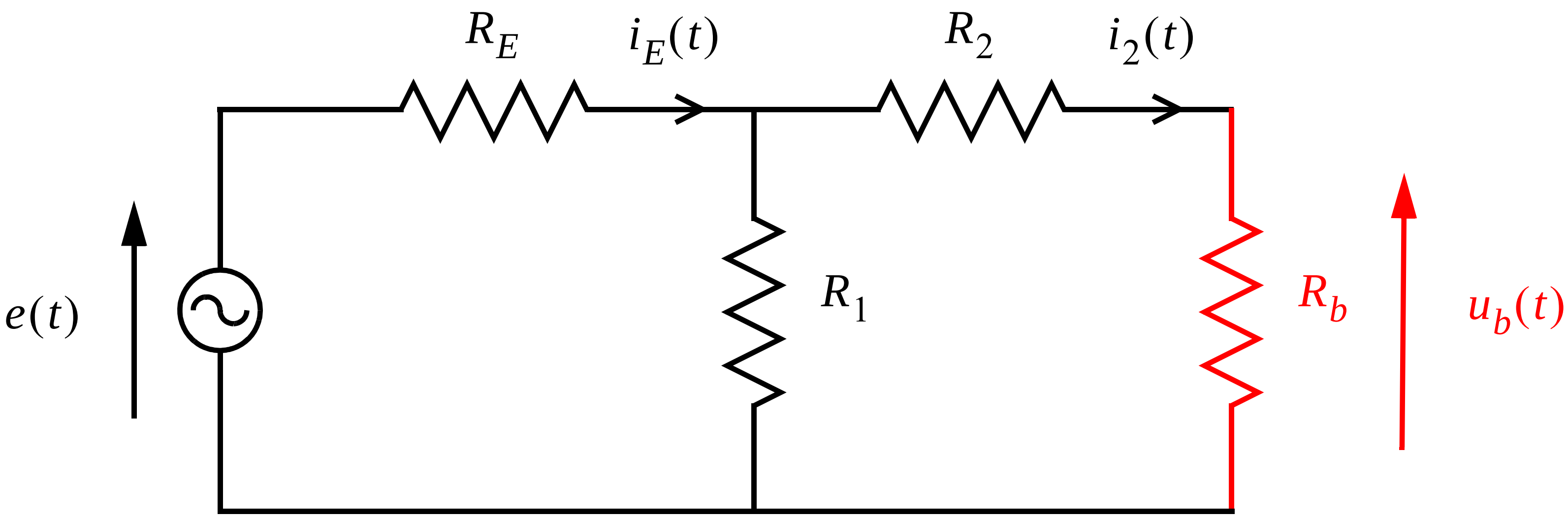
Experiment II

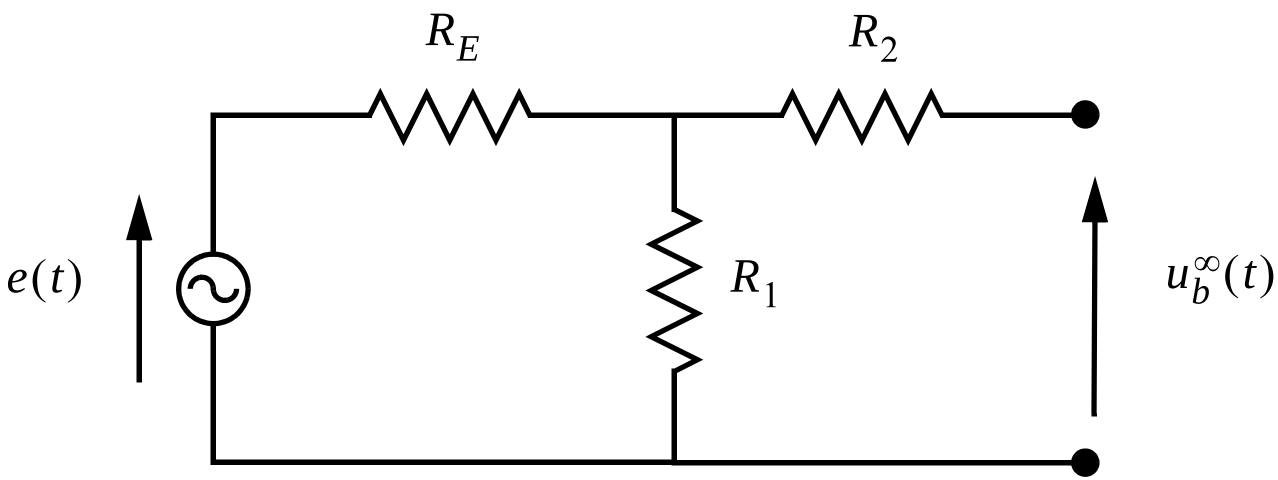


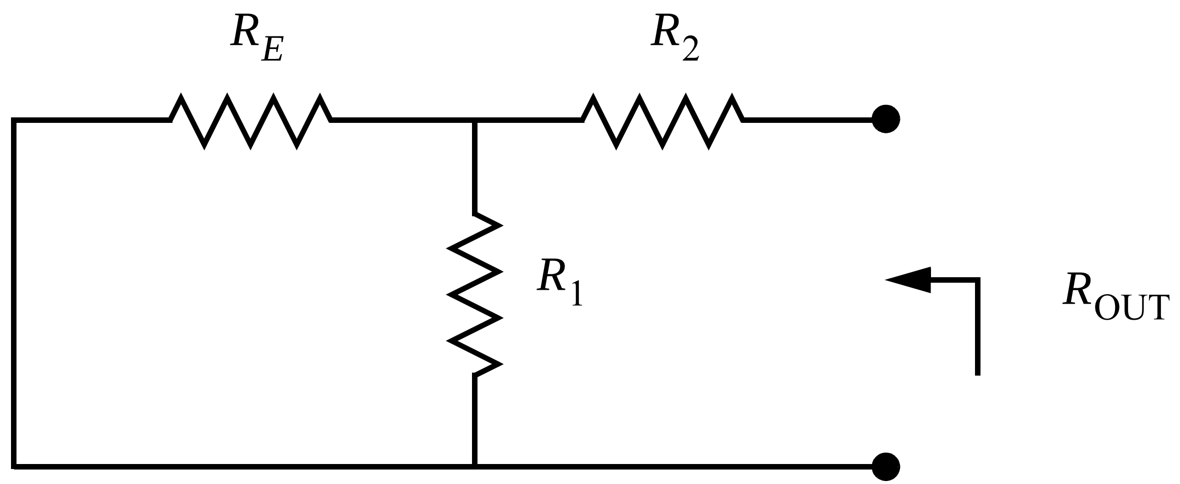


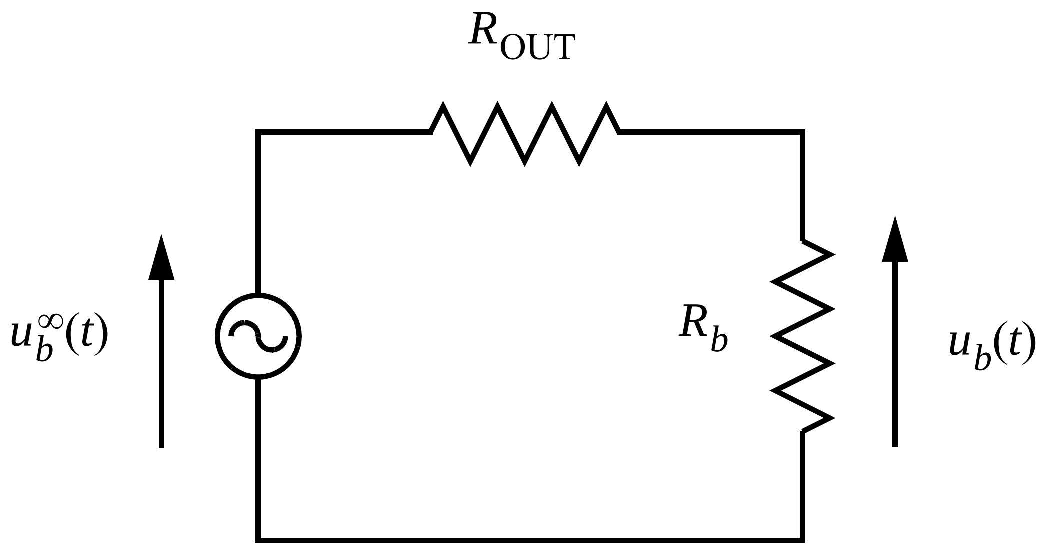
I_x
I_x^0
R_{OUT}
U_x

Voorbeeld

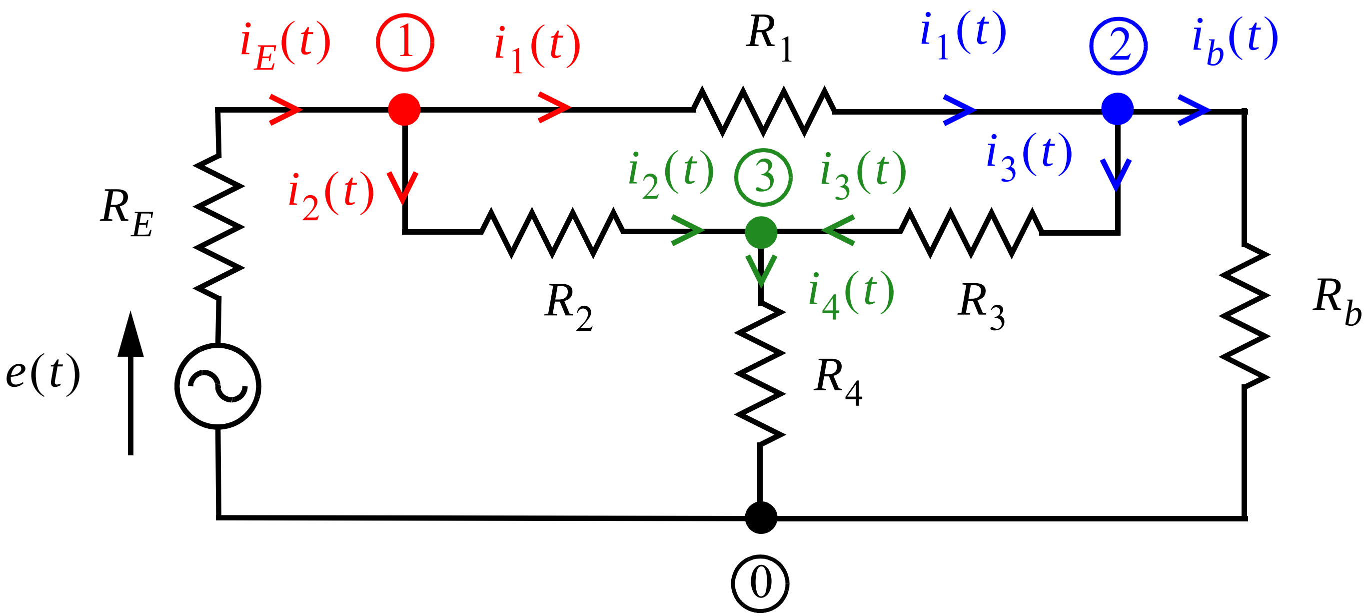


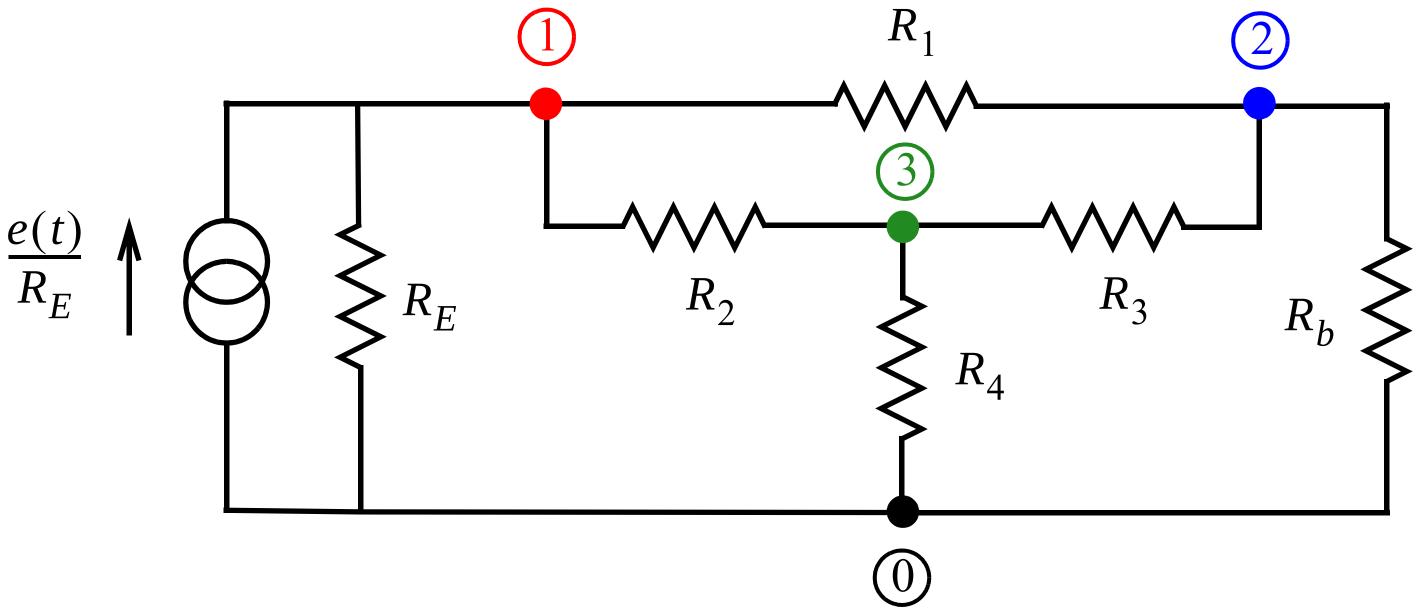

R_E
R_2
$e(t)$
R_1
$u_b^{\infty}(t)$


R_E
R_2
R_1
R_{OUT}


R_{OUT}
$u_b^\infty(t)$
R_b
$u_b(t)$

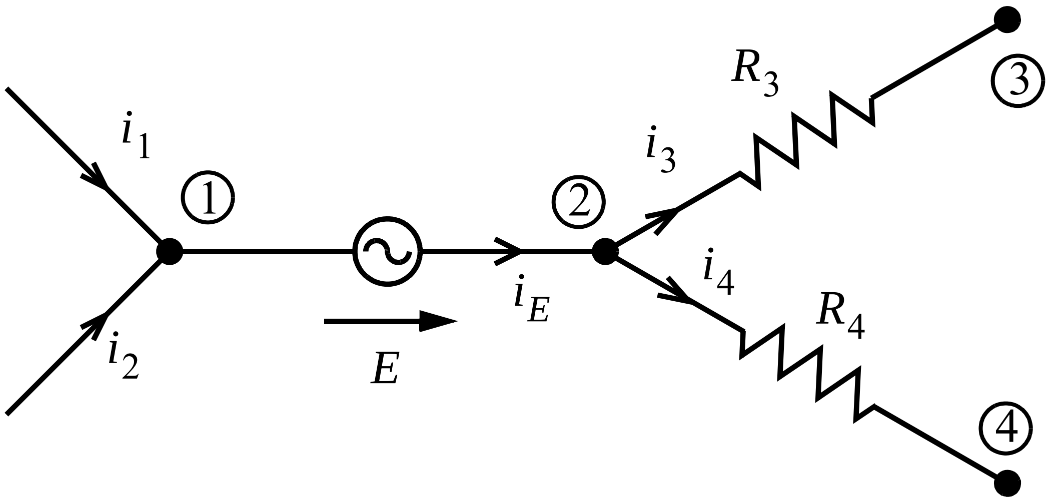
Methode van de knooppuntpotentialen

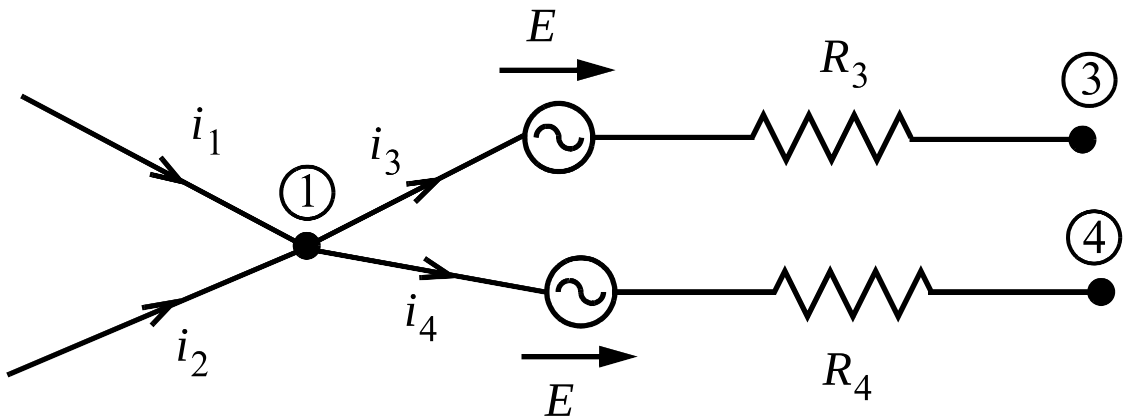




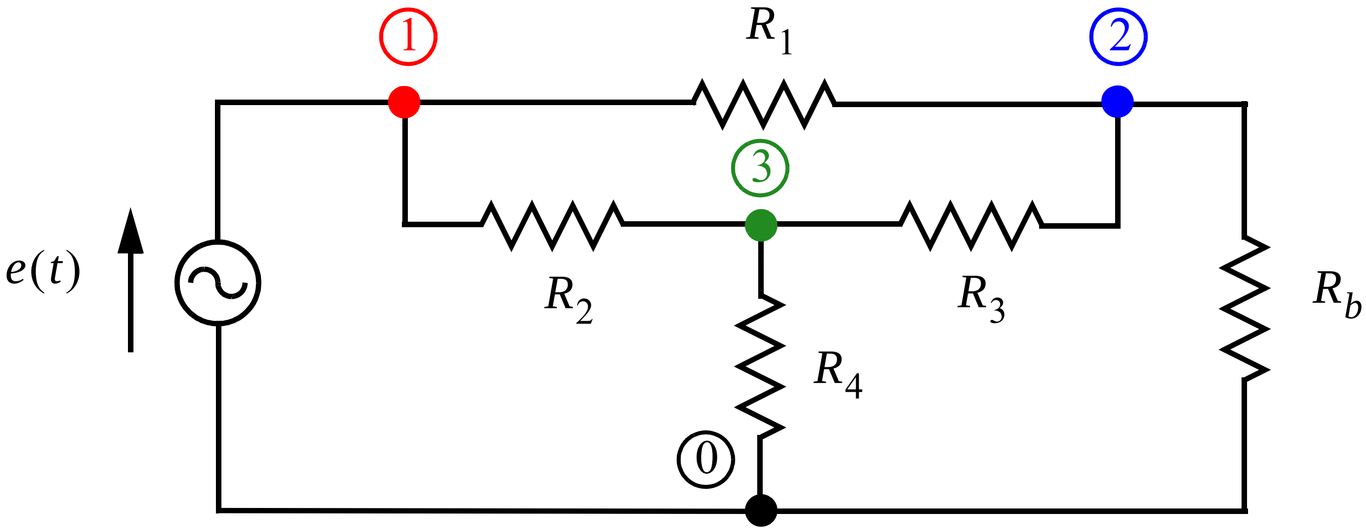
1
R_1
2
3
$\frac{e(t)}{R_E}$
R_E
R_2
R_3
R_b
R_4
0

