System Identification - A Frequency Domain Approach

Errata list

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
xxxv	-3, -8	Richardson	Richards
20	(1-32)	$\sum_{k=1}^{N} e^2(k,\theta)$	$\frac{1}{2}\sum_{k=1}^{N} e^2(k,\theta)$
21	(1-33)	$\frac{1}{N}\sum_{k=1}^{N} e(k,\theta)\ldots$	$\sum_{k=1}^{N} e(k,\theta)\ldots$
23	(1-40)	$e^T(\theta)We(\theta)$	$\frac{1}{2}e^T(\theta)We(\theta)$
63	-14	exp($j$rand(255, 1))	exp($2\pi j$rand(255, 1))
73	2, 10	$E\{\ \}$	$\mathscr{E}\{\ \}$
120	11	$b = 2\pi k_1 f_0^2$	$b = 2\pi k_1 f_0$
161 (see also Note a)	(5-78c)	$L_r\frac{1}{\sqrt{\pi t}} + \lambda_r e^{\lambda_r^2 t}\mathrm{erfc}(-\lambda_r\sqrt{t})$	$L_r\left(\frac{1}{\sqrt{\pi t}} + \lambda_r e^{\lambda_r^2 t}\mathrm{erfc}(-\lambda_r\sqrt{t})\right)$
161 (see also Note a)	-3	$\mathrm{Re}(\lambda_r^2) < 0$ or $\lvert\mathrm{Re}(\lambda_r)\rvert < \lvert\mathrm{Im}(\lambda_r)\rvert$	$\lvert\angle\lambda_r\rvert > \pi/4$
163	-3	$X_N(s) = X(z) - \tilde{X}(z)$	$X_N(s) = X(s) - \tilde{X}(s)$
171	(5-131)	$S(j2\pi g)$	$S(j2\pi g)e^{j2\pi(f-g)t}$
287	(8-21)	$\mathscr{E}\{\lvert\hat{\varepsilon}(\Omega_k, \tilde{\theta}_{\mathrm{ML}}(Z_0), \hat{Z}(k))\rvert^2\}$	$(\mathscr{E}\{\lvert\hat{\varepsilon}(\Omega_k, \tilde{\theta}_{\mathrm{ML}}(Z_0), \hat{Z}(k))\rvert^2\})^2$
263	2	$\frac{-\Omega_k^r U(k)}{\sigma_e(\Omega_k,\theta)} + \ldots$	$\frac{-\Omega_k^r U(k)}{\sigma_e(\Omega_k,\theta)} - \ldots$
263	(7-220)	$e_{\mathrm{re}}(\theta^{(i-1)}, Z)$	$\varepsilon_{\mathrm{re}}(\theta^{(i-1)}, Z)$
263	(7-221)	$e_{\mathrm{re}}(\theta^{(i-1)}, Z)$	$\varepsilon_{\mathrm{re}}(\theta^{(i-1)}, Z)$
284	(8-10)	$\hat{\varepsilon}(\Omega_k, \theta, \hat{Z}(k))$	$\lvert\hat{\varepsilon}(\Omega_k, \theta, \hat{Z}(k))\rvert^2$

