

# System Identification - A Frequency Domain Approach, 2nd edition

## Errata list

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
xxxvi	23	$\times$ -norm of an $n \times m$ matrix $A$	$\infty$ -norm of an $n \times m$ matrix $A$
xxxvi	30	$\mathbb{C}^m$	$\mathbb{C}^m$
xxxvi	32	$\mathbb{C}^m$	$\mathbb{C}^m$
54	(2-50)	$\hat{\sigma}_G^2(k) \approx$	$(M-1)\hat{\sigma}_G^2(k) \approx$
56	(2-57)	$G_0(j\omega_k)(N_1(k) - N_2(k))$	$G_0(j\omega_k)(1 + N_1(k) - N_2(k))$
59	line -14	DTF	DFT
78	3 <sup>rd</sup> line after (3-8)	... of the nonlinear distortion ...	... of the nonlinear system ...
110	1 <sup>st</sup> line of (3-77)	$g_\alpha(\tau_0, \dots, \tau_{\alpha-1})u(t - \tau_0)\dots u(t - \tau_{\alpha-1})$	$g_\alpha(\tau_1, \dots, \tau_\alpha)u(t - \tau_1)\dots u(t - \tau_\alpha)$
146	-6	$(M = TF_0/(2P))$	$(M = Tf_0/(2P))$
163	5	... w.r.t. for increasing ...	... w.r.t. $\phi$ for increasing ...
185	2 <sup>nd</sup> line after (6-33)	... $\max(n_a, n_b) - 1$ ...	... $\max(n_a, n_b) - 1$ for $\Omega = z^{-1}$ and $n_i = \max(n_a, n_b)$ for $\Omega = s, \sqrt{s}$ ...
188	1 <sup>st</sup> line before Section 6.3.3	... $n_i \geq \max(n_a, n_b) - 1$ .	... $n_i > \max(n_a, n_b) - 1$ . (proof: see Note a)
189	2 <sup>nd</sup> line before Section 6.3.4	... $\geq \max(n_a, n_b) - 1$ .	... $> \max(n_a, n_b) - 1$ . (proof: see Note a)
190	line 3	... $\geq \max(n_a, n_b) - 1$ .	... $> \max(n_a, n_b) - 1$ . (proof: see Note a)
199	1 <sup>st</sup> line after (6-66)	... $n_i = (\max(n_c, n_d) - 1)$ ...	... $n_i = \max(n_c, n_d) - 1$ for $\Omega = z^{-1}$ and $n_i = \max(n_c, n_d)$ for $\Omega = s, \sqrt{s}$ ...
199	line -3	$n_j = \max(n_c, n_d) - 1$	$n_j$

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
208	(6-99)	$X(k) = \frac{1}{T_s \sqrt{N}} \sum_{n=-\infty}^{+\infty} X_N(s_k - nj\omega_s)$	$X(k) = \frac{1}{T_s \sqrt{N}} \sum_{n=-\infty}^{+\infty} X_N(s_k - nj\omega_s)$ $- \frac{x(NT_s) - x(0)}{2\sqrt{N}}$ (proof: see Note a)
208	1 <sup>st</sup> line after (6-100)	$I(s) = \frac{I_1(s) - I_2(s)}{\sqrt{NT_s}}$ is a polynomial of order $\max(n_a, n_b) - 1$	add the follow terms to $I(s)$ $B(s) \frac{u(NT_s) - u(0)}{2\sqrt{N}} - \dots$ $A(s) \frac{y(NT_s) - y(0)}{2\sqrt{N}}$ and replace “ $\max(n_a, n_b) - 1$ ” by “ $\max(n_a, n_b)$ ” (proof: see Note a)
240	(7-48)	$G_{rz}(\Omega_k) = \begin{bmatrix} G_{ry}(\Omega_k) \\ G_{ry}(\Omega_k) \end{bmatrix}$	$G_{rz}(\Omega_k) = \begin{bmatrix} G_{ry}(\Omega_k) \\ G_{ru}(\Omega_k) \end{bmatrix}$
250	(7-69)	$T_{H_U}(kP)$	$T_{H_U}(\Omega_{kP})$
253	-13	Examples of such signals are one column of the Hadamard or orthogonal multisines of Section 2.7.2 on page 65, where $r_{\text{siso}}(t)$ is a random phase multisine, or one column of the (full) random orthogonal multisines of Section 3.7 on page 92.	Examples of such signals are $n_u$ different realizations of a random phase multisine, or one column of the (full) random orthogonal multisines of Section 3.7 on page 92.
270	(7-113)	$E\{V_{n[j,:]}^H V_{n[i,:]} \}$	$\mathbb{E}\{V_{n[j,:]}^H V_{n[i,:]} \}$
273	(7-134)	$\Delta \hat{G}_{ru}$	$\Delta \hat{G}_{rz}$
281	5	... constraint optimization ...	... constrained optimization ...
425	line after (10-119)	the sum of $F(z^{-1})$ over can ...	the sum of $F(z_k^{-1})$ over $k$ can ...
448	-12	... for models ...	... for $s$ -domain models ...
449	caption Figure 11-8	... measure-ment ...	... measurement ...
529	-19	... can be sued for ...	... can be used for ...
683	Exercise 19.7	$\text{Re}(jX) = \text{Im}(X)$	$\text{Re}(jX) = -\text{Im}(X)$