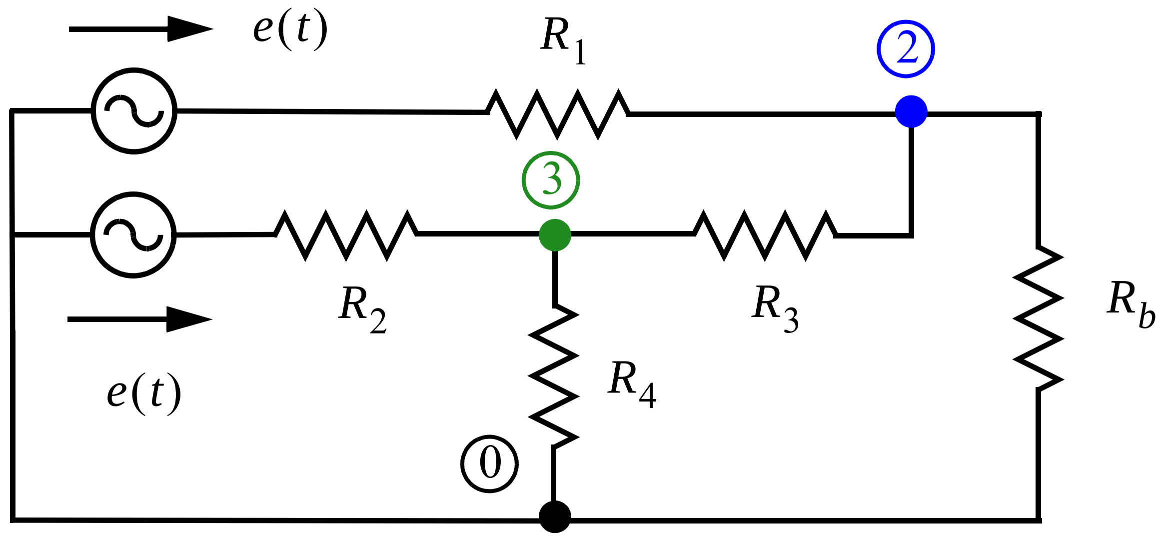
V-shift





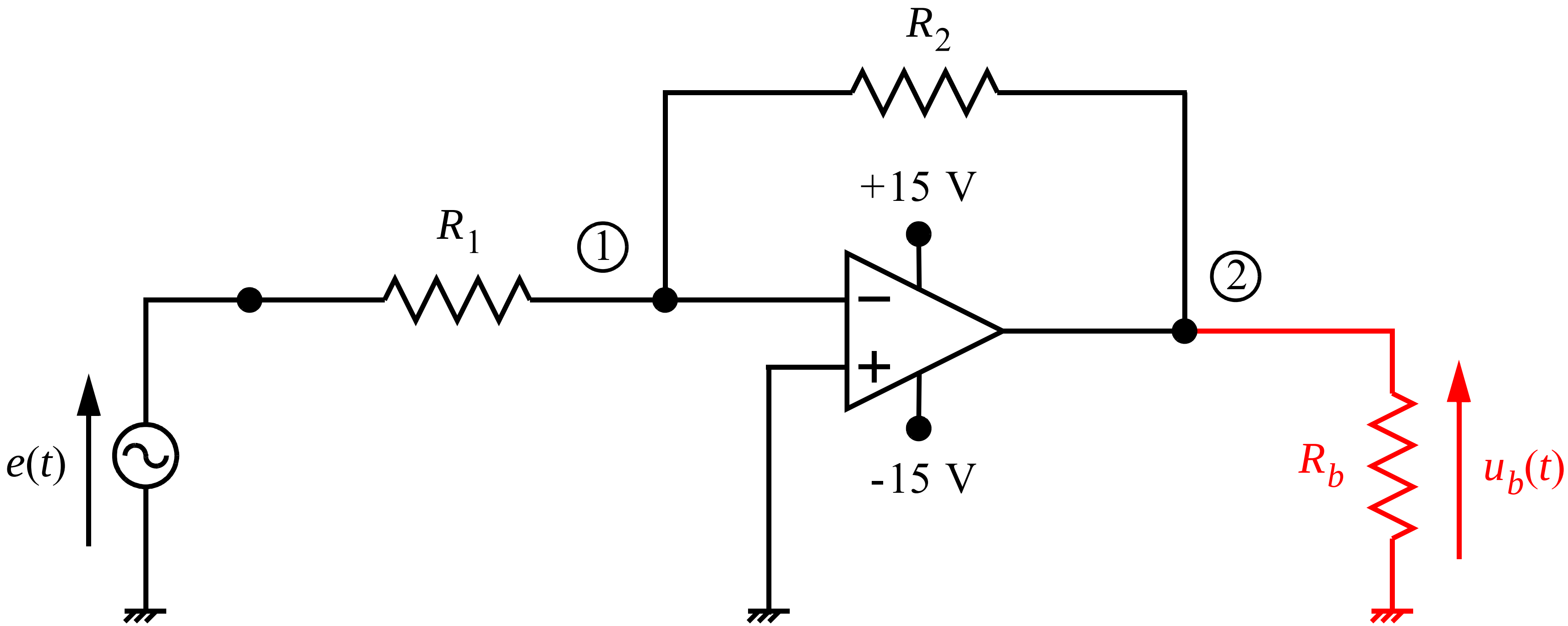
Toepassing V-shift

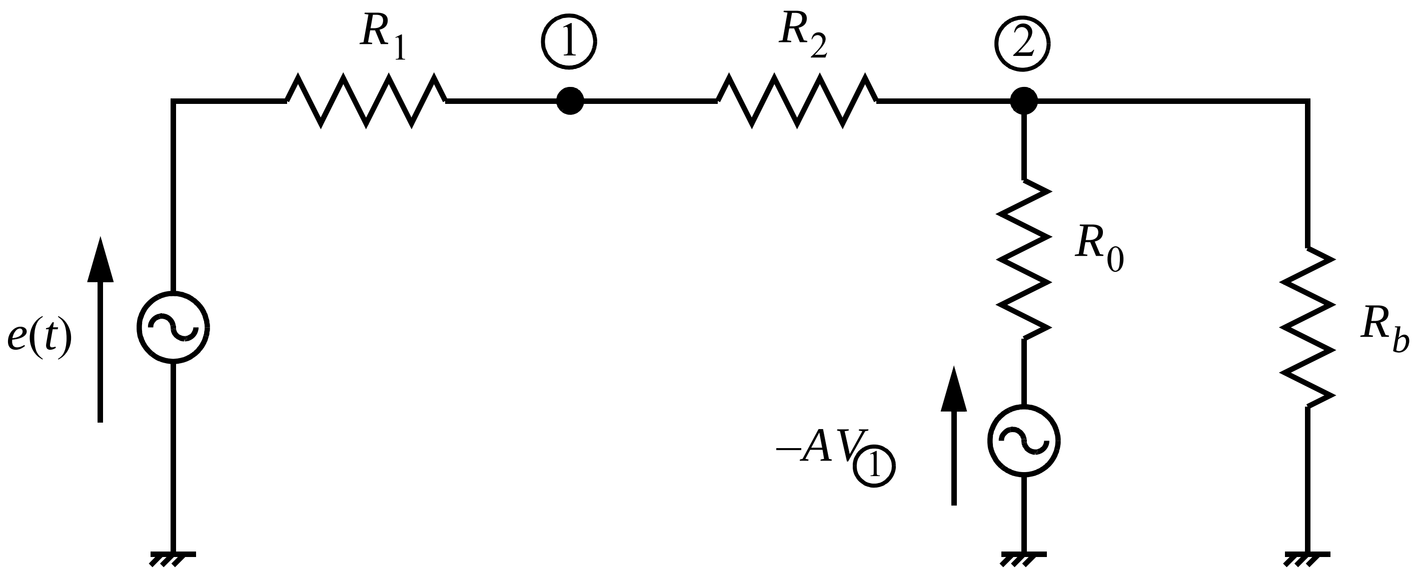


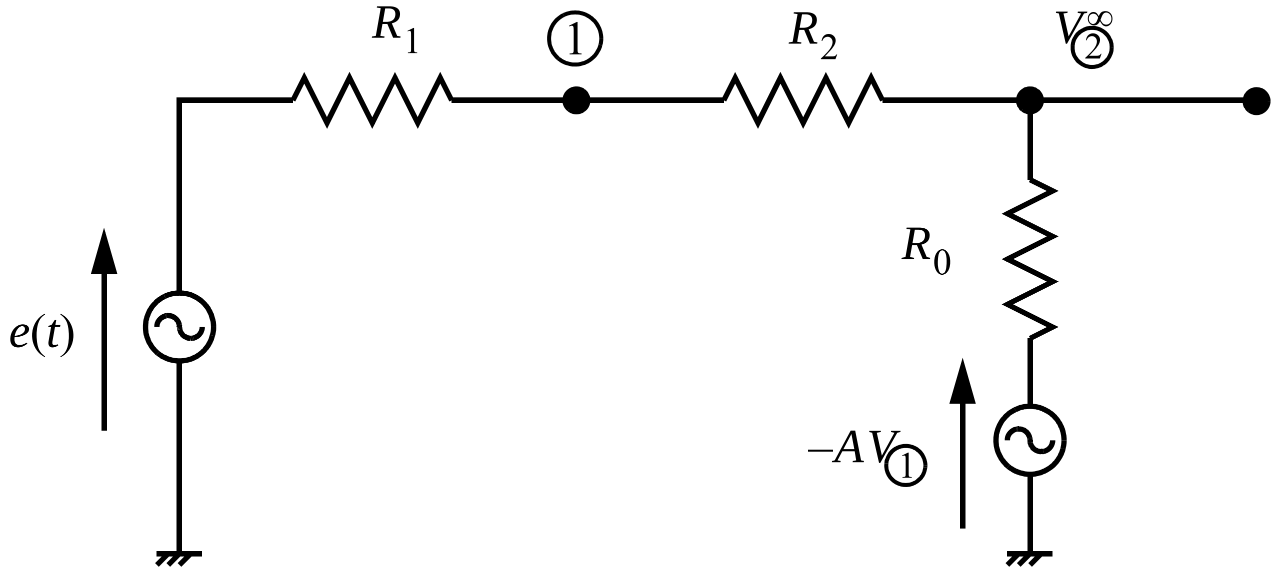


e(t)
R1
2
3
R2
R3
Rb
e(t)
R4
0

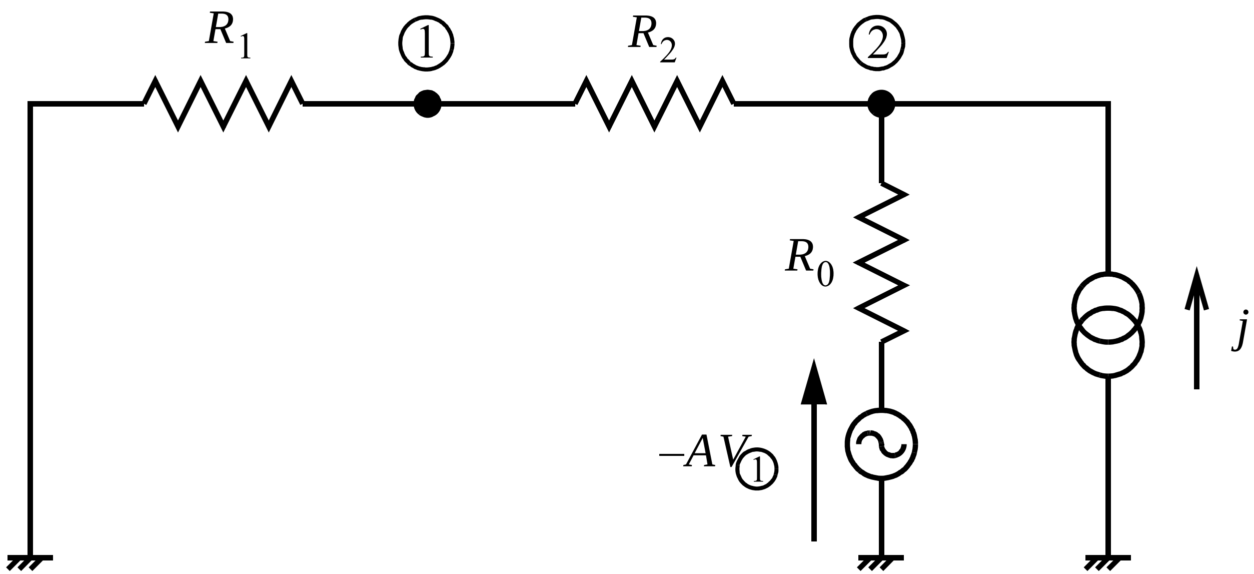
Illustratie op een actief netwerk

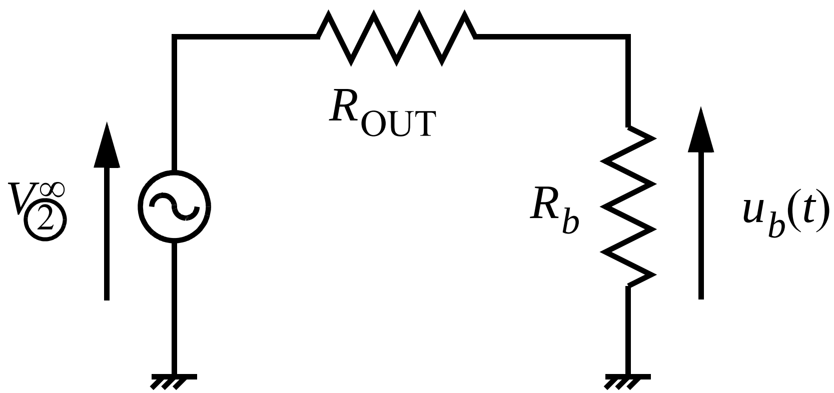



R_1
①
R_2
②
R_0
R_b
$e(t)$
$-AV_{①}$

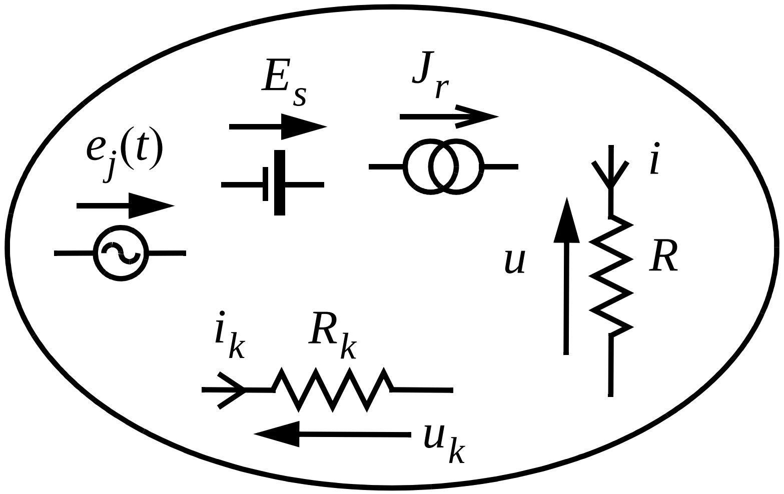


R1
①
R2
V∞
②
R0
e(t)
–AV①

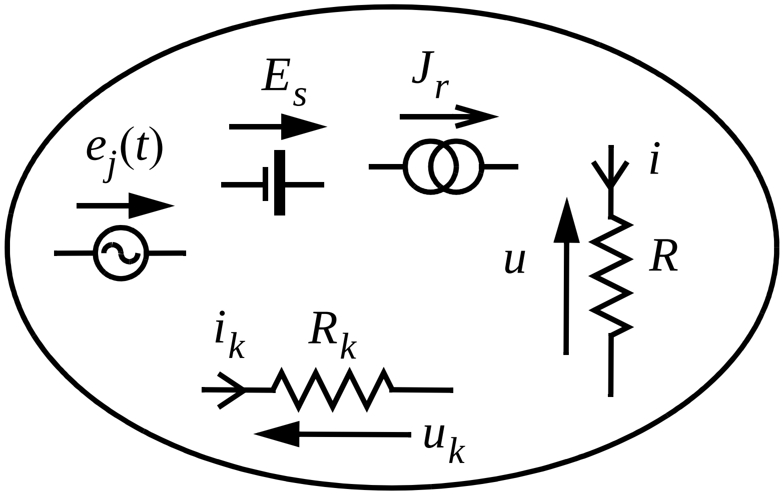

R1
1
R2
2
R0
j
−AV1

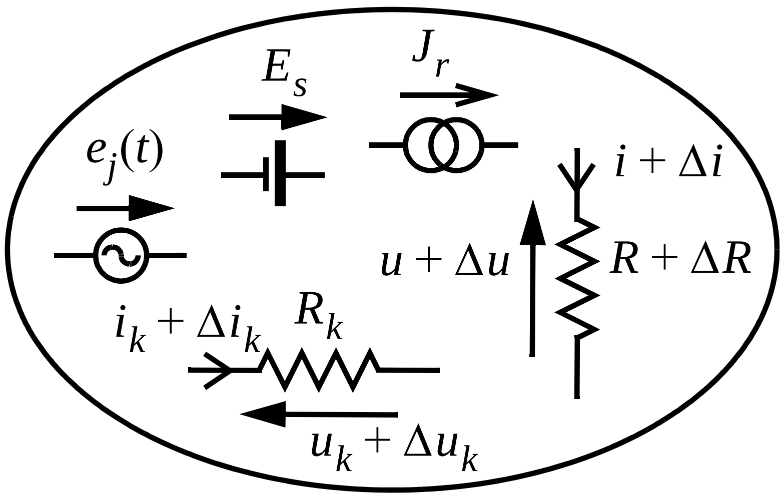

R_{OUT}
$V_{\infty 2}$
R_b
$u_b(t)$

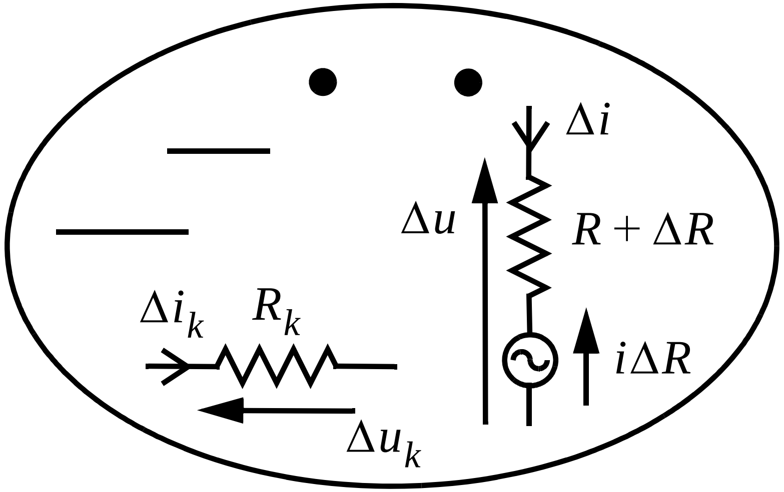
Compensatiestelling



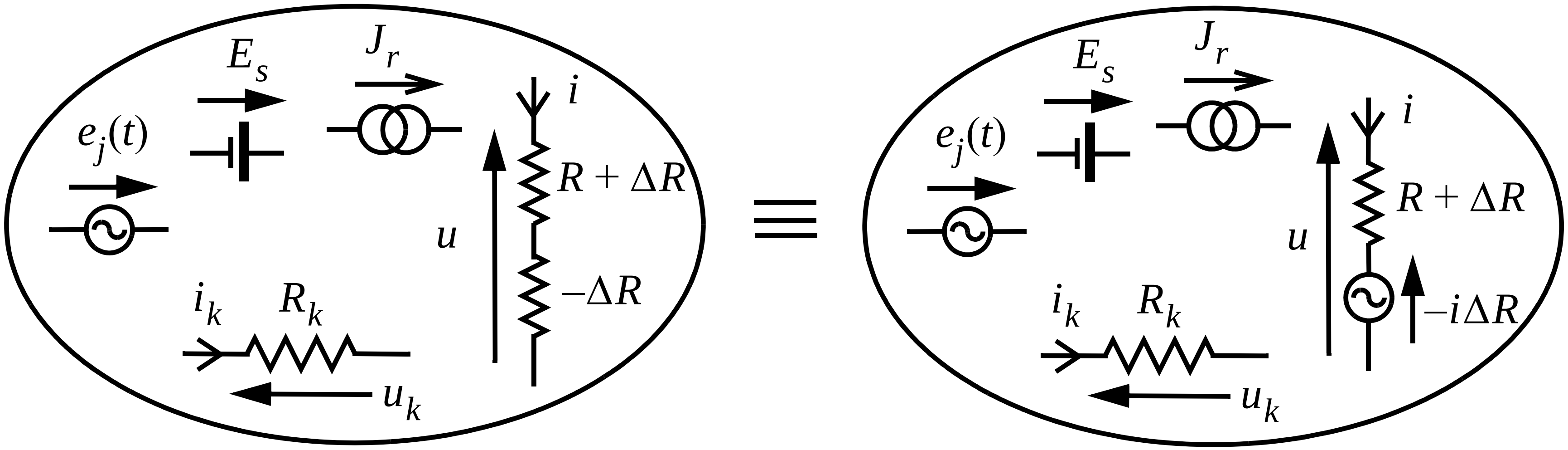
Compensatiestelling





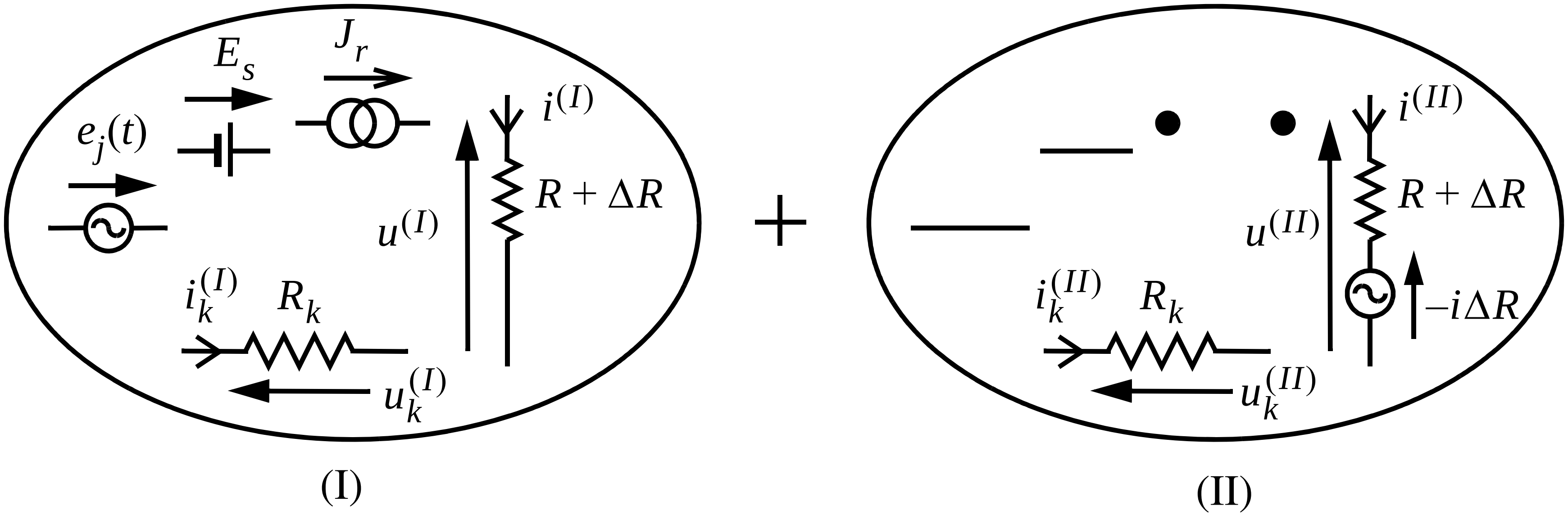


Δi
Δu
$R + \Delta R$
Δi_k
R_k
$i\Delta R$
Δu_k



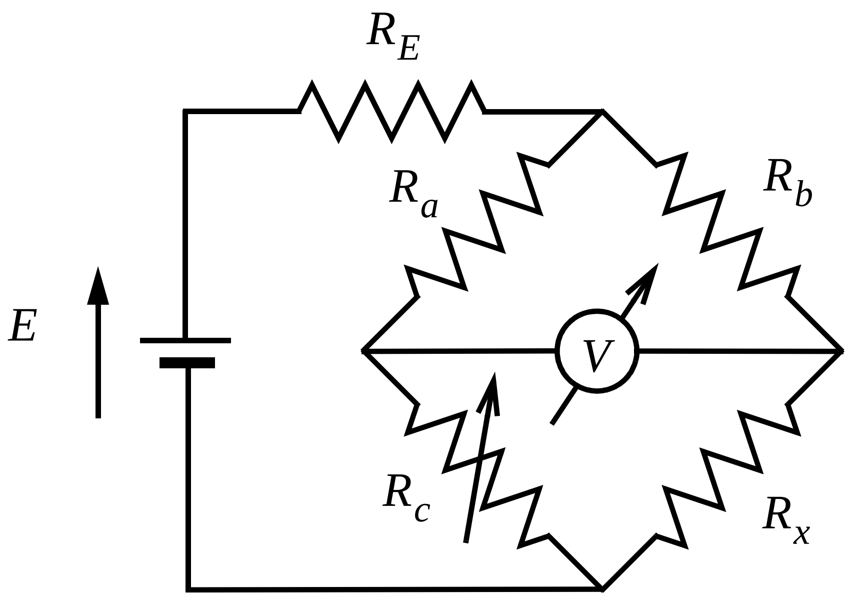
E_s
J_r
$e_j(t)$
i
$R + \Delta R$
u
$-\Delta R$
i_k
R_k
u_k
$\equiv$
E_s
J_r
$e_j(t)$
i
$R + \Delta R$
u
$-i\Delta R$
i_k
R_k
u_k

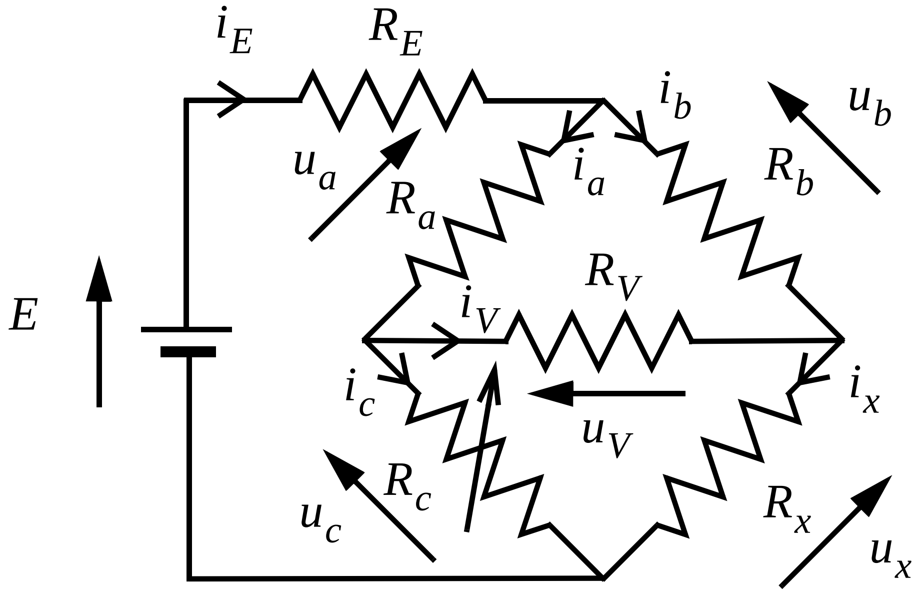