PAGE	LINE or EQ.	IN BOOK	SHOULD BE
308	(8-66)	$V_{\mathrm{ML}}{'}^{T}(\tilde{\theta}(Z_0))$	$V_{\mathrm{SML}}{'}^{T}(\tilde{\theta}(Z_0))$
310	(8-79)	$\mathscr{E}\{\lvert\hat{\varepsilon}(\Omega_k, \tilde{\theta}_{\mathrm{ML}}(Z_0), \hat{Z}(k))\rvert^2\}$	$(\mathscr{E}\{\lvert\hat{\varepsilon}(\Omega_k, \tilde{\theta}_{\mathrm{ML}}(Z_0), \hat{Z}(k))\rvert^2\})^2$
316 (see also Note b)	(8-106)	$\ldots = Ff_0 = 0$	$\ldots = O(F\lvert\lambda\rvert^F)$ with $\lvert\lambda\rvert < 1$
317 (see also Note b)	(8-113)	$\ldots = Ff_0 = 0$	$\ldots = O(F\lvert\lambda\rvert^F)$ with $\lvert\lambda\rvert < 1$
317	-9	(8-109)	(8-108)
318	-13	very	every
328	(9-10)	... Fi_{22}^{-1} ...	... $(Fi_{22} - Fi_{21}Fi_{11}^{-1}Fi_{12})^{-1}$ ...
354	9 and (10-3)	$g_{\mathrm{d}}(k)$	$g_{\mathrm{ZOH}}(k)$
370	(10-17)	$N\lambda$	$N\ln(\lambda)$
375	(10-22)	$\dfrac{\exp\left(-\sum_{k=0}^{N-1}\dfrac{\lvert Y(k) - G(z_k^{-1}, \theta)U(k)\rvert^2}{\lambda\lvert H(z_k^{-1}, \theta)\rvert^2}\right)}{\pi^N\lambda^N\prod_{k=0}^{N-1}\lvert H(z_k^{-1}, \theta)\rvert^2}$	$\dfrac{\exp\left(-\sum_{k=0}^{N-1}\dfrac{\lvert Y(k) - G(z_k^{-1}, \theta)U(k)\rvert^2}{2\lambda\lvert H(z_k^{-1}, \theta)\rvert^2}\right)}{2\pi^{N/2}\lambda^{N/2}\prod_{k=0}^{N-1}\lvert H(z_k^{-1}, \theta)\rvert}$
375 (see also Note b)	(10-25)	$\ldots = Nf_0 = 0$	$\ldots = O(\lvert\lambda\rvert^N)$ with $\lvert\lambda\rvert < 1$
391	(11-9)	$T_{\mathrm{c}}(z_k^{-1})$	$I_{\mathrm{c}}(z_k^{-1})$
426	-17	$r = 1, 2, \ldots, k-1$	$r = 1, 2, \ldots, s-1$
472 (Corollary 14.30)	12	Lemma 14.31	Theorem 14.29
542	-5	$\tilde{\theta}(z_o)$	$\tilde{\theta}(\mathbf{z_0})$
585	-10	… for frequency domain identification.	… using periodic input signals.
585	17	Levi	Levy
589	-21	Söderström, T., and B. Carlsson (2000)	Söderström, T., and M. Mossberg (2000)

Note a: stability analysis of the impulse response in the $\sqrt{s}$-domain (eq. (5-78c) p. 161)

The asymptotic behavior ($t \to \infty$) of

$$f(t) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t}\operatorname{erfc}(-\lambda\sqrt{t}) \qquad \text{(E-1)}$$

with $\operatorname{erfc}(z)$ the complementary error function, is analyzed as a function of λ. Using $\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$ and $\operatorname{erf}(-z) = -\operatorname{erf}(z)$, with $\operatorname{erf}(z)$ the error function, (E-1) can be rewritten as

$$f(t) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t}(1 + \operatorname{erf}(\lambda\sqrt{t})) = \frac{1}{\sqrt{\pi t}} + \lambda e^{\lambda^2 t}(2 - \operatorname{erfc}(\lambda\sqrt{t})) \qquad \text{(E-2)}$$

Applying the following asymptotic expansions ($z \to \infty$) of $\operatorname{erfc}(z)$

$$\begin{aligned} \operatorname{erfc}(-z) &= -\frac{e^{-z^2}}{z\sqrt{\pi}}\left(1 + \sum_{m=1}^{\infty}(-1)^m\frac{(2m-1)!!}{(2z^2)^m}\right) \qquad |\angle z| > \pi/4 \\ \operatorname{erfc}(z) &= \frac{e^{-z^2}}{z\sqrt{\pi}}\left(1 + \sum_{m=1}^{\infty}(-1)^m\frac{(2m-1)!!}{(2z^2)^m}\right) \qquad |\angle z| < 3\pi/4 \end{aligned} \qquad \text{(E-3)}$$

(see Abramowitz and Stegun, 1970) to (E-1) and (E-2) gives

$$f(t) = \begin{cases} 2\lambda e^{\lambda^2 t} + O(t^{-3/2}) & |\angle\lambda| \le \pi/4 \\ \dfrac{1}{2\lambda^2\sqrt{\pi}t^{3/2}} + O(t^{-5/2}) & |\angle\lambda| > \pi/4 \end{cases} \qquad \text{(E-4)}$$

It shows that $f(t)$ (i) grows exponentially for $|\angle\lambda| < \pi/4$ ($\Rightarrow \operatorname{Re}(\lambda^2) > 0$), (ii) decreases algebraically to zero as $O(t^{-3/2})$ for $|\angle\lambda| > \pi/4$, and (iii) converges to a periodic solution for $|\angle\lambda| = \pi/4$ ($\Rightarrow \operatorname{Re}(\lambda^2) = 0$) as $O(t^{-3/2})$. With the property $\operatorname{erf}(\bar{z}) = \overline{\operatorname{erf}(z)}$ the results for the complex conjugate root $\bar{\lambda}$ can easily be derived from equations (E-1) to (E-4).