## Note a: Relationship between the DFT and the Continuous Fourier Spectrum

Consider a continuous-time signal  $x(t)$  sampled at the rate  $f_s = 1/T_s$  and observed during  $T = NT_s$  (see Fig. 1). Using the notation on p. 39, the relationship between the DFT  $X_{\text{DFT}}(k)$  of the observed samples  $x(nT_s)$ ,  $n = 0, 1, \dots, N-1$ ,

$$X_{\text{DFT}}(k) = \sum_{n=0}^{N-1} x(nT_s) e^{-j2\pi nk/N} \quad (\text{E-1})$$

and the continuous Fourier spectrum  $X_T(j\omega) = F\{x(t)w_T(t)\}$ , with  $w_T(t) = 1$  for  $t \in [0, T]$  and zero elsewhere, is given by

$$X_{\text{DFT}}(k) + (x(NT_s) - x(0))/2 = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X_T(j(\omega_k - n\omega_s)) \quad (\text{E-2})$$

where  $\omega_k = 2\pi kf_s/N$ .

*Proof:* The relationship between the Fourier spectrum  $X(e^{j\omega T_s})$  of the discrete-time samples  $x(nT_s)$ ,  $n \in \mathbb{Z}$ ,

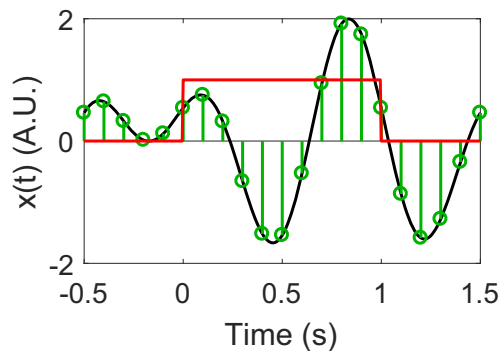
$$X(e^{j\omega T_s}) = F\left\{ \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT_s) \right\} = \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-jn\omega T_s} \quad (\text{E-3})$$

and the Fourier spectrum  $X(j\omega) = F\{x(t)\}$  of the continuous-time signal  $x(t)$  is given by

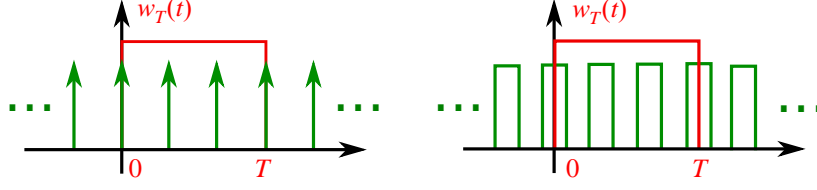
$$X(e^{j\omega T_s}) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X(j(\omega - n\omega_s)) \quad (\text{E-4})$$

(see pp. 82-83 of A.V. Oppenheim and R.W. Schaffer (1989). *Discrete-Time Signal Processing*. Englewood Cliffs (NJ)).

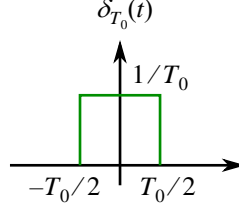
While (E-4) remains valid for the windowed signal  $x(t)w_T(t)$



**Figure 1.** Continuous-time signal  $x(t)$  (black line) sampled at the rate  $f_s$  (green circles) and observed during  $T = 1$  s (red rectangular window).