(a) (b)



$e_j(t)$
E_s
J_r
$i^{(I)}$
$R + \Delta R$
$u^{(I)}$
$i_k^{(I)}$
R_k
$u_k^{(I)}$
(I)
$i^{(II)}$
$R + \Delta R$
$u^{(II)}$
$i_k^{(II)}$
R_k
$-i\Delta R$
$u_k^{(II)}$
(II)

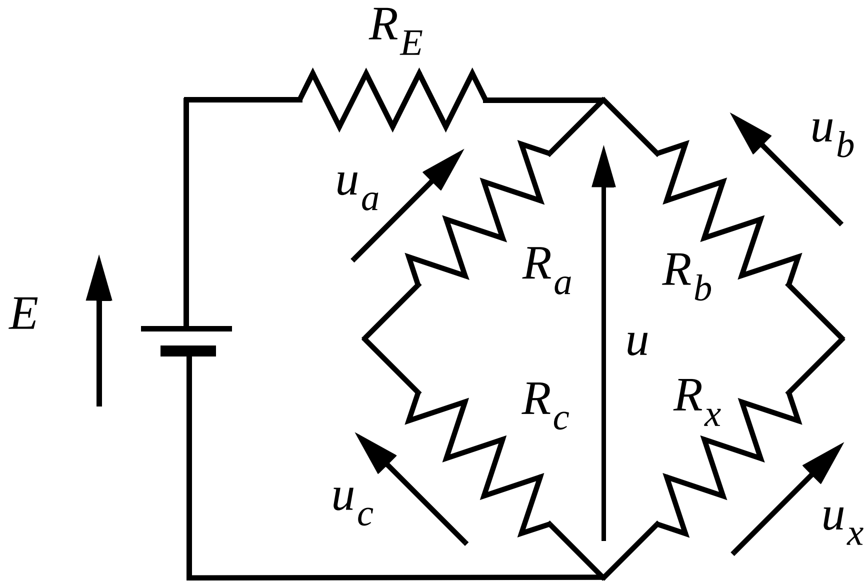
Gevoeligheid brug van Wheatstone rond evenwicht





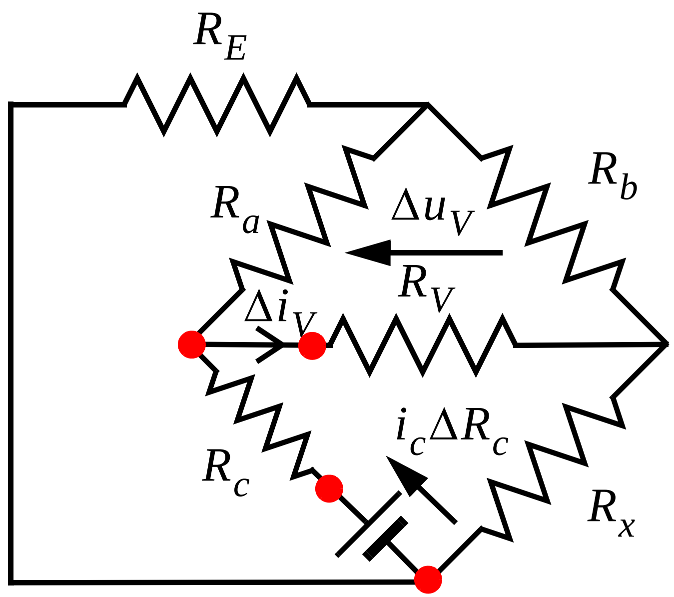
i_E
R_E
i_b
u_b
u_a
i_a
R_a
R_b
E
R_V
i_V
i_c
i_x
u_V
R_c
u_c
R_x
u_x

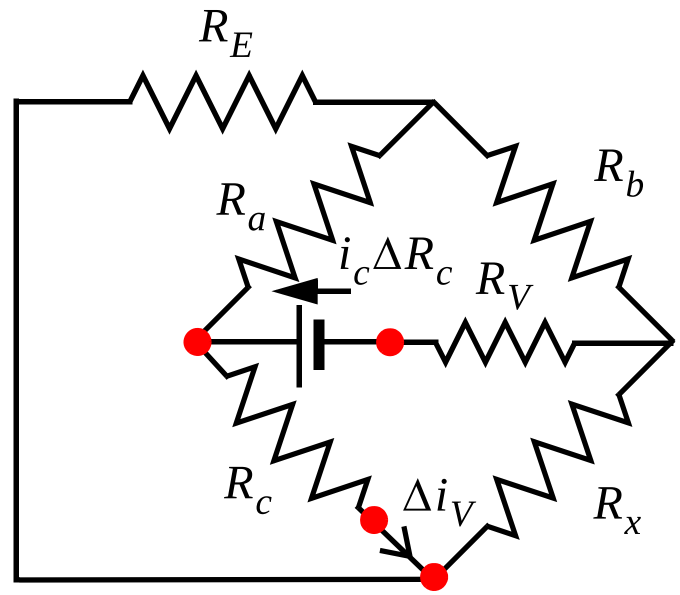
brug in evenwicht: $u_V = 0$ en $i_V = 0$

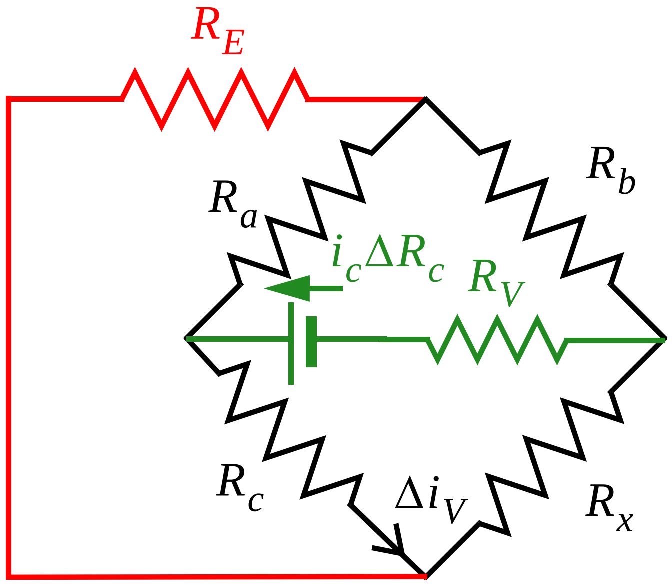


$u_c = u_x$

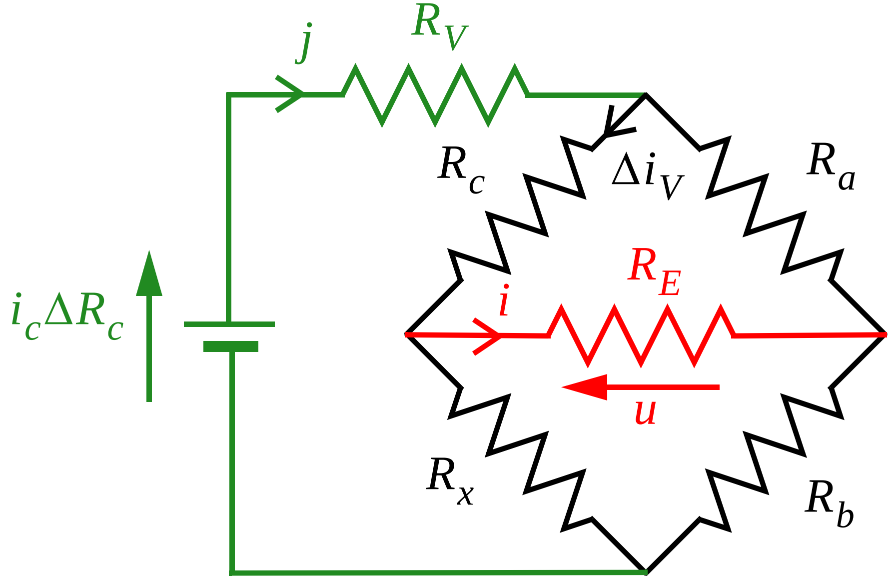
$u_a = u_b$


R_E
R_b
R_a
Δu_V
R_V
Δi_V
$i_c \Delta R_c$
R_c
R_x


R_E
R_b
R_a
$i_c \Delta R_c$
R_V
R_c
Δi_V
R_x



R_E
R_a
R_b
$i_c \Delta R_c$
R_V
R_c
Δi_V
R_x



j
R_V
R_c
Δi_V
R_a
R_E
i
$i_c \Delta R_c$
u
R_x
R_b

Numeriek voorbeeld

$$\Delta u_V = \frac{R_a R_b R_V E}{[(R_a + R_c)R_V + (R_a + R_b)R_c][(R_a + R_b)R_E + (R_a + R_c)R_b]}\Delta R_c$$

R_V » $\max(R_a, R_b, R_c)$

$$\Delta u_V \approx \frac{R_a R_b E}{(R_a + R_c)[(R_a + R_b)R_E + (R_a + R_c)R_b]}\Delta R_c$$