References

Abramowitz M. and I.A. Stegun (1970). *Handbook of Mathematical Functions.* Dover Publications, New York (USA).

Note b: sum over the unit circle of functions that are analytic outside the unit circle and zero at infinity (eq. (8-106) p. 316, eq. (8-113) p. 317, and eq. (10-25) p. 375)

Let $F(z^{-1})$ be a rational function of z^{-1} that is analytic outside the unit circle ($|z| \geq 1$) and zero at $z = \infty$ ($F(0) = 0$). The Taylor series of $F(z^{-1})$ w.r.t. z^{-1} at $z^{-1} = 0$ equals

$$F(z^{-1}) = \sum_{r=1}^{\infty} f_r z^{-r} \text{ for any } |z| \geq 1 \tag{E-5}$$

Using

$$\sum_{k=0}^{N-1} z_k^{-r} = \begin{cases} 0 & \text{for } r \neq nN \\ N & \text{for } r = nN \end{cases} \text{ with } n = 0, 1, \ldots \tag{E-6}$$

the sum of $F(z^{-1})$ over a uniform grid on the unit circle ($z_k = \exp(j2\pi k/N)$, $k = 0, 1, \ldots, N-1$) can be written as

$$\sum_{k=0}^{N-1} F(z_k^{-1}) = \sum_{r=1}^{\infty} f_r \sum_{k=0}^{N-1} z_k^{-r} = N \sum_{n=1}^{\infty} f_{nN} \tag{E-7}$$

Since $|f_r| \leq K|\lambda|^r$, with $z = \lambda$ the pole of $F(z^{-1})$ closest to the unit circle ($|\lambda| < 1$) and K a constant, the absolute value of (E-7) can be bounded above as

$$\left|\sum_{k=0}^{N-1} F(z_k^{-1})\right| \leq N \sum_{n=1}^{\infty} K|\lambda|^{nN} \leq O(N|\lambda|^N) \tag{E-8}$$

It shows that contribution of (8-106) to (8-107) and of (8-113) to (8-114) converges ($F \to \infty$) exponentially to zero at the rate of the dominant pole of $\Delta H(z^{-1}, \theta)/H(z^{-1}, \theta)$ and $F(z^{-1}, z^{-1}, \theta)$ respectively. Hence, all consistency claims remain valid.

Let $H(z^{-1})$ be a stable and inversely stable monic rational form in z^{-1}. The natural logarithm of $H(z^{-1})$ equals

$$\ln(H(z^{-1})) = \sum_r \ln(1 - \beta_r z^{-1}) - \sum_l \ln(1 - \alpha_l z^{-1}) \tag{E-9}$$

with α_l, β_r the poles and zeros of $H(z^{-1})$ satisfying $|\alpha_l| < 1$ and $|\beta_r| < 1$. Using eq. (E-6) and the Taylor series expansion

$$\ln(1 - \lambda z^{-1}) = -\sum_{r=1}^{\infty} (\lambda z^{-1})^r / r, \tag{E-10}$$

the sum of $\ln(1-\lambda z^{-1})$ over a uniform grid on the unit circle ($z_k = \exp(j2\pi k/N)$, $k = 0, 1, \ldots, N-1$) can be written as

$$\sum_{k=0}^{N-1} \ln(1-\lambda z_k^{-1}) = -\sum_{r=1}^{\infty} \frac{\lambda^r}{r} \sum_{k=0}^{N-1} z_k^{-r} = -N \sum_{n=1}^{\infty} \frac{\lambda^{nN}}{nN} \qquad \text{(E-11)}$$

The absolute value of (E-11) can be bounded above as

$$\left| \sum_{k=0}^{N-1} \ln(1-\lambda z_k^{-1}) \right| \leq \sum_{n=1}^{\infty} \frac{|\lambda|^{nN}}{n} \leq O(|\lambda|^N) \qquad \text{(E-12)}$$

Collecting (E-9) and (E-12) gives

$$\left| \sum_{k=0}^{N-1} \ln(H(z_k^{-1})) \right| \leq O(|\lambda|^N) \qquad \text{(E-13)}$$

where $z = \lambda$ is the pole or zero of $H(z^{-1})$ closest to the unit circle ($|\lambda| < 1$). It shows that (10-25) converges exponentially to zero at the rate of the dominant pole of $\ln(H(z^{-1}))$. Hence, the asymptotic ($N \to \infty$) equivalence between the time and frequency domain maximum likelihood cost functions remains valid.