**Figure 2.** Dirac train  $\sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$  (left) and train of block functions  $\sum_{n=-\infty}^{+\infty} \delta_{T_0}(t - nT_s)$  (right), where  $\delta_{T_0}(t)$  is defined in Fig. 3. At the edges of the observation window  $w_T(t)$  the block function is halved.



**Figure 3.** Block function  $\delta_{T_0}(t)$ .

$$X_T(e^{j\omega T_s}) = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} X_T(j(\omega - n\omega_s)) \quad (\text{E-5})$$

this is no longer true for the right hand side of (E-3) because the Dirac functions at the edges of the window  $w_T(t)$  are multiplied by a discontinuity (see Fig. 2, left plot). To quantify the result of the multiplication of a Dirac function with a discontinuity in the Fourier spectrum

$$X_T(e^{j\omega T_s}) = F \left\{ \sum_{n=-\infty}^{+\infty} x(t) w_T(t) \delta(t - nT_s) \right\} \quad (\text{E-6})$$

we replace the Dirac function  $\delta(t)$  by the block function  $\delta_{T_0}(t)$  defined in Fig. 3 and take the limit for  $T_0 \rightarrow 0$

$$X_T(e^{j\omega T_s}) = \lim_{T_0 \rightarrow 0} F \left\{ \sum_{n=-\infty}^{+\infty} x(t) w_T(t) \delta_{T_0}(t - nT_s) \right\} \quad (\text{E-7})$$

From the right plot of Fig. 2 it can be seen that half of the area of the block functions  $\delta_{T_0}(t)$  and  $\delta_{T_0}(t - T)$  is cut away by the window  $w_T(t)$ . Therefore,

$$\int_{-\infty}^{+\infty} \delta_{T_0}(t) w_T(t) dt = \int_{-\infty}^{+\infty} \delta_{T_0}(t - T) w_T(t) dt = \frac{1}{2} \quad (\text{E-8})$$

and, hence, the samples  $x(0)$  and  $x(NT_s)$  only contribute with a factor 1/2 to the spectrum (E-7)

$$X_T(e^{j\omega T_s}) = \lim_{T_0 \rightarrow 0} F \left\{ \sum_{n=-\infty}^{+\infty} x(t) w_T(t) \delta_{T_0}(t - nT_s) \right\} = \frac{x(0)}{2} + \sum_{n=1}^{N-1} x(nT_s) e^{-nj\omega T_s} + \frac{x(NT_s)}{2} e^{-Nj\omega T_s} \quad (\text{E-9})$$

Note that this result is consistent with the property that the inverse Fourier transform of  $X_T(j\omega)$  evaluated at  $t = 0$  and  $t = NT_s = T$  gives, respectively,  $x(0)/2$  and  $x(T)/2$ . Combining (E-1)

with (E-5) and (E-9) evaluated at  $\omega = \omega_k = 2\pi k f_s / N$  proves (E-2).

*Discussion:* The consequence of (E-2) is that Eq. (6-99) on p. 208 should be replaced by

$$X(k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} x(tT_s) z_k^{-t} = \frac{1}{T_s \sqrt{N}} \sum_{n=-\infty}^{+\infty} X_N(s_k - nj\omega_s) - \frac{x(NT_s) - x(0)}{2\sqrt{N}} \quad (\text{E-10})$$

and that the following terms should be added to  $I(s_k)$  in Eq. (6-100)

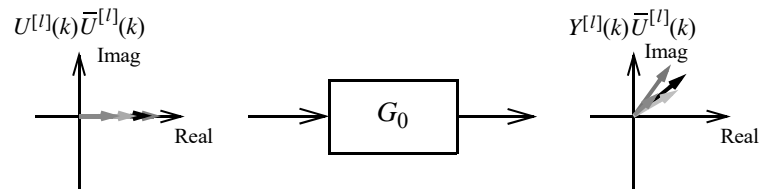
$$B(s_k) \frac{u(NT_s) - u(0)}{2\sqrt{N}} - A(s_k) \frac{y(NT_s) - y(0)}{2\sqrt{N}} \quad (\text{E-11})$$

Hence,  $I(s)$  is a polynomial of order  $\max(n_a, n_b)$  instead of  $\max(n_a, n_b) - 1$ .

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## Errors in figures

page 55, Figure 2-20 should be replaced by



**Figure 2-20.** Successive realizations of  $U^{[l]}(k)\bar{U}^{[l]}(k)$  and  $Y^{[l]}(k)\bar{U}^{[l]}(k)$ .