$R_a = 1\ \text{k}\Omega$

$R_b = 200\ \Omega$

$R_c = 3{,}05\ \text{k}\Omega$

$R_x = 610\ \Omega$

$R_E = 50\ \Omega$

$R_V = 100\ \text{M}\Omega$

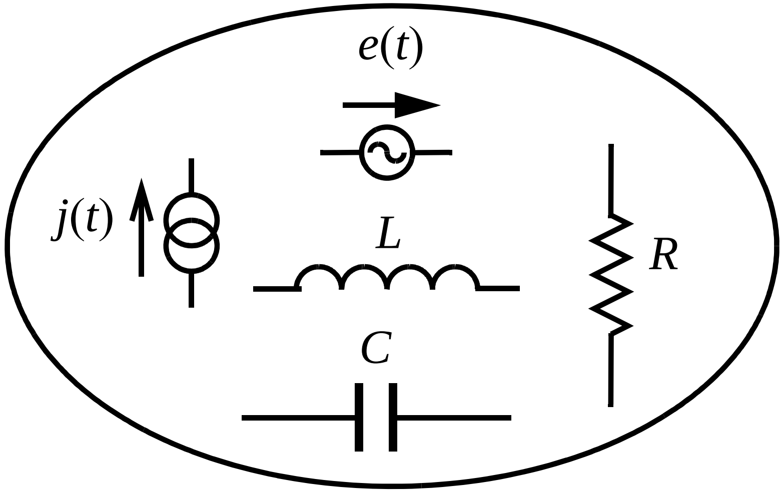
$E = 18\ \text{V}$

$\Delta R_x / R_x = 10^{-3} \Rightarrow \Delta R_c \approx 3\ \Omega$

$\Delta u_V \approx 1\ \text{mV}/\Omega\ \ \Delta R_c$

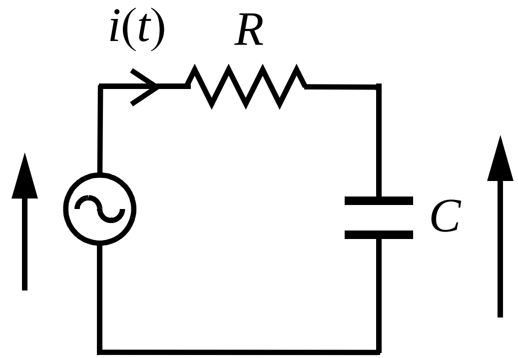
$\Rightarrow \Delta u_V \approx 3\ \text{mV}$

RLC-netwerken

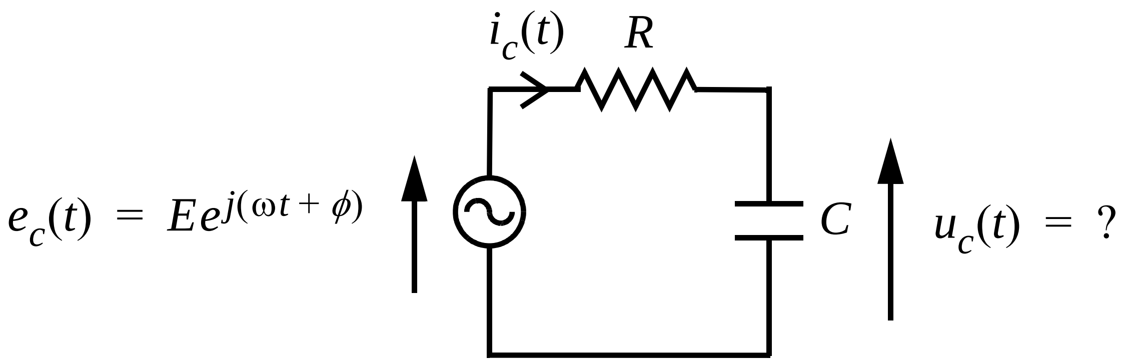


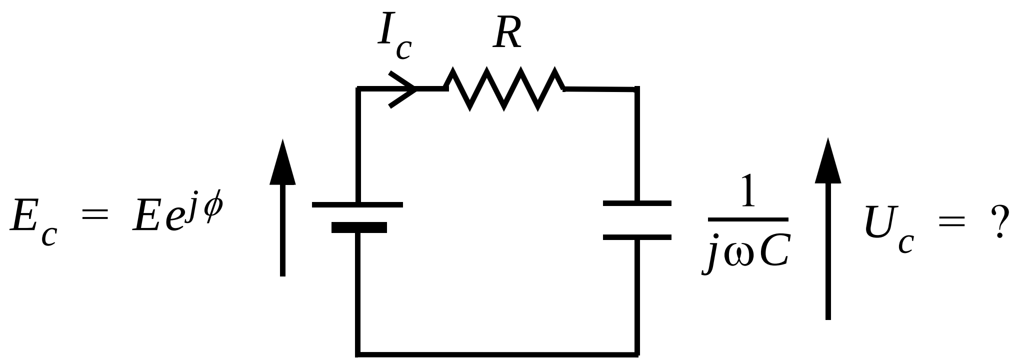
Éénvoudig voorbeeld

$e(t) = E\cos(\omega t + \phi)$



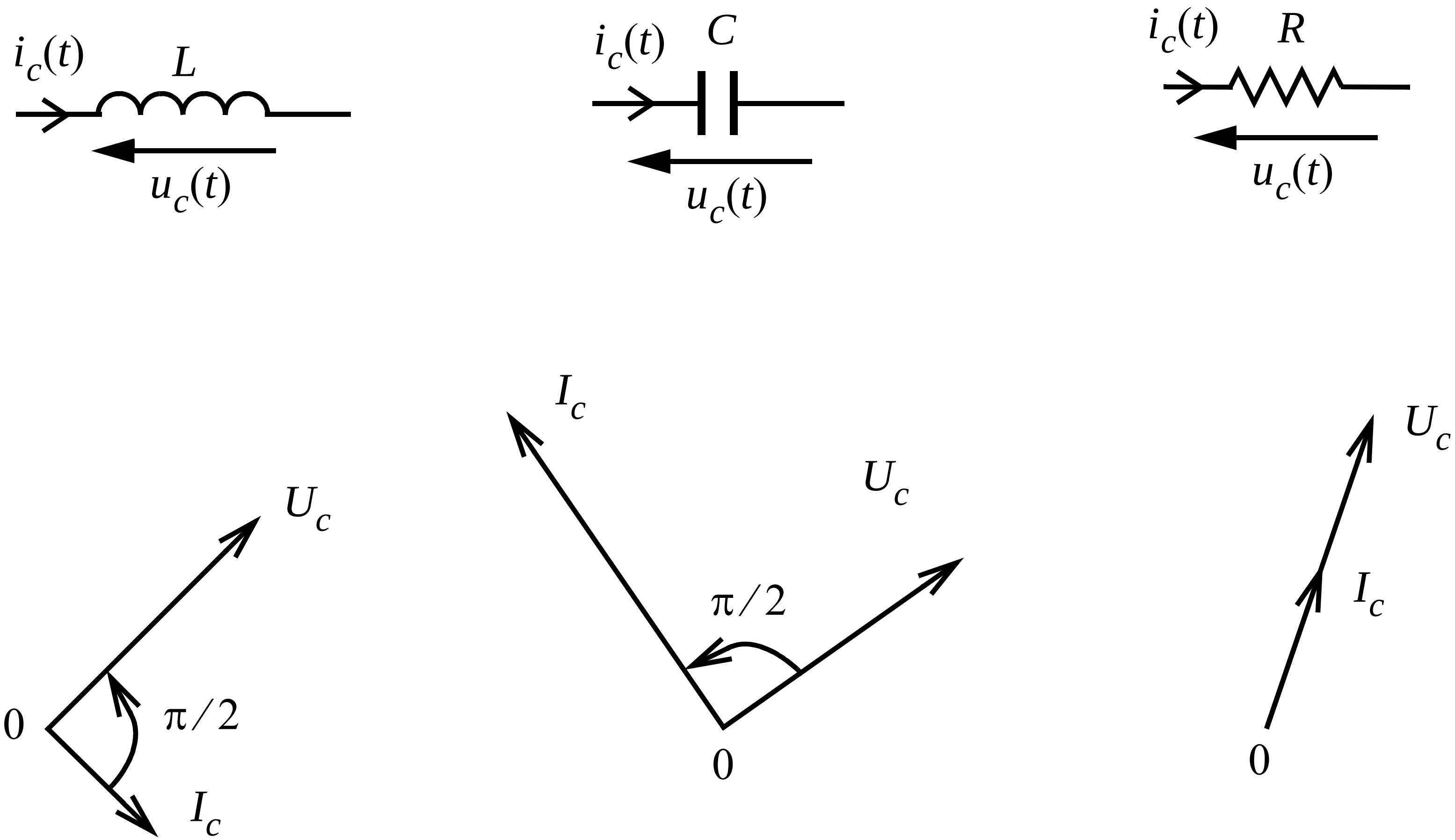
$u(t) = ?$


$e_c(t) = Ee^{j(\omega t + \phi)}$
$i_c(t)$
R
C
$u_c(t) = ?$

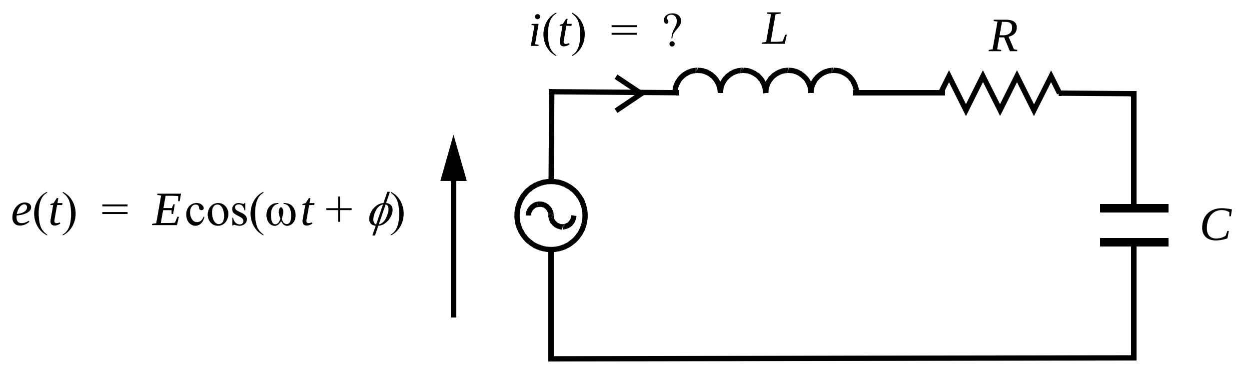


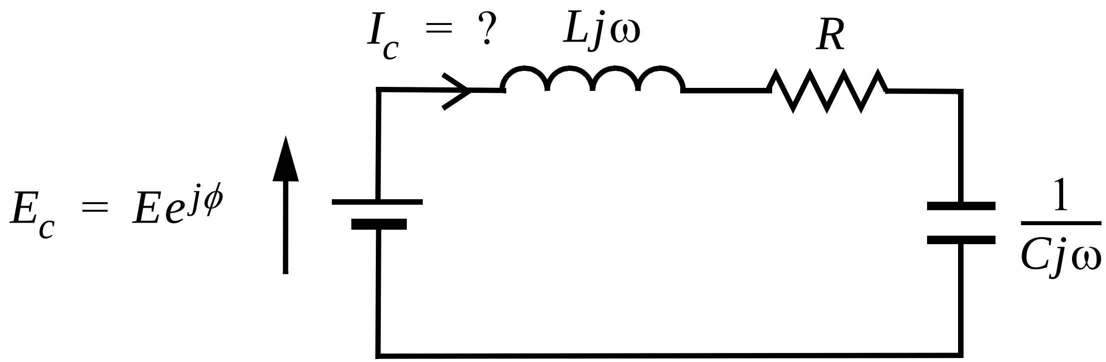
I_c
R
$E_c = Ee^{j\phi}$
$\frac{1}{j\omega C}$
$U_c = ?$

Impedantie onder sinusoïdaal regime



Voorbeeld 1

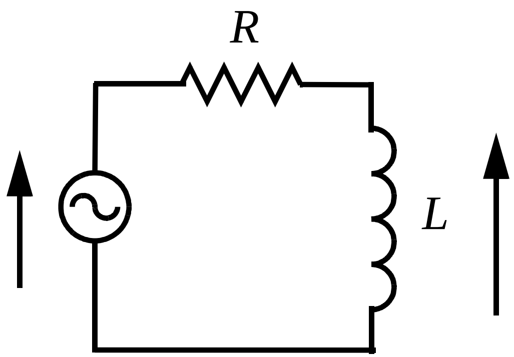




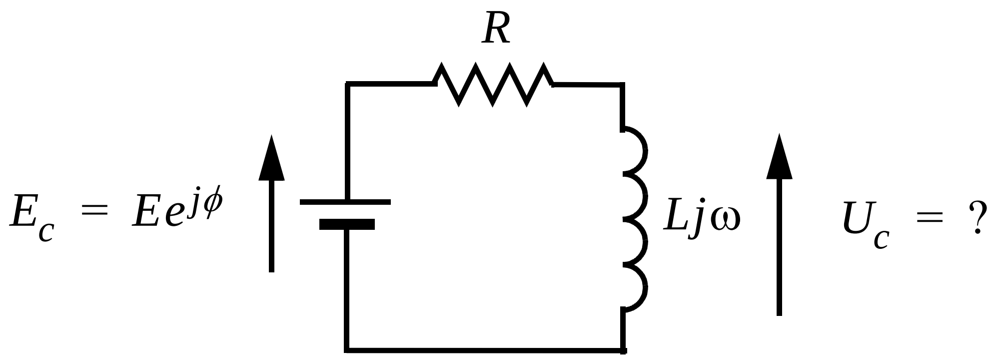
$I_c = ?$
$Lj\omega$
R
$E_c = Ee^{j\phi}$
$\frac{1}{Cj\omega}$

Voorbeeld 2

$e(t) = E\cos(\omega t + \phi)$

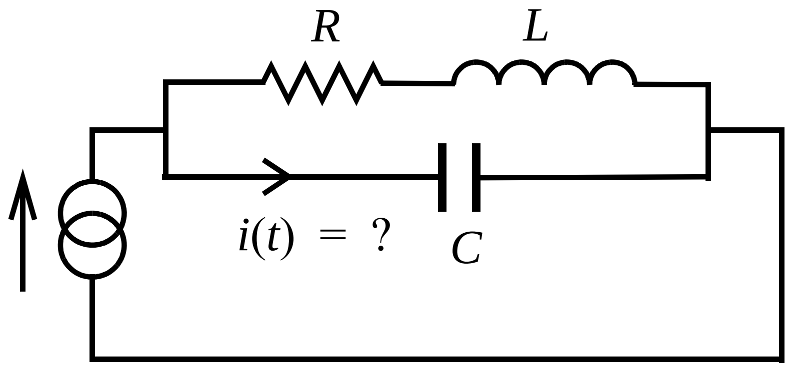


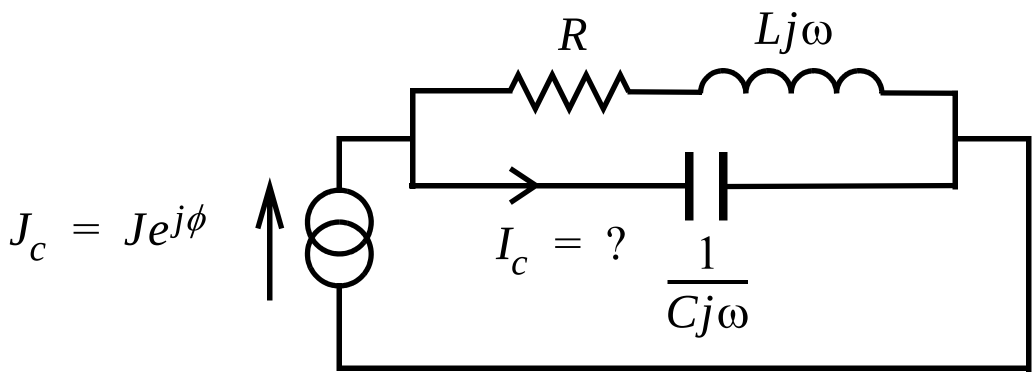
$u(t) = ?$


R
Ec = Eejϕ
Ljω
Uc = ?

Voorbeeld 3

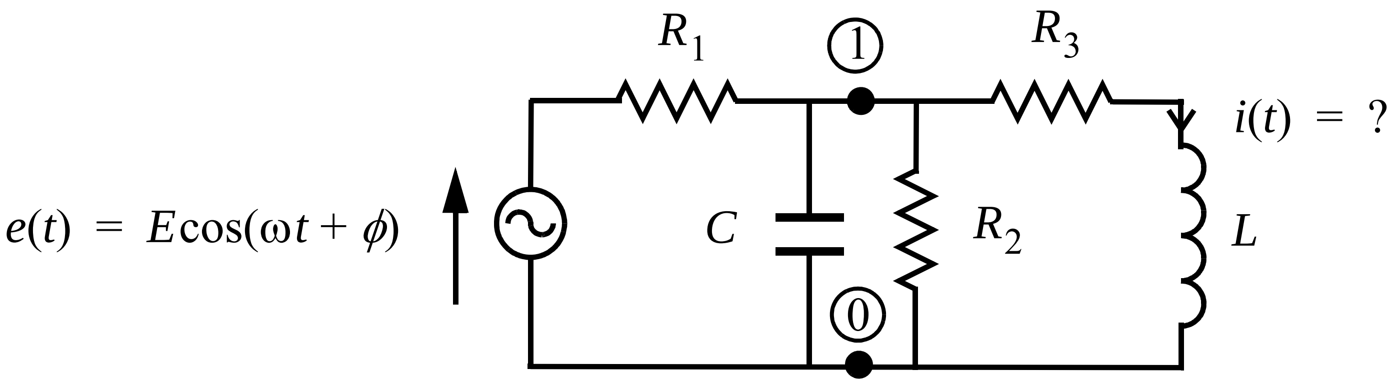
$$j(t) = J\sin(\omega t + \phi)$$

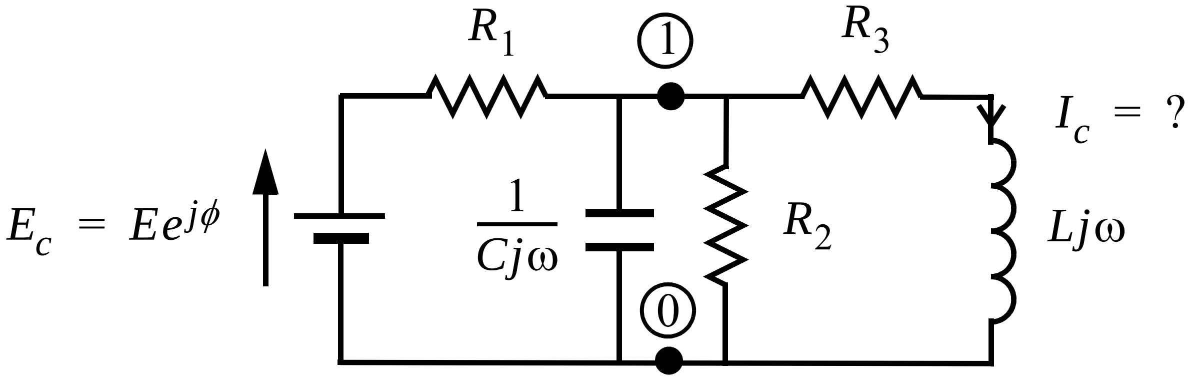




$J_c = Je^{j\phi}$
R
$Lj\omega$
$I_c = ?$
$\frac{1}{Cj\omega}$

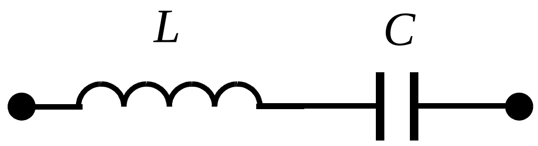
Voorbeeld 4

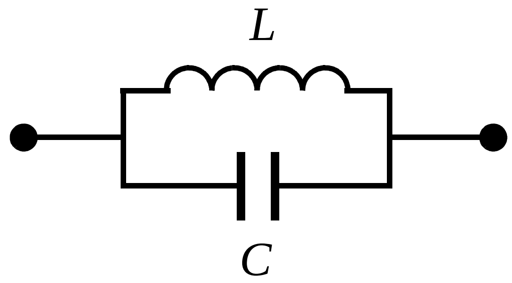


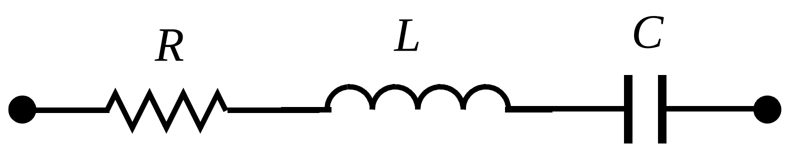


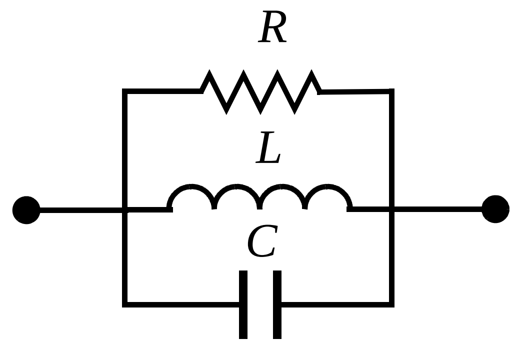
R_1
1
R_3
$I_c = ?$
$E_c = Ee^{j\phi}$
$\frac{1}{Cj\omega}$
R_2
$Lj\omega$
0