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System Identification - A Frequency Domain Approach

Errors in figures

page 62, Figure 2-27 should be replaced by

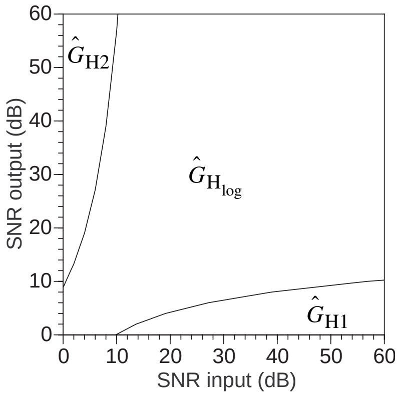

Figure 2-27. Selection between $G_{H_{log}}(j\omega_k)$, $\hat{G}_{H1}(j\omega_k)$, and $\hat{G}_{H2}(j\omega_k)$ as a function of the SNR in case the repeated measurements are not well synchronized.

page 143, Figure 5-5 should be replaced by

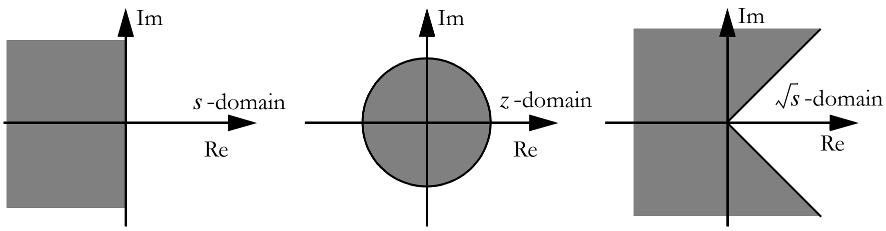

FIGURE 5-5. Grey area: stable and minimum phase regions of, respectively, the poles and zeros. s -domain: $\mathrm{Re}(s) < 0$, z -domain: $|z| < 1$, and $\sqrt{s}$ -domain: $|\angle\sqrt{s}| > \pi/4$.

Errors in figures - continued

page 338, Figure 9-8 should be replaced by

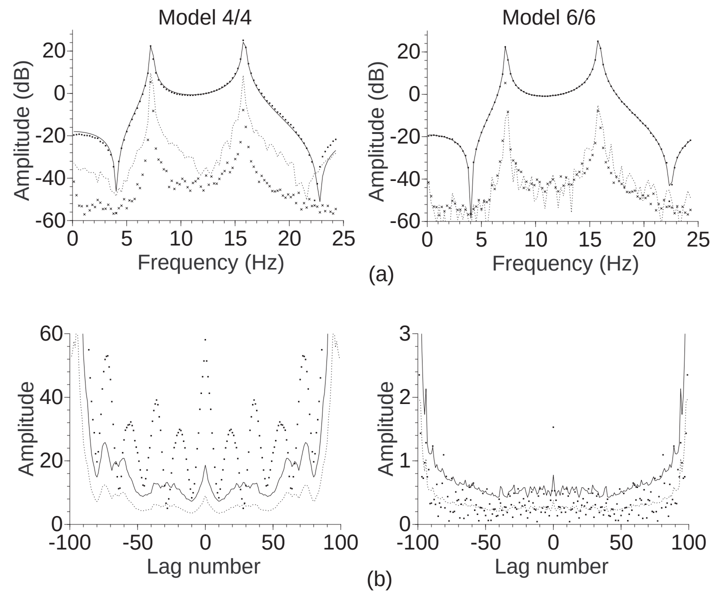

Figure 9-8. Illustration of model error detection and qualification on a vibrating robot arm. (a) the identified transfer function: dots: measurements, — model, ··· model errors, x measurement uncertainty σ_G; (b) $R_{\varepsilon\varepsilon}(m)$, dots: measurement, ··· 50% bound, — 95% bound.

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