Resonantiekringen

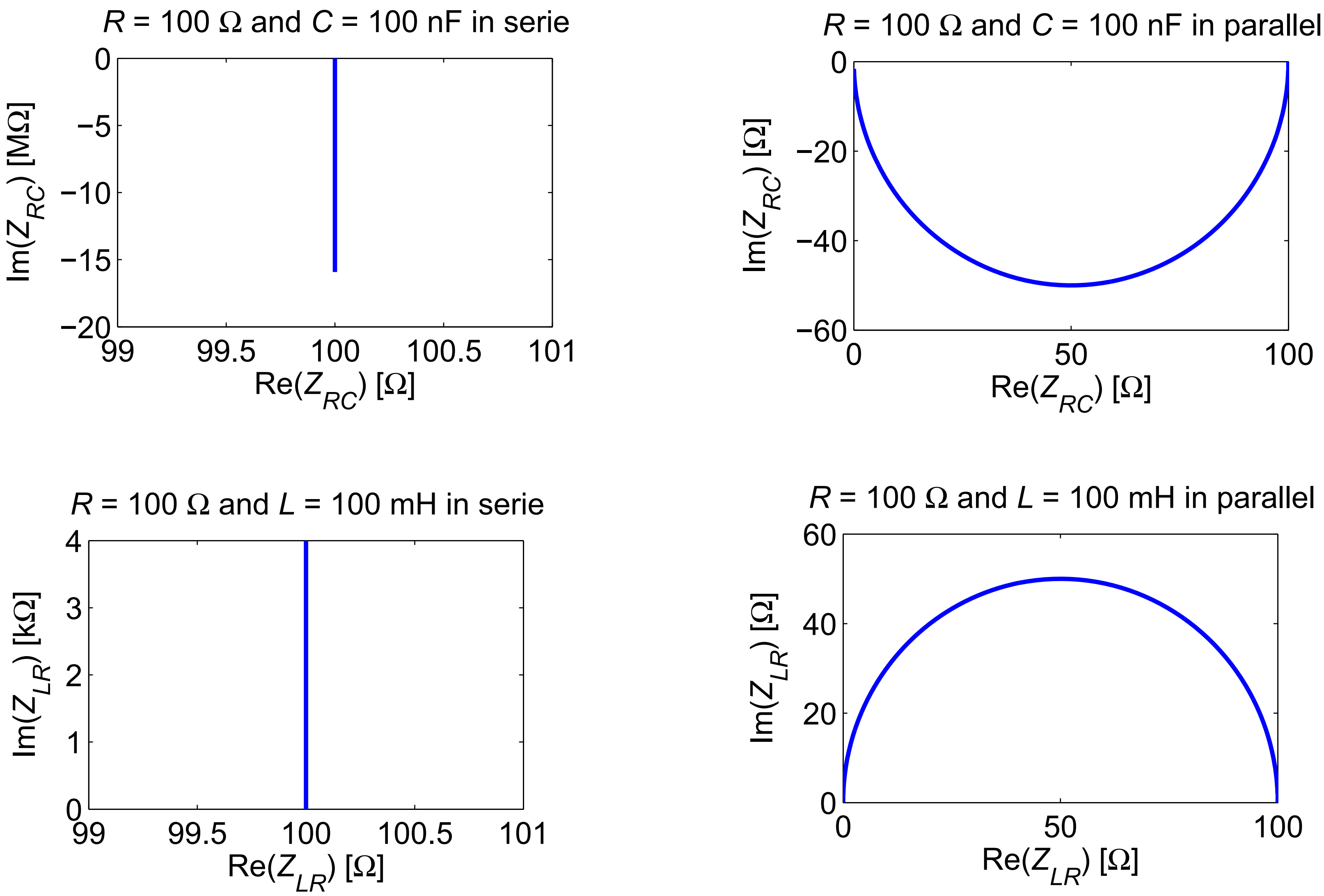


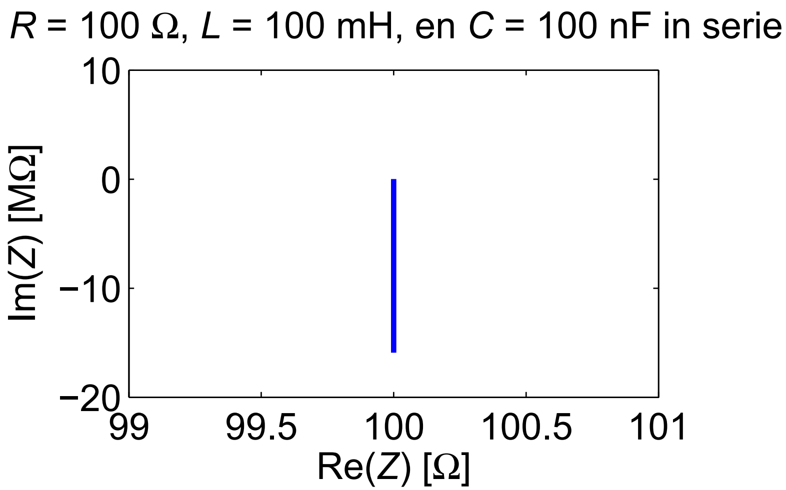


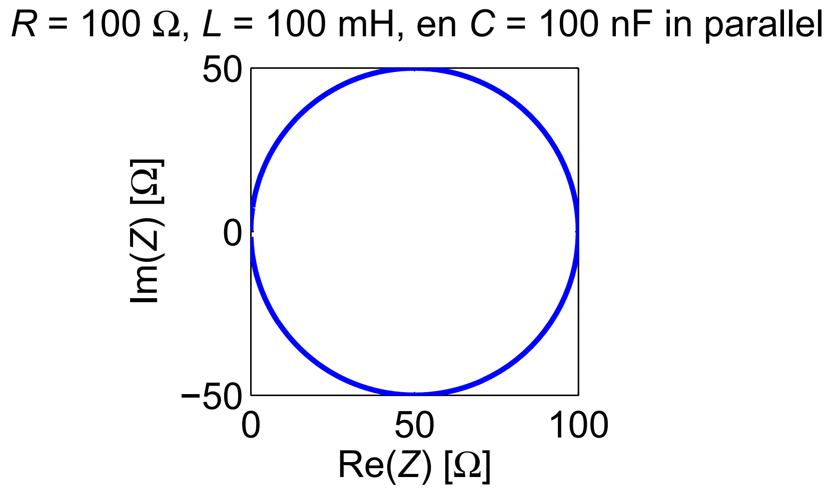


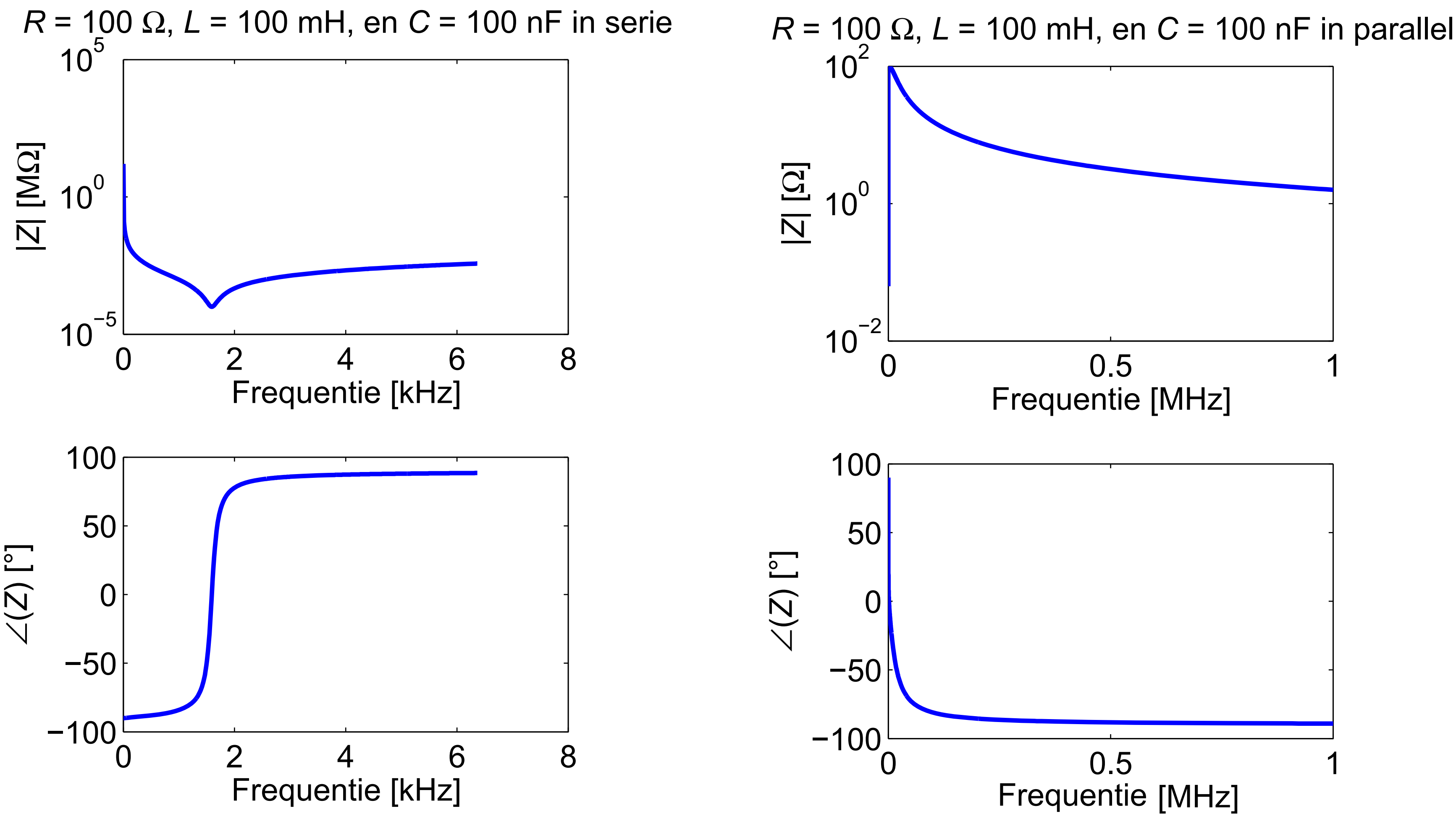


Voorstelling impedanties in complexe vlak



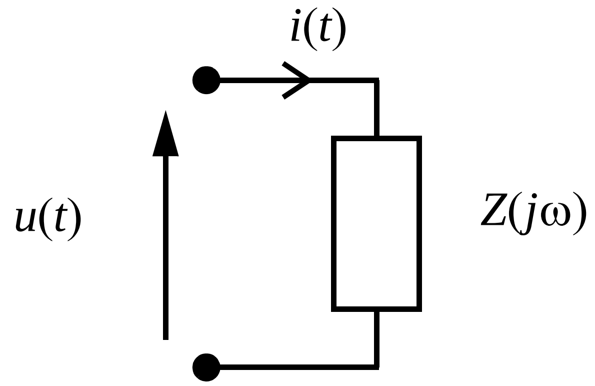

R = 100 Ω, L = 100 mH, en C = 100 nF in serie
10
0
−10
−20
99
99.5
100
100.5
101
Re(Z) [Ω]
Im(Z) [MΩ]


R = 100 Ω, L = 100 mH, en C = 100 nF in parallel
50
0
−50
0
50
100
Re(Z) [Ω]
Im(Z) [Ω]

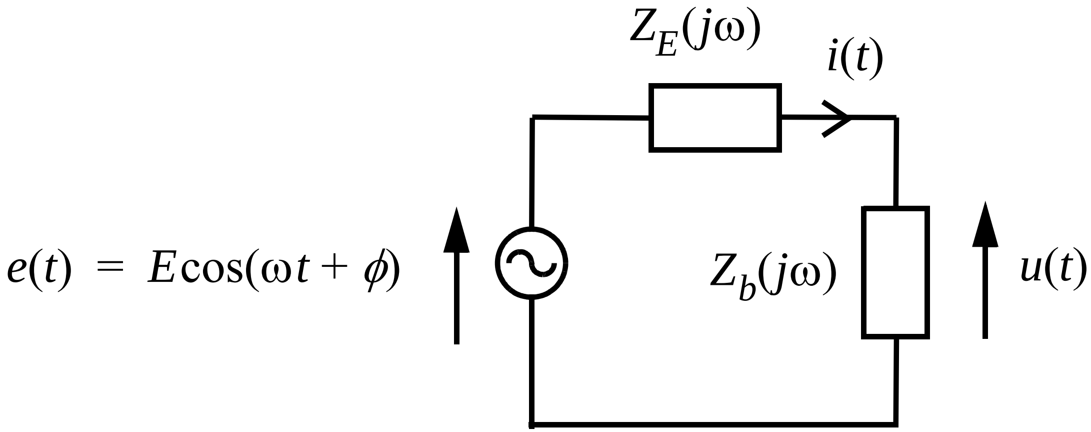


R = 100 Ω, L = 100 mH, en C = 100 nF in serie
|Z| [MΩ]
10^5
10^0
10^−5
0
2
4
6
8
Frequentie [kHz]
∠(Z) [°]
100
50
0
−50
−100
0
2
4
6
8
Frequentie [kHz]
R = 100 Ω, L = 100 mH, en C = 100 nF in parallel
|Z| [Ω]
10^2
10^0
10^−2
0
0.5
1
Frequentie [MHz]
∠(Z) [°]
100
50
0
−50
−100
0
0.5
1
Frequentie [MHz]

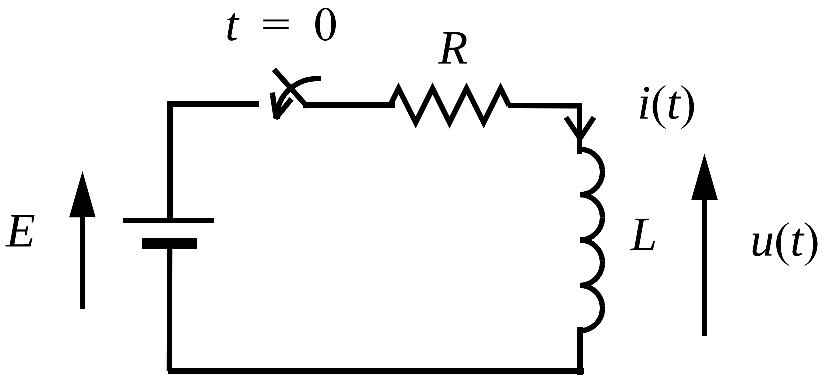
Gemiddeld vermogen onder sinusoïdaal regime



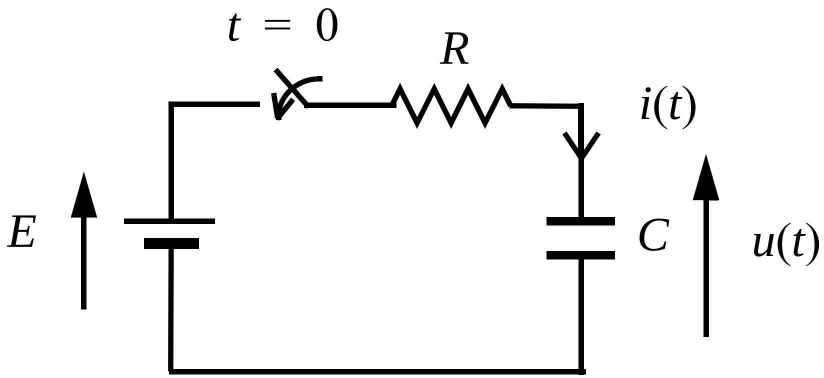
Maximale vermogenoverdracht



Overgangsverschijnselen - tijdsdomein

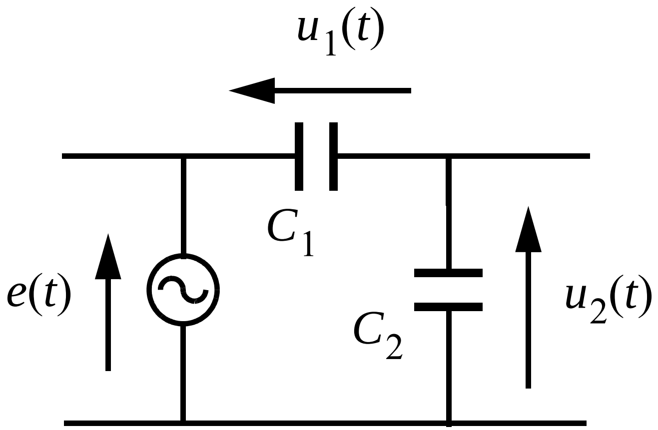


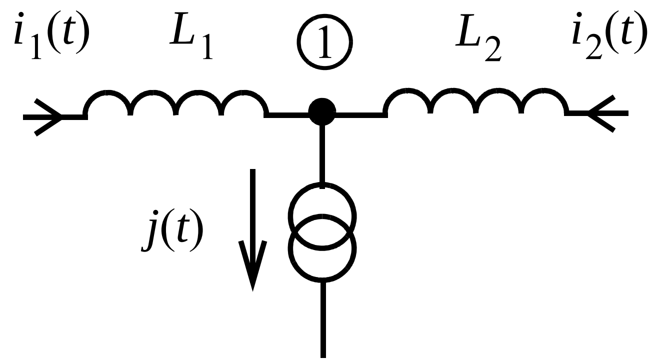
$i(0) = 0$



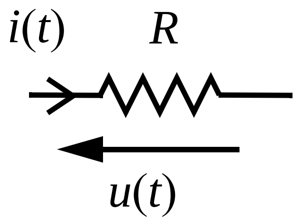
$u(0) \neq 0$

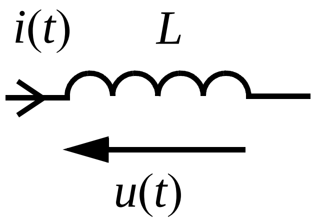
Bepaling van de orde

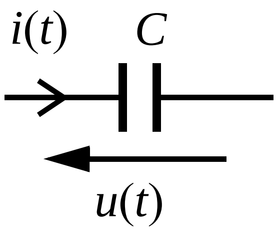




Overgangsverschijnselen - Laplace domein







$$u(t) = Ri(t)$$

$$\Downarrow$$

$$U(p) = RI(p)$$

$$u(t) = L\frac{di(t)}{dt}$$

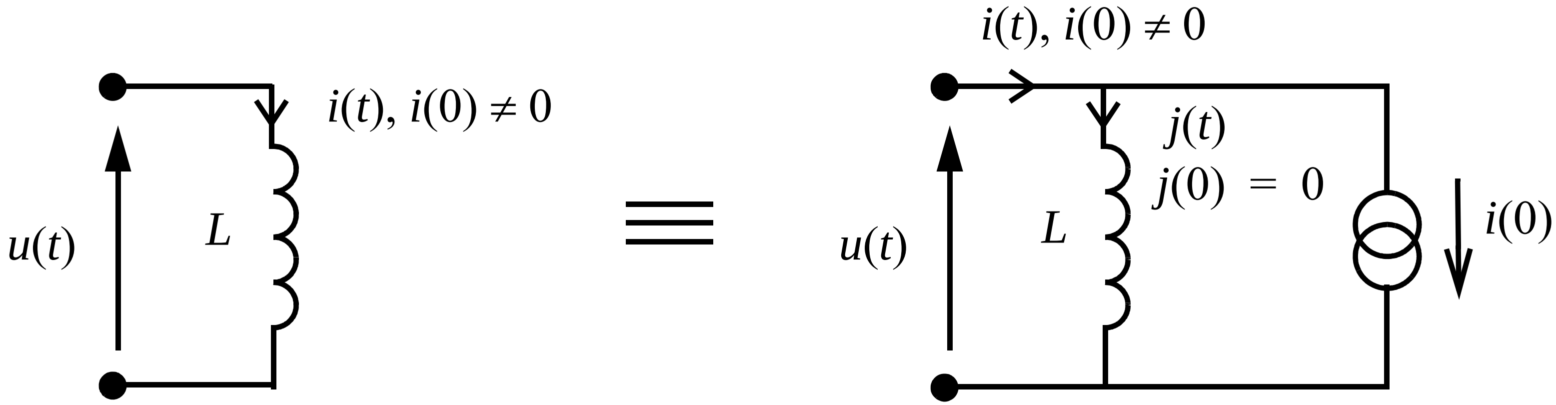
$$\Downarrow$$

$$U(p) = LpI(p) - Li(0)$$

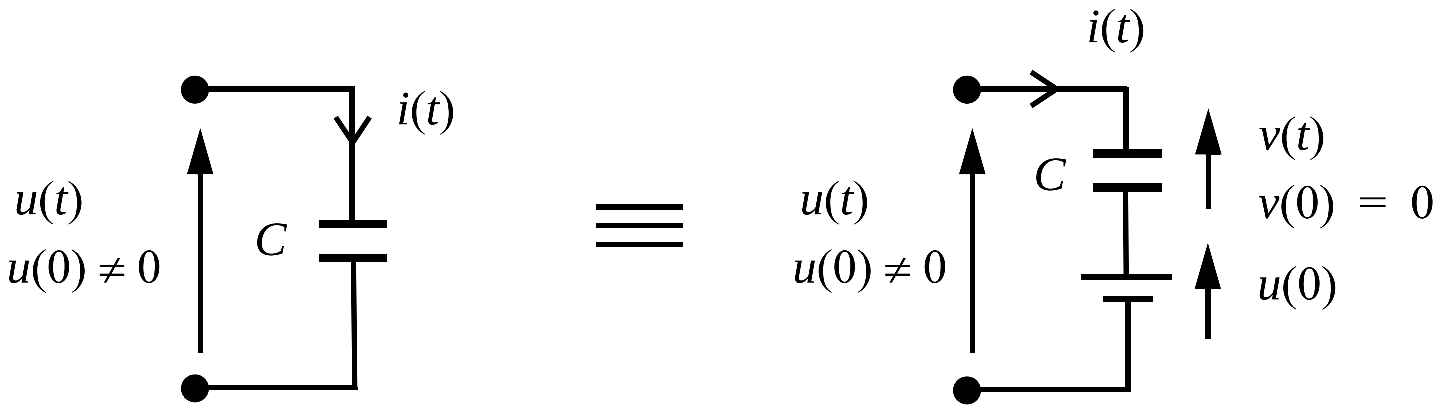
$$i(t) = C\frac{du(t)}{dt}$$

$$\Downarrow$$

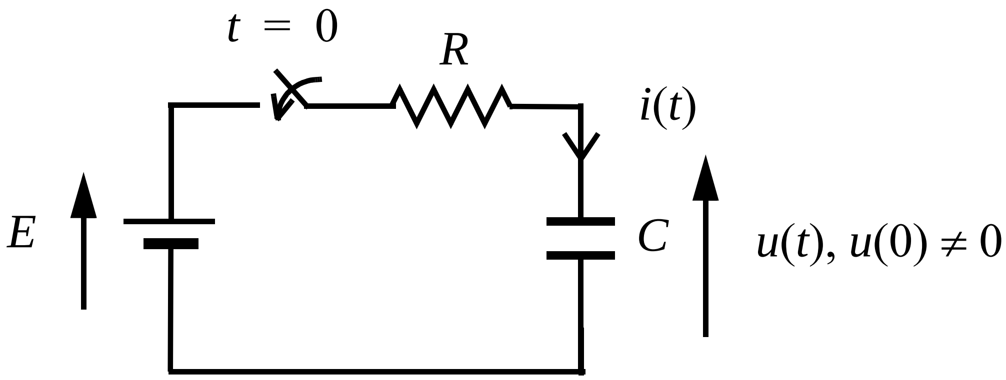
$$U(p) = \frac{1}{Cp}I(p) + \frac{u(0)}{p}$$



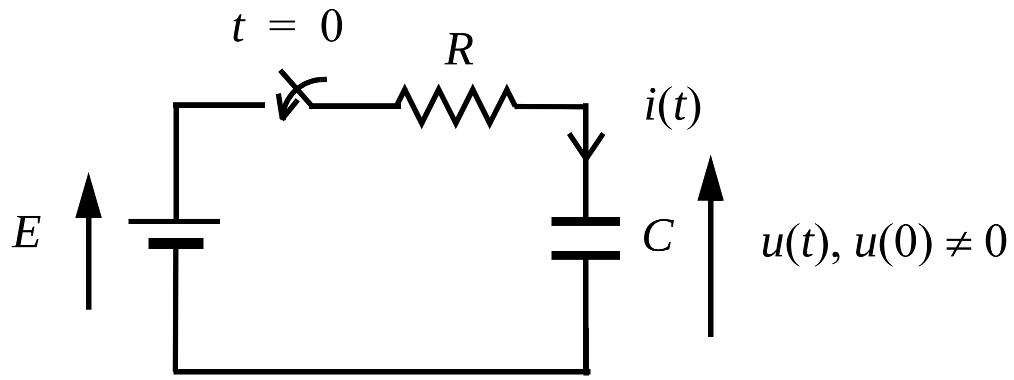
i(t), i(0) ≠ 0
u(t)
L
≡
i(t), i(0) ≠ 0
j(t)
j(0) = 0
u(t)
L
i(0)

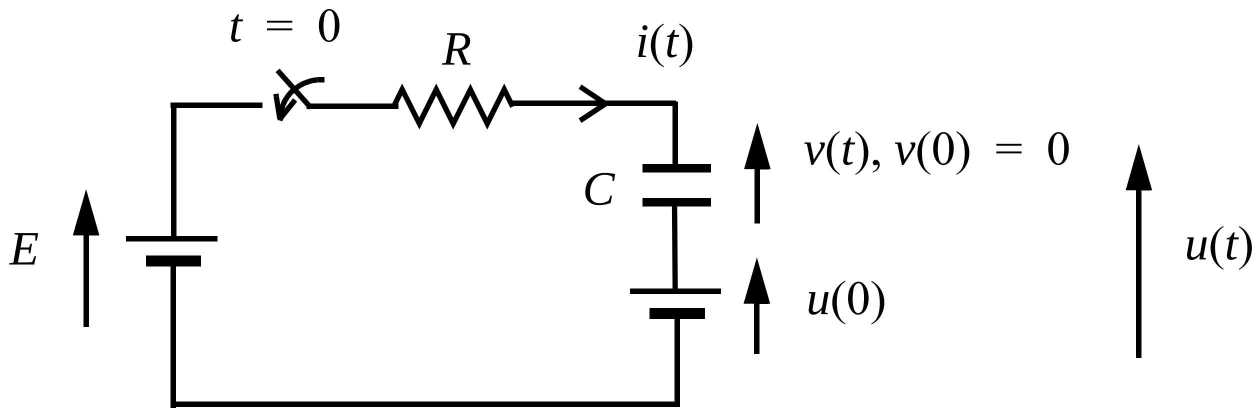

u(t)
u(0) ≠ 0
C
i(t)
≡
i(t)
u(t)
u(0) ≠ 0
C
v(t)
v(0) = 0
u(0)

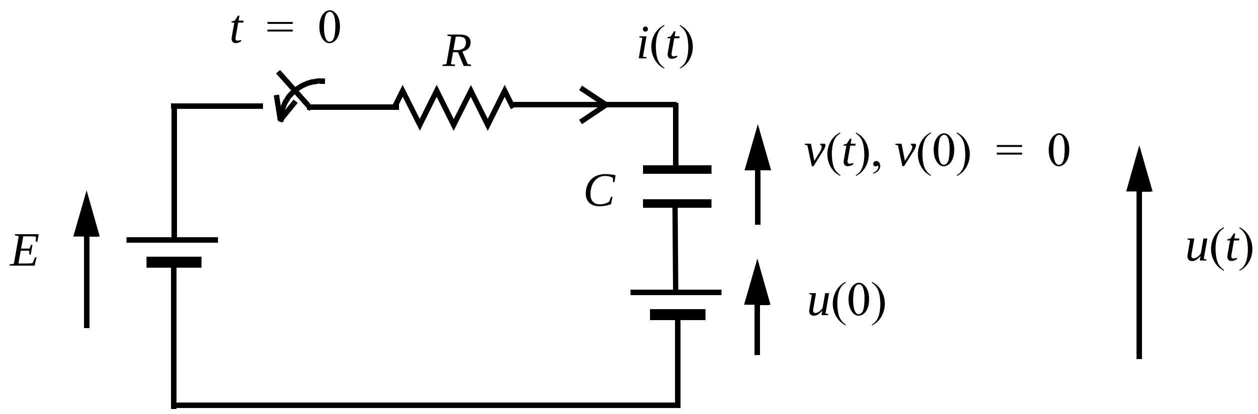
Voorbeeld 1



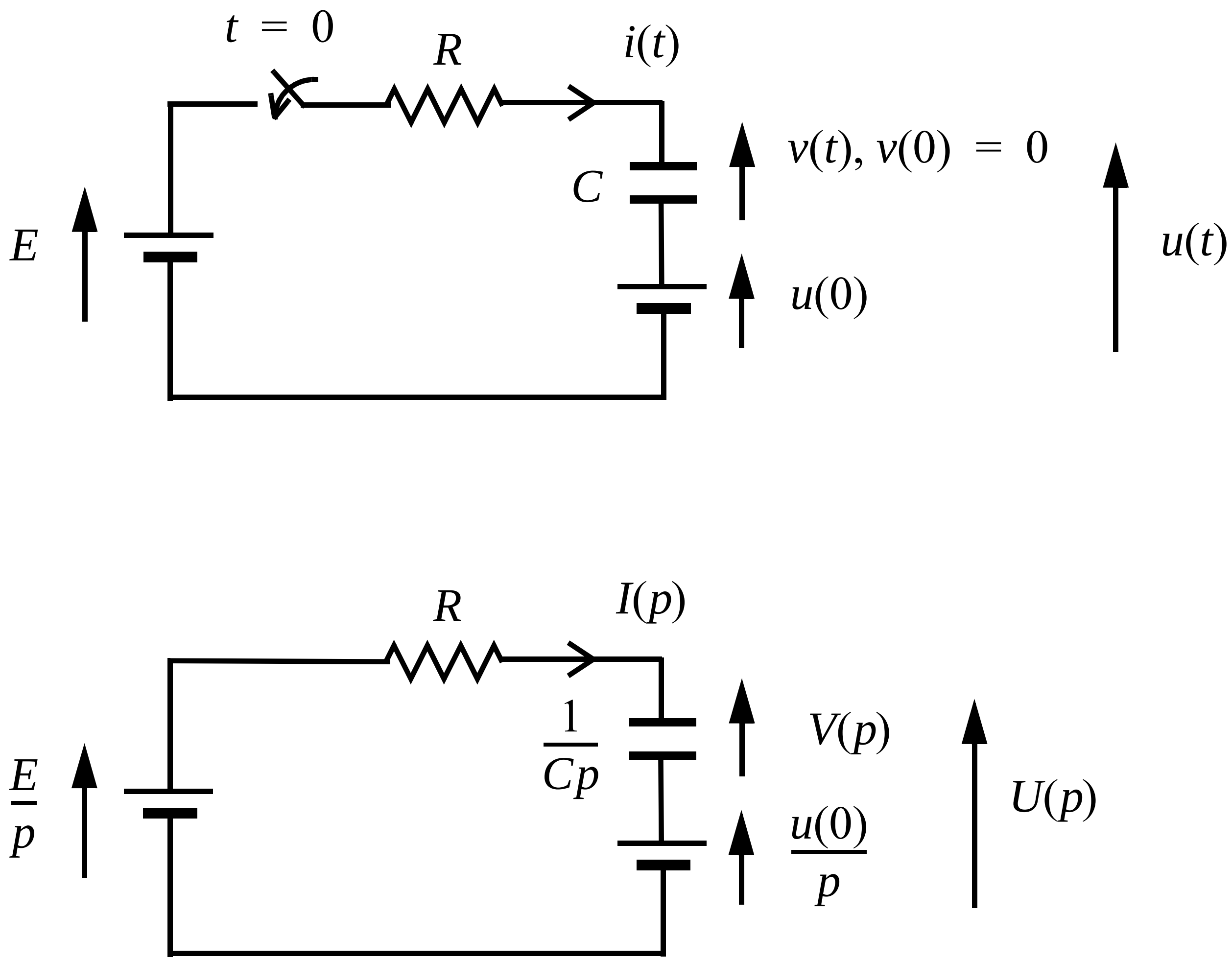
Voorbeeld 1





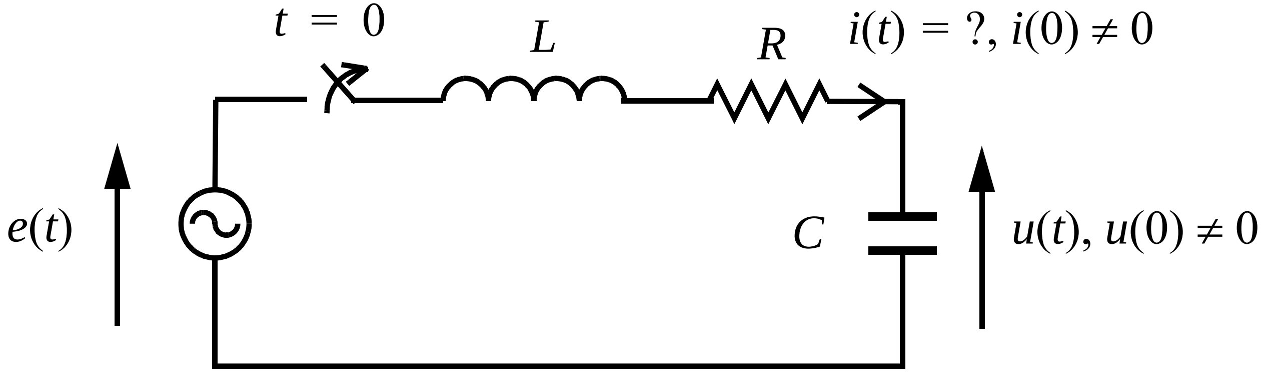


$t = 0$
R
$i(t)$
$v(t), v(0) = 0$
C
E
$u(t)$
$u(0)$

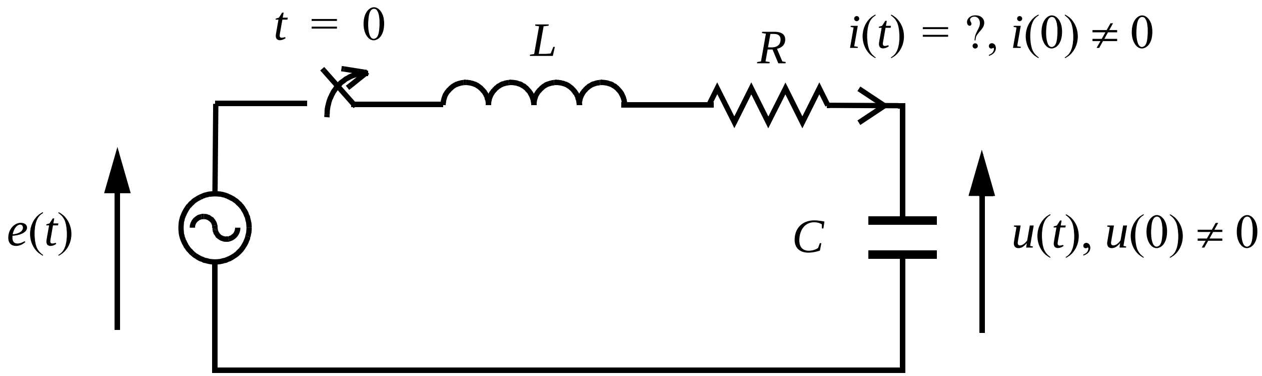


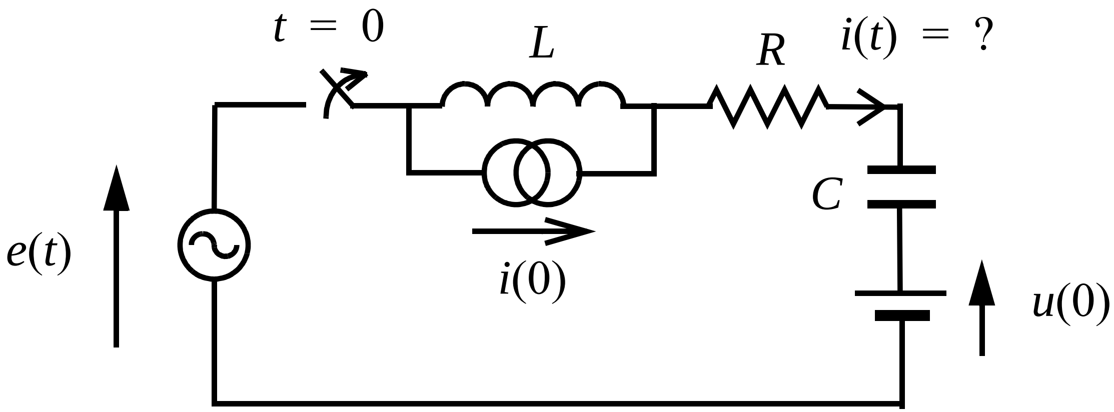
t = 0
R
i(t)
v(t), v(0) = 0
C
E
u(0)
u(t)
R
I(p)
1/Cp
V(p)
E/p
u(0)/p
U(p)

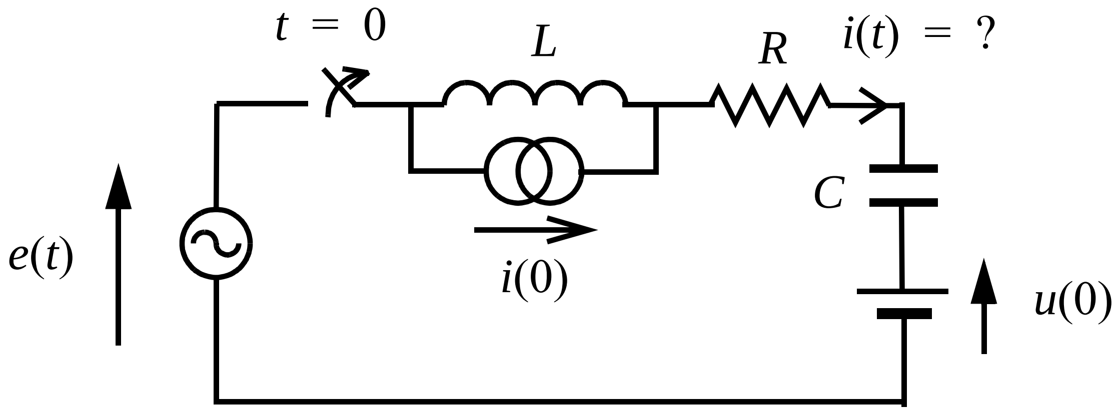
Voorbeeld 2



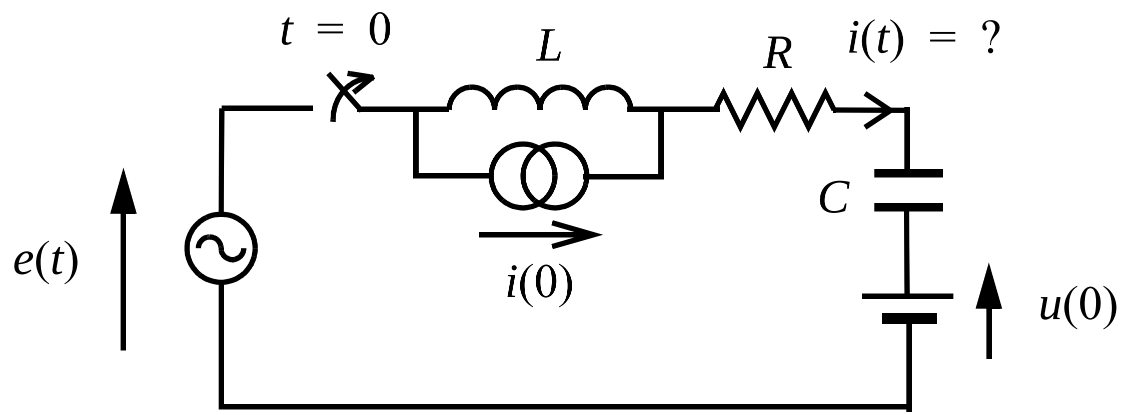
Voorbeeld 2

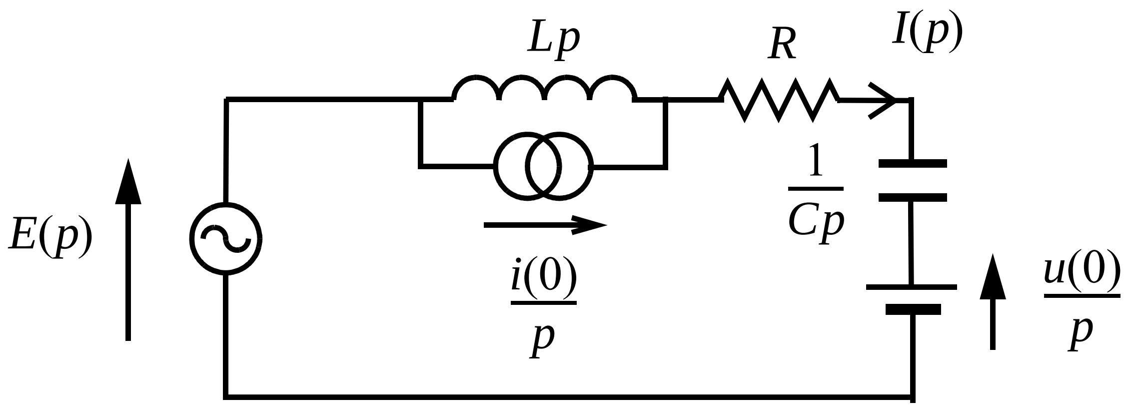


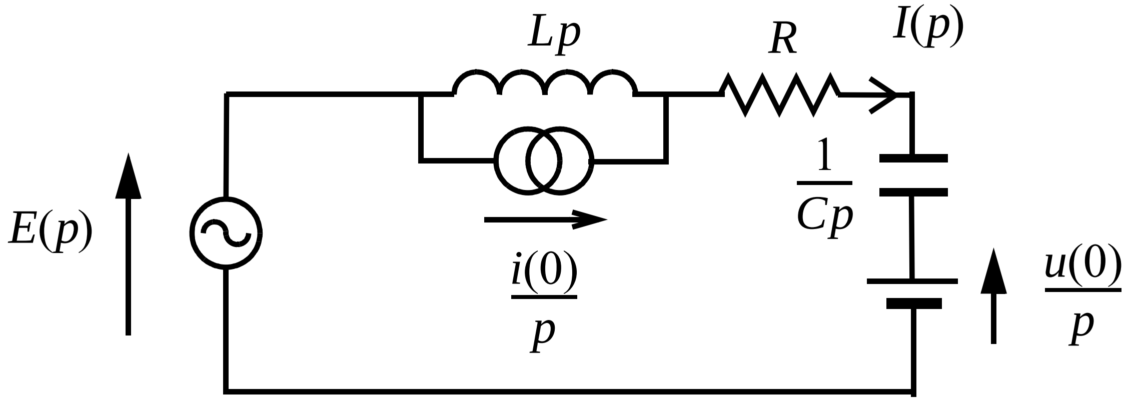




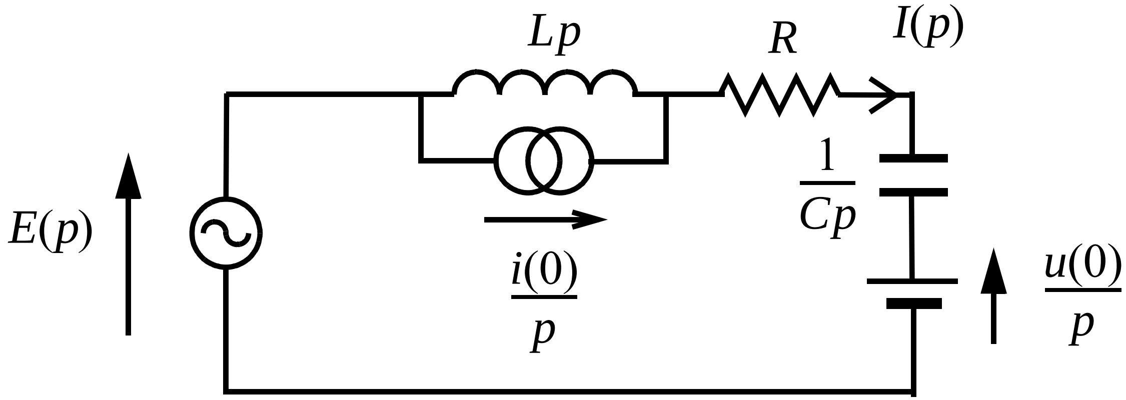
t = 0
L
R
i(t) = ?
C
e(t)
i(0)
u(0)

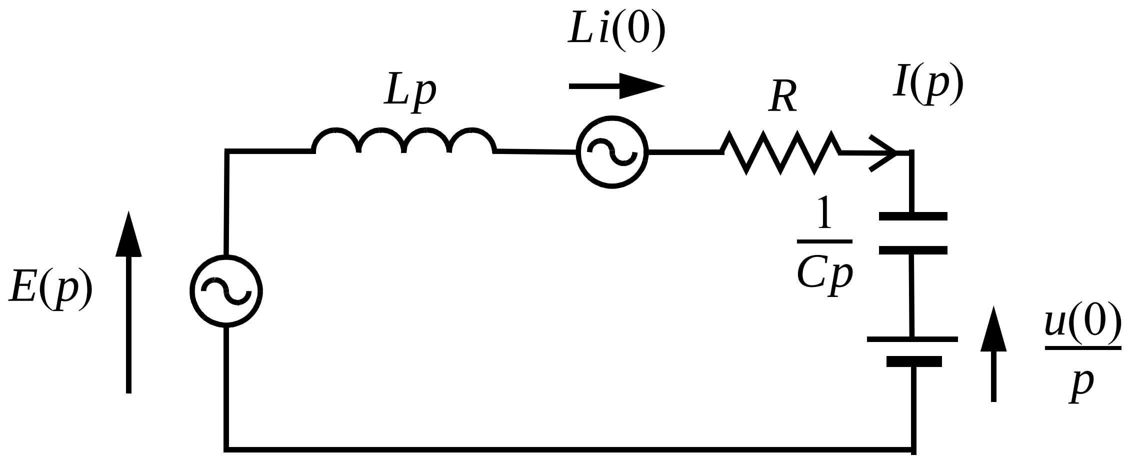

t = 0
L
R
i(t) = ?
C
e(t)
i(0)
u(0)


Lp
R
I(p)
1/Cp
E(p)
i(0)/p
u(0)/p

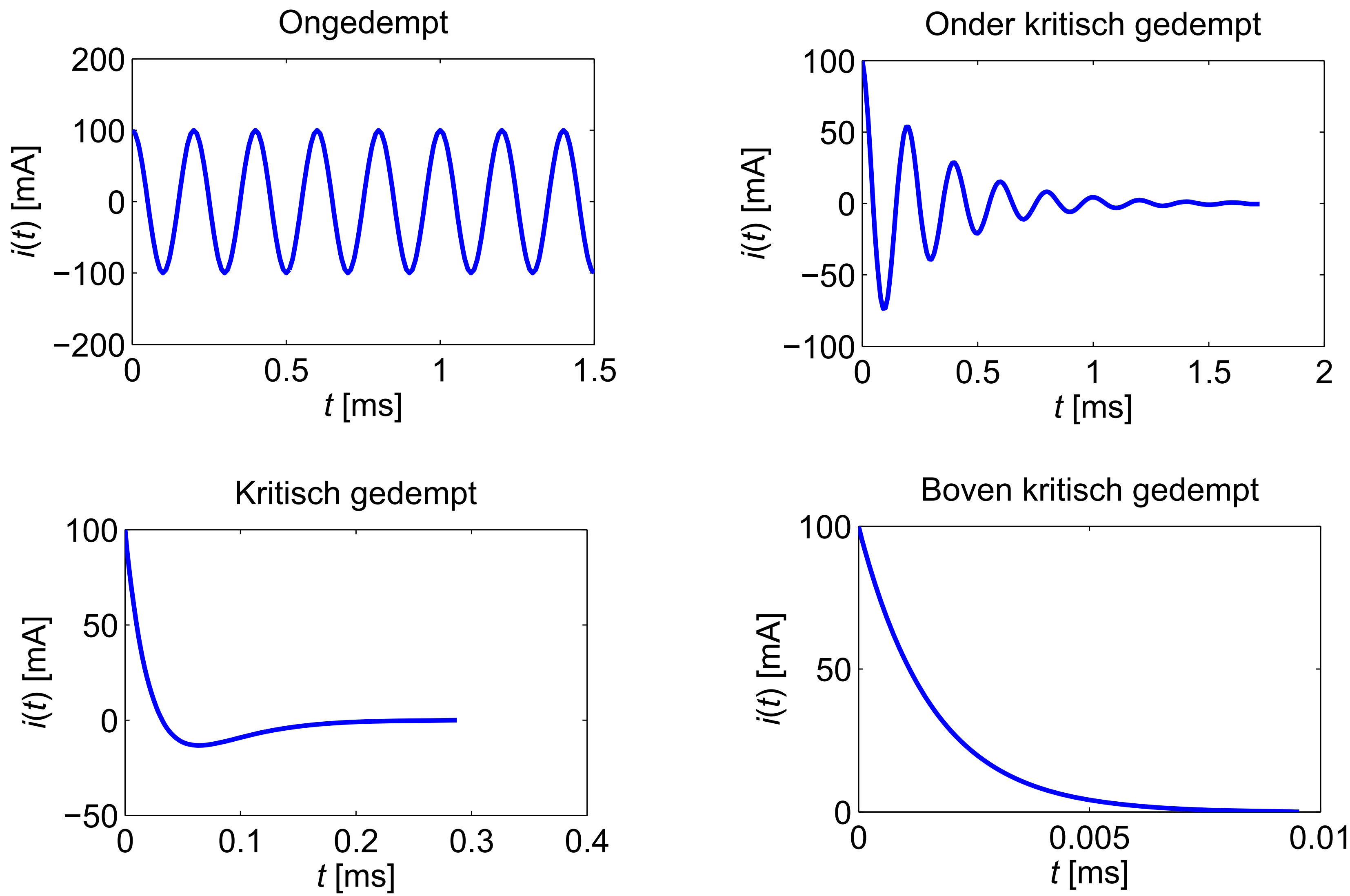


Lp
R
$I(p)$
$\frac{1}{Cp}$
$E(p)$
$\frac{i(0)}{p}$
$\frac{u(0)}{p}$


Lp
R
$I(p)$
$E(p)$
$\frac{1}{Cp}$
$\frac{i(0)}{p}$
$\frac{u(0)}{p}$

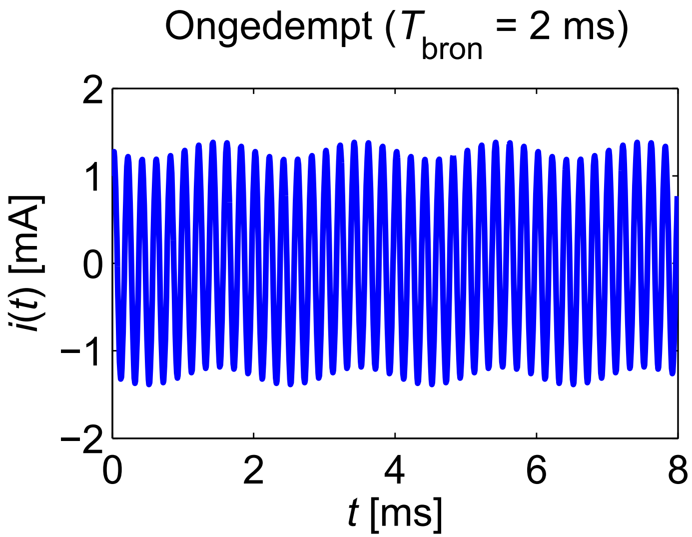

$Li(0)$
Lp
R
$I(p)$
$E(p)$
$\frac{1}{Cp}$
$\frac{u(0)}{p}$

DC bron

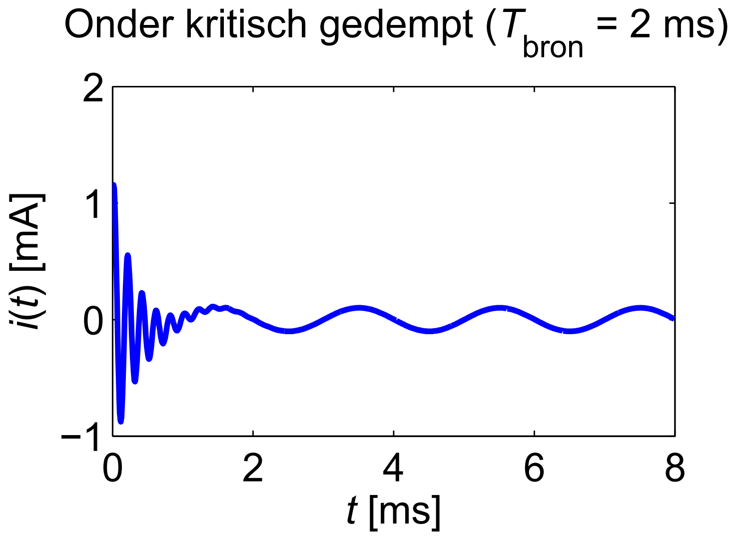


Ongedempt
200
100
0
−100
−200
0
0.5
1
1.5
i(t) [mA]
t [ms]
Onder kritisch gedempt
100
50
0
−50
−100
0
0.5
1
1.5
2
i(t) [mA]
t [ms]
Kritisch gedempt
100
50
0
−50
0
0.1
0.2
0.3
0.4
i(t) [mA]
t [ms]
Boven kritisch gedempt
100
50
0
0
0.005
0.01
i(t) [mA]
t [ms]

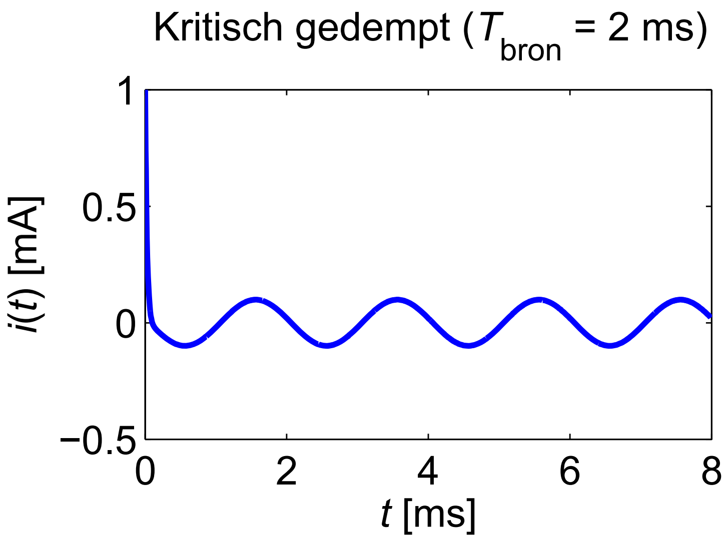
AC bron



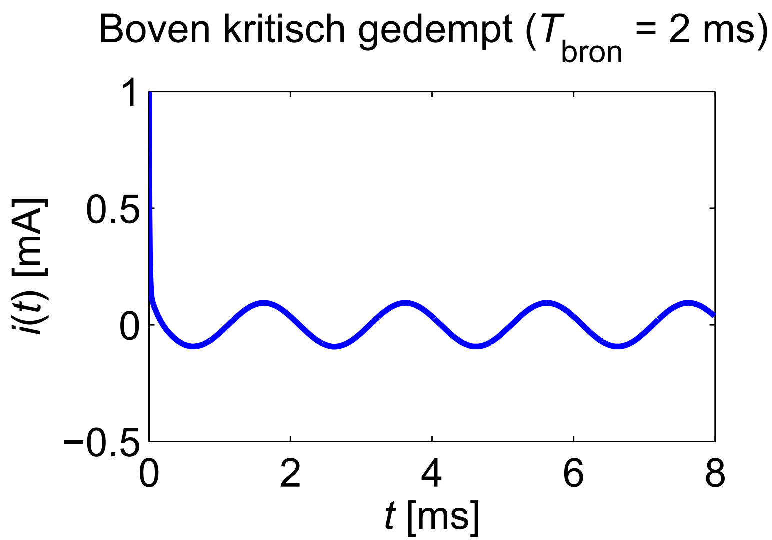
Ongedempt (T_{bron} = 2 ms)
2
1
0
−1
−2
0
2
4
6
8
i(t) [mA]
t [ms]



Onder kritisch gedempt (T_{bron} = 2 ms)
2
1
0
−1
0
2
4
6
8
i(t) [mA]
t [ms]

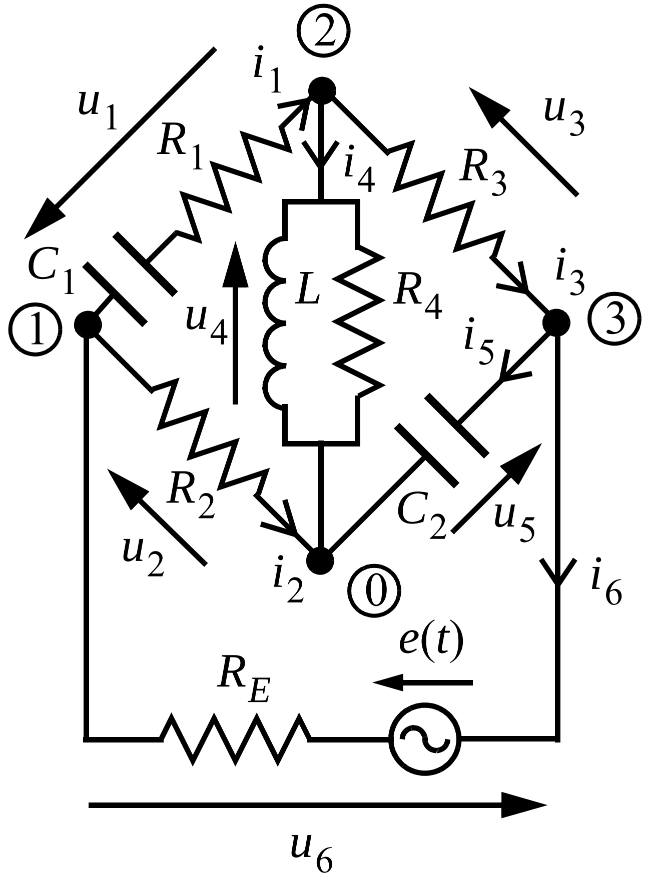


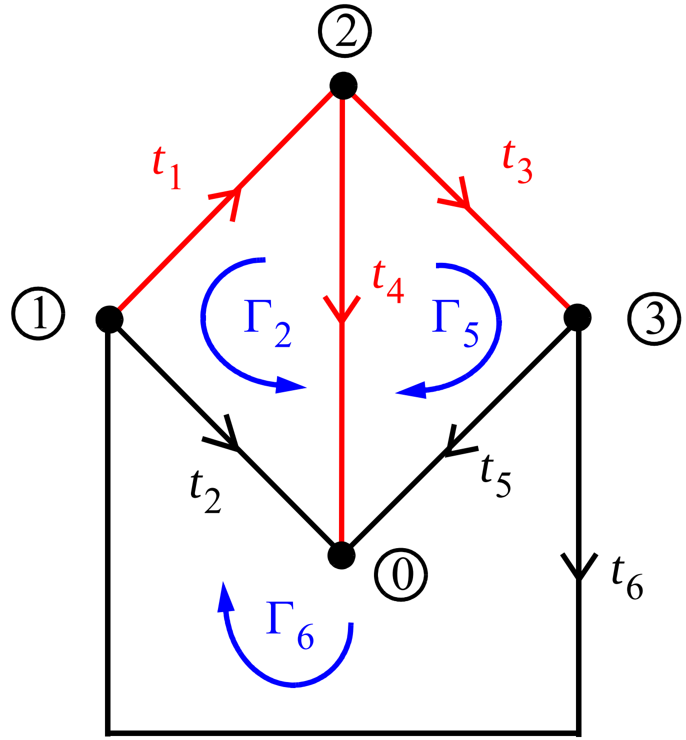
Kritisch gedempt (T_{bron} = 2 ms)
1
0.5
0
−0.5
0
2
4
6
8
i(t) [mA]
t [ms]



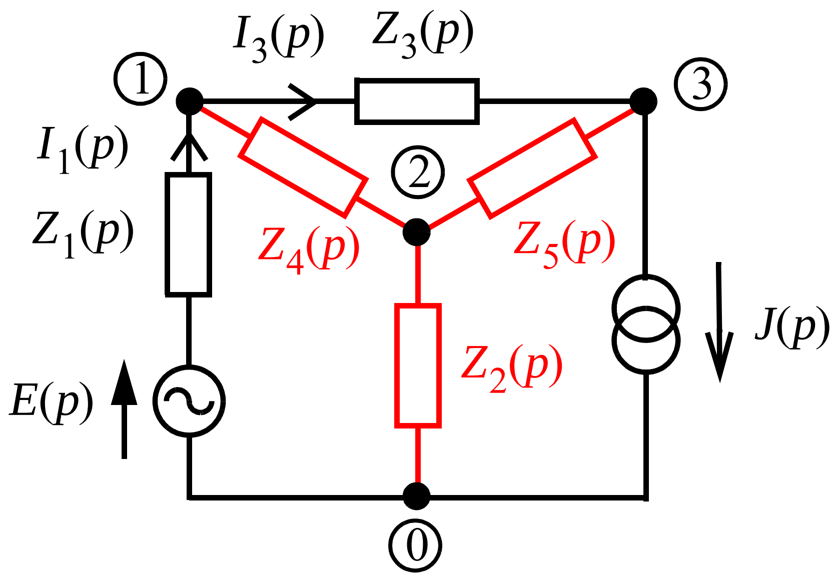
Boven kritisch gedempt (T_{bron} = 2 ms)
1
0.5
0
−0.5
0
2
4
6
8
i(t) [mA]
t [ms]

Methode van de maasstromen

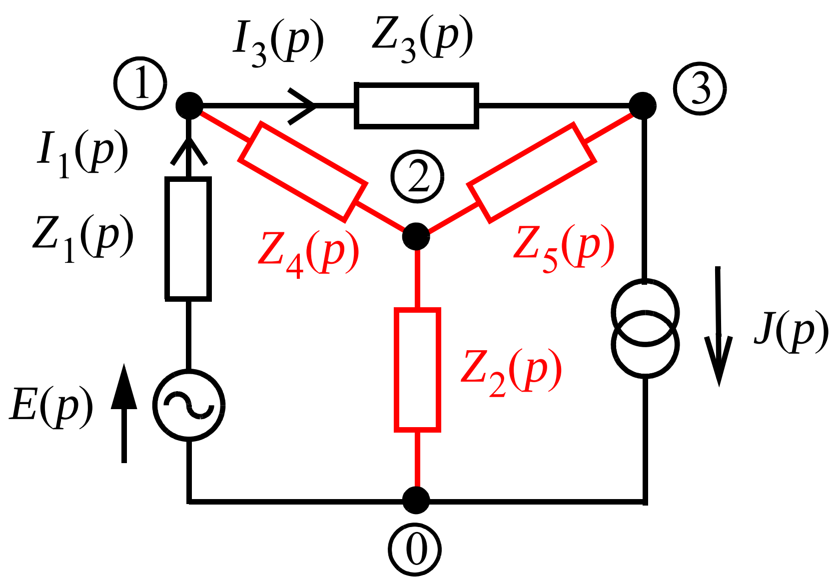




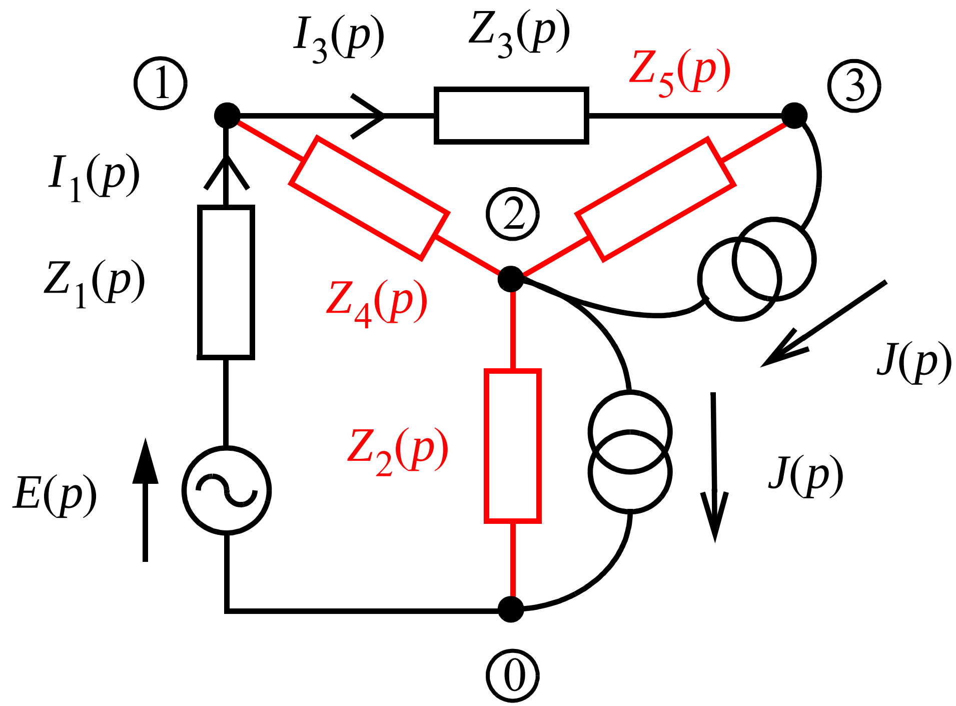
I-shift



I-shift

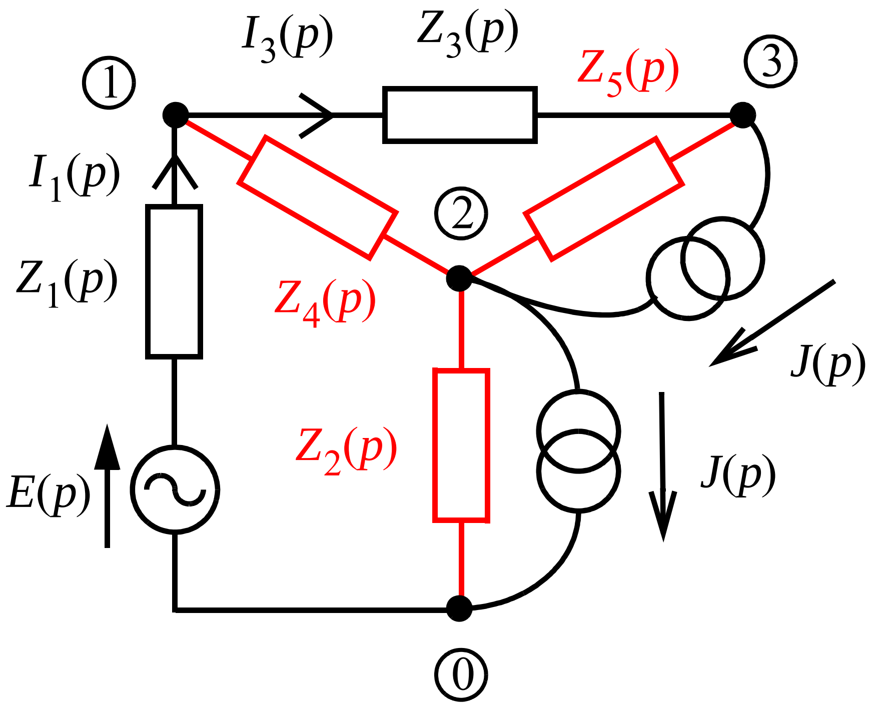


①
②
③
⓪
$I_3(p)$
$Z_3(p)$
$I_1(p)$
$Z_1(p)$
$Z_4(p)$
$Z_5(p)$
$Z_2(p)$
$J(p)$
$E(p)$

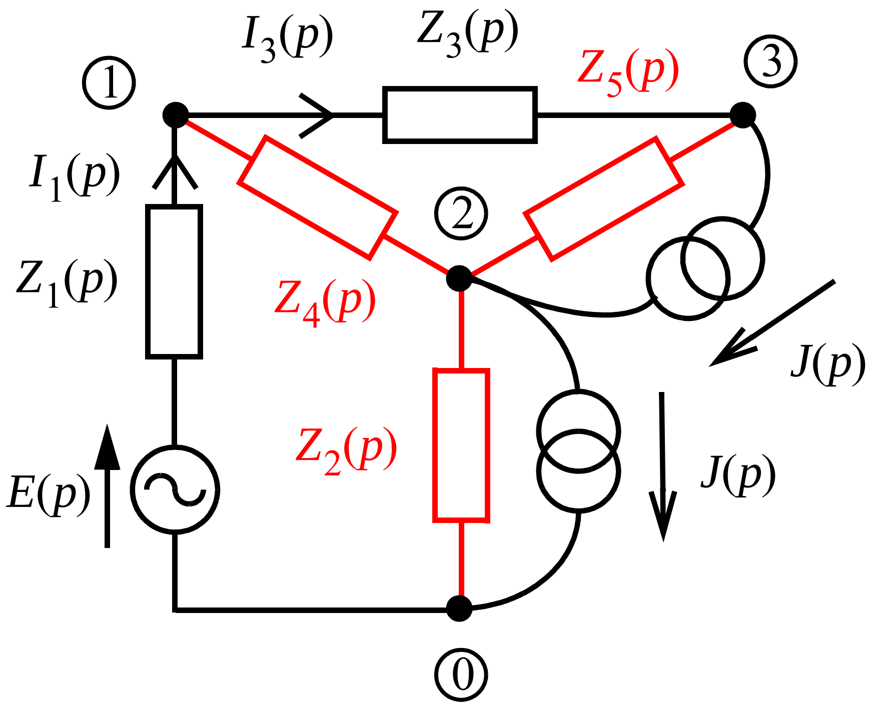


①
②
③
⓪
$I_3(p)$
$Z_3(p)$
$Z_5(p)$
$I_1(p)$
$Z_1(p)$
$Z_4(p)$
$Z_2(p)$
$J(p)$
$J(p)$
$E(p)$

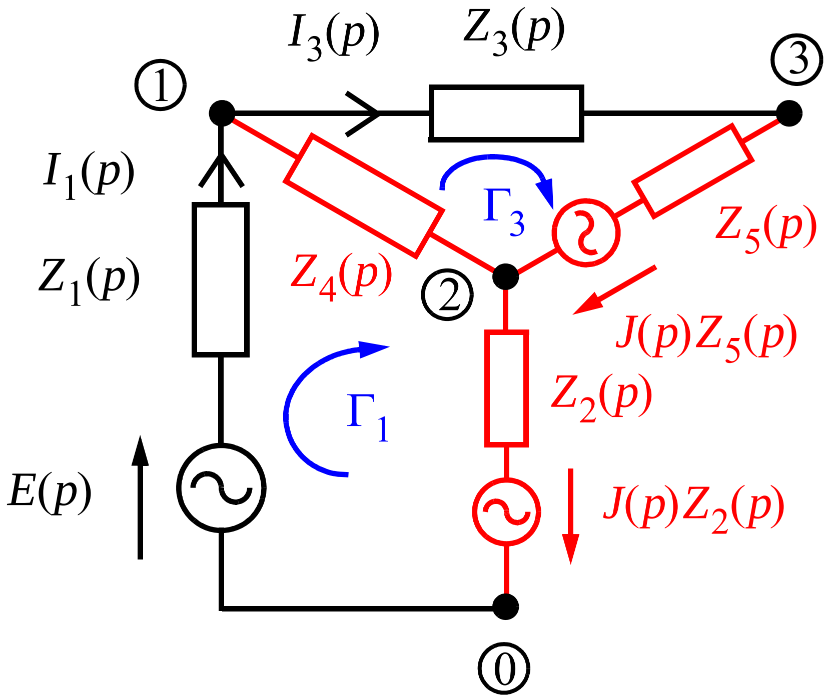
I-shift



I-shift



①
②
③
⓪
$I_3(p)$
$Z_3(p)$
$Z_5(p)$
$I_1(p)$
$Z_1(p)$
$Z_4(p)$
$Z_2(p)$
$E(p)$
$J(p)$
$J(p)$



①
②
③
⓪
$I_3(p)$
$Z_3(p)$
$I_1(p)$
$Z_1(p)$
$Z_4(p)$
$Z_5(p)$
Γ_3
Γ_1
$J(p)Z_5(p)$
$Z_2(p)$
$J(p)Z_2(p)$
$E(p